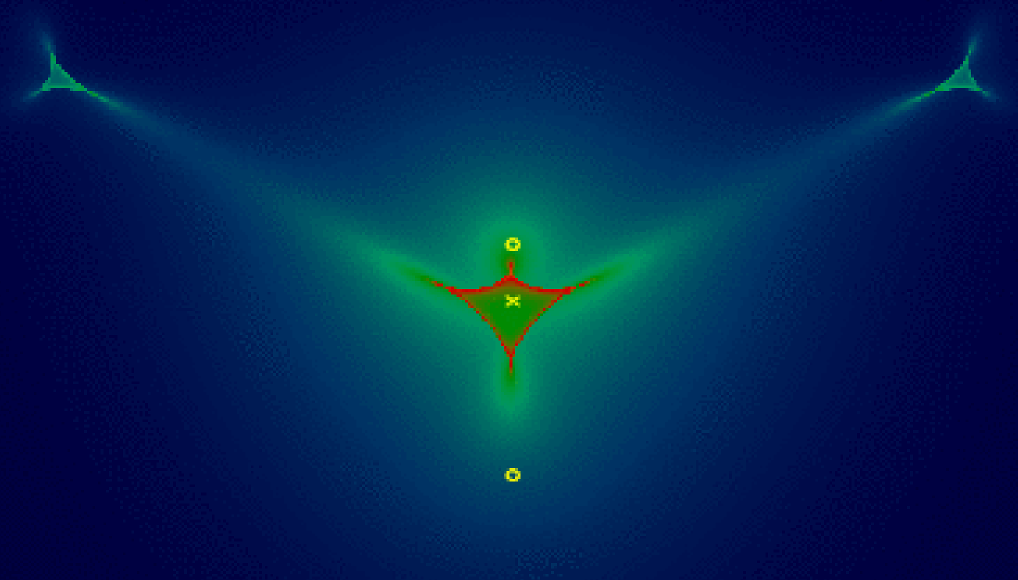
Observational and modeling techniques in microlensing planet searches



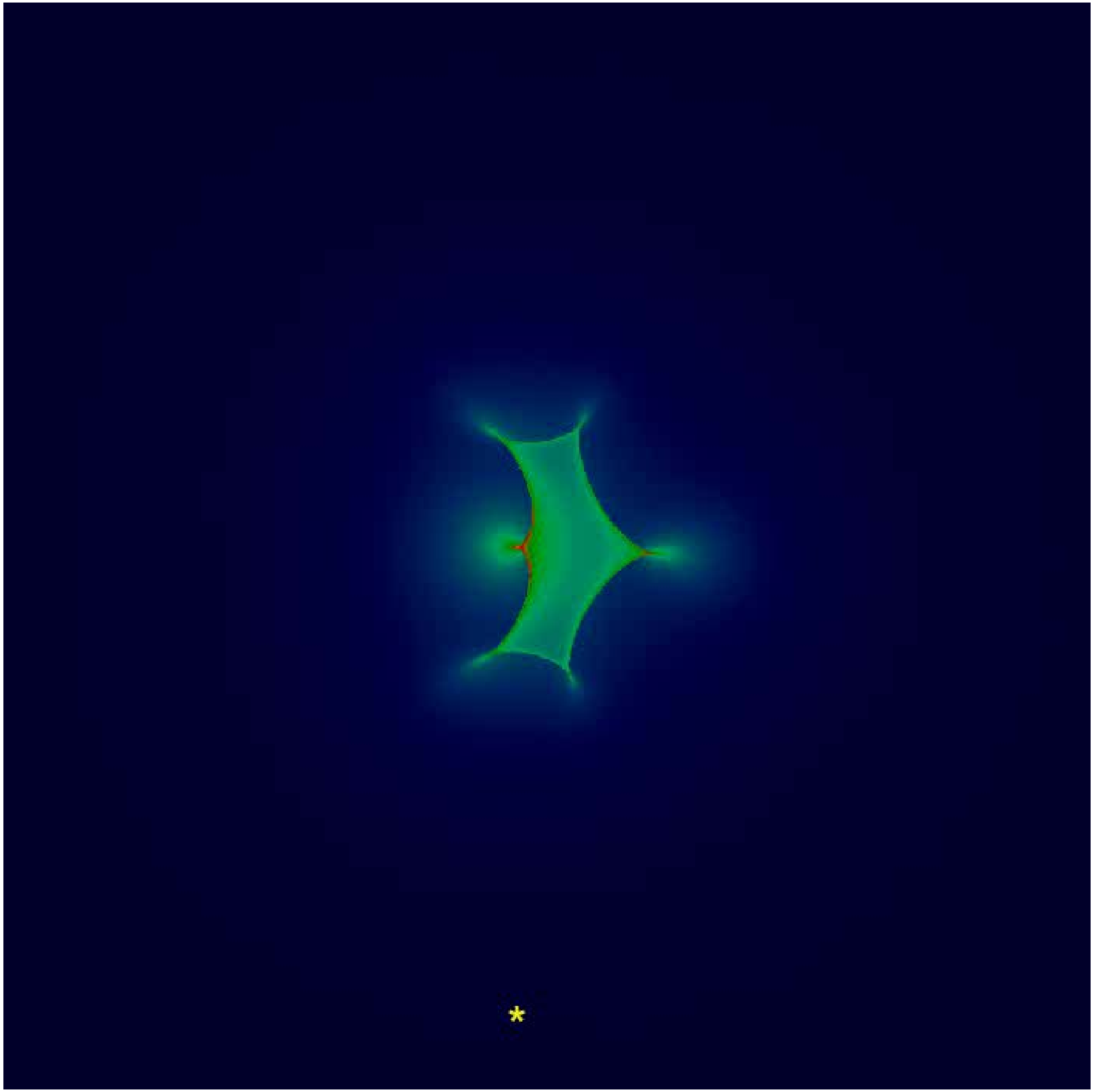
Dijana Dominis Prester

7.8.2007, Belgrade

Simulating synthetic data

Fitting

Optimizing



$q = 0.3$

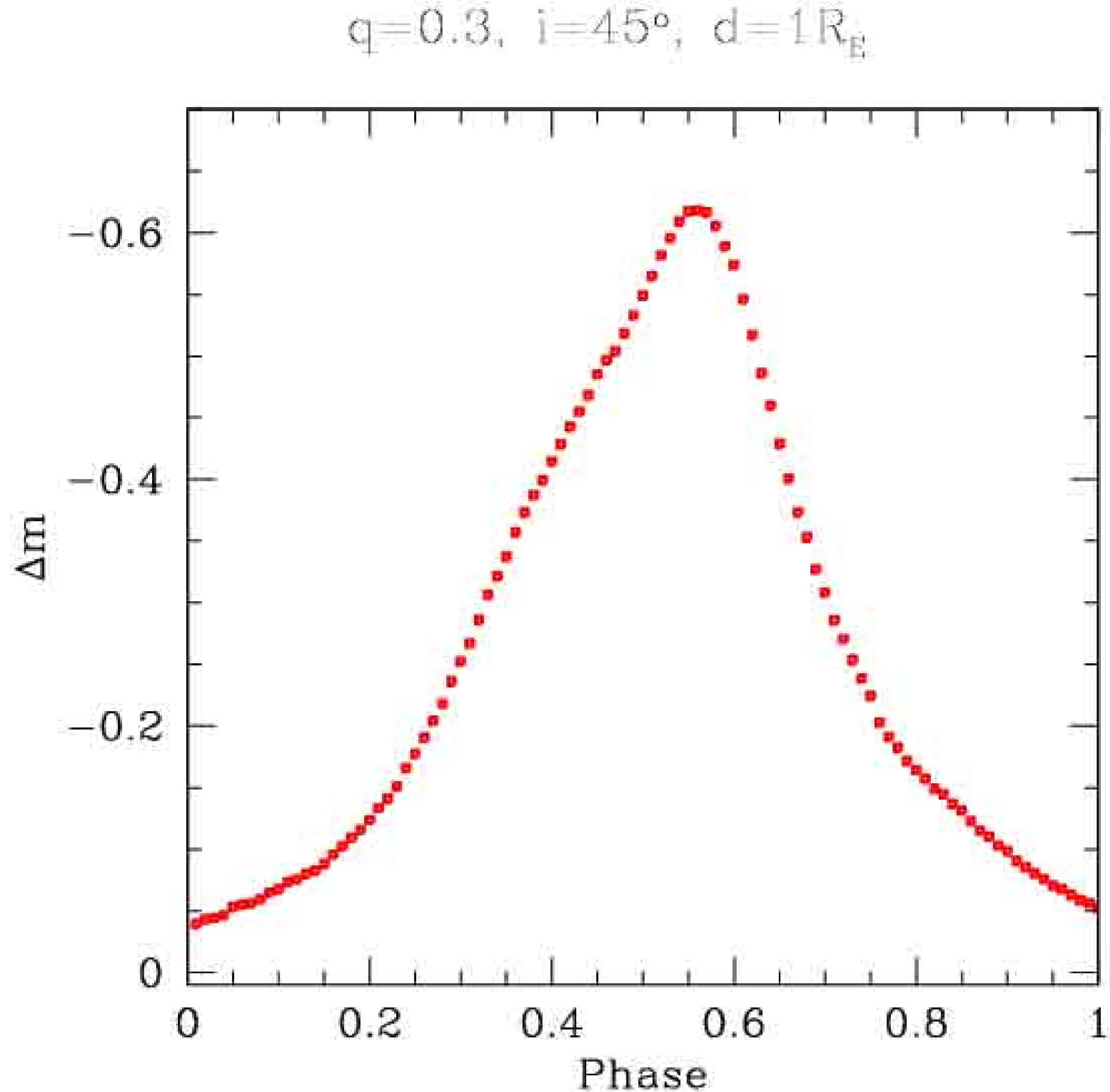
$i = 45^o$

$d = 1R_E$

$R_* = 10R_{Sun}$

$(x_1, y_1) = (460,65)$

$(x_2, y_2) = (285,853)$



q=0.3, i=45°, d=1R_E
−0.6
−0.4
−0.2
0
Δm
0
0.2
0.4
0.6
0.8
1
Phase

Modeling a Synthetic Light Curve

Standard deviation (errorbars):

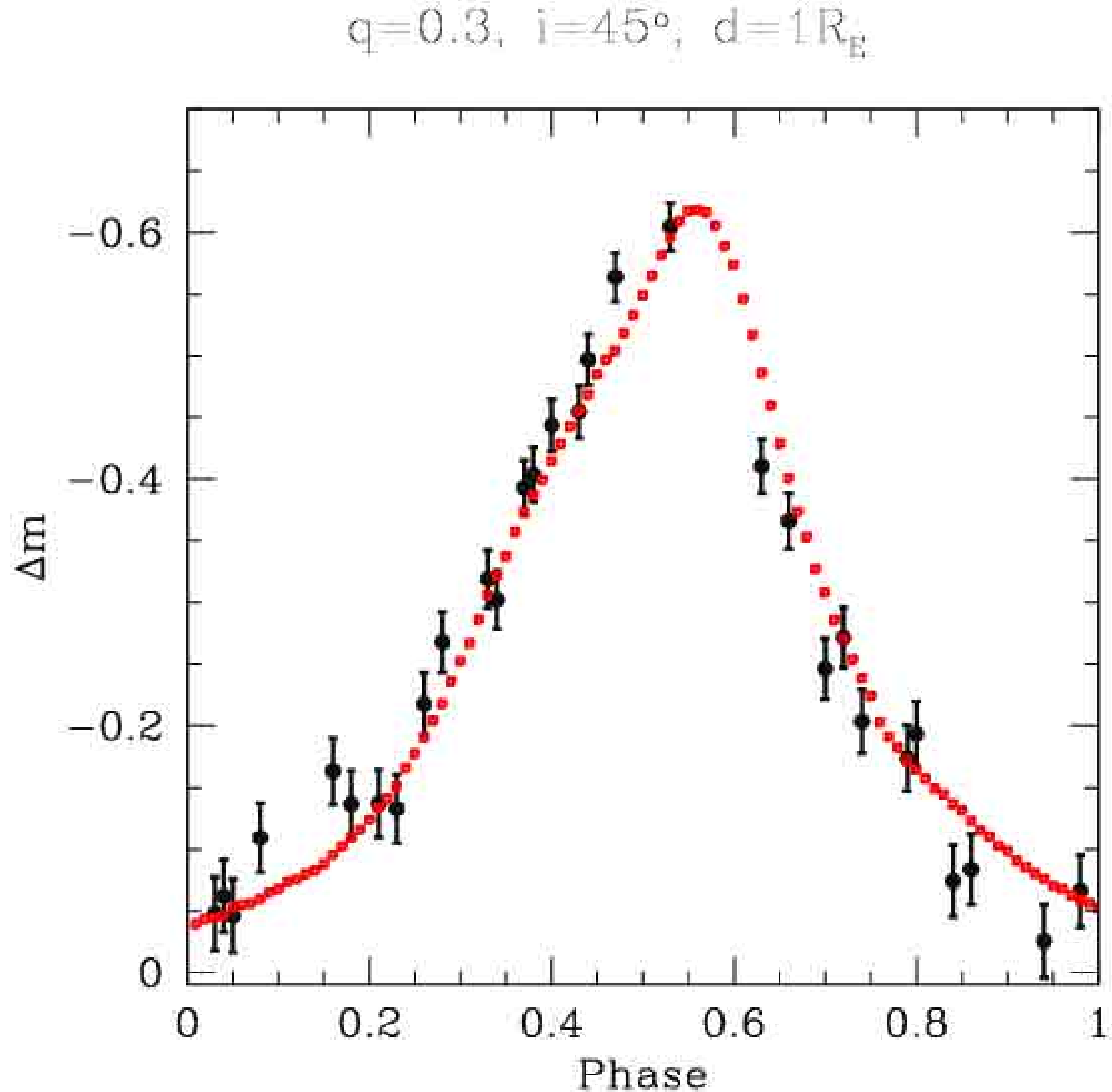
$$\sigma_m = \sigma_0 \frac{1}{1+\Delta m}$$

$$\sigma \in [\sigma_0, \sigma_{\min}]$$

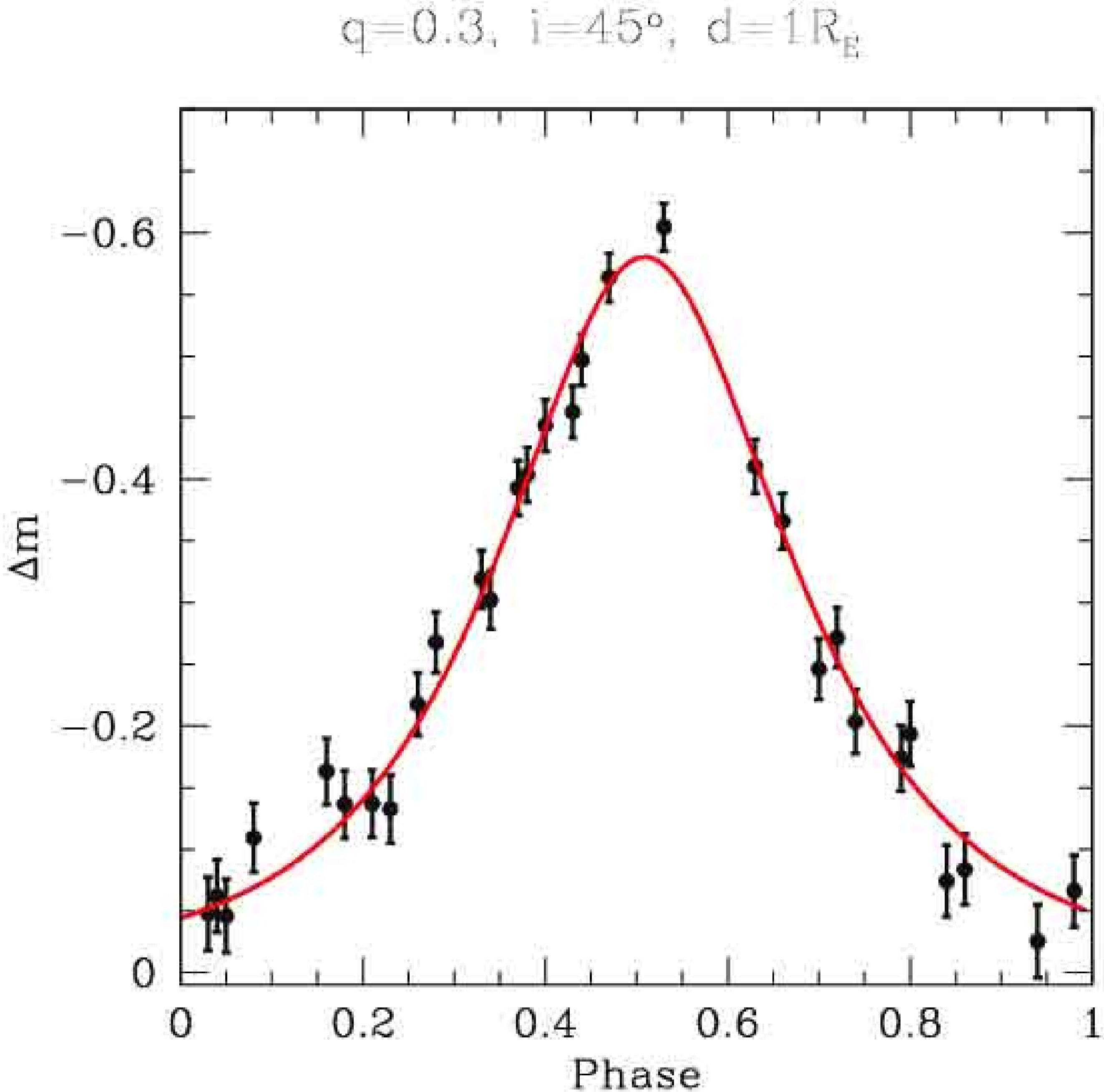
Gaussian scatter → **Noisy data**

Picking random data points (i.e. 30 out of 100) → **Irregular data coverage**

Fitting the light curve by using an inverse χ^2 optimization method)



q=0.3, i=45°, d=1R_E
Δm
−0.6
−0.4
−0.2
0
0
0.2
0.4
0.6
0.8
1
Phase



q=0.3, i=45°, d=1R$_E$
−0.6
−0.4
−0.2
0
Δm
0
0.2
0.4
0.6
0.8
1
Phase

Chi square test

Chi squared per degrees of freedom:

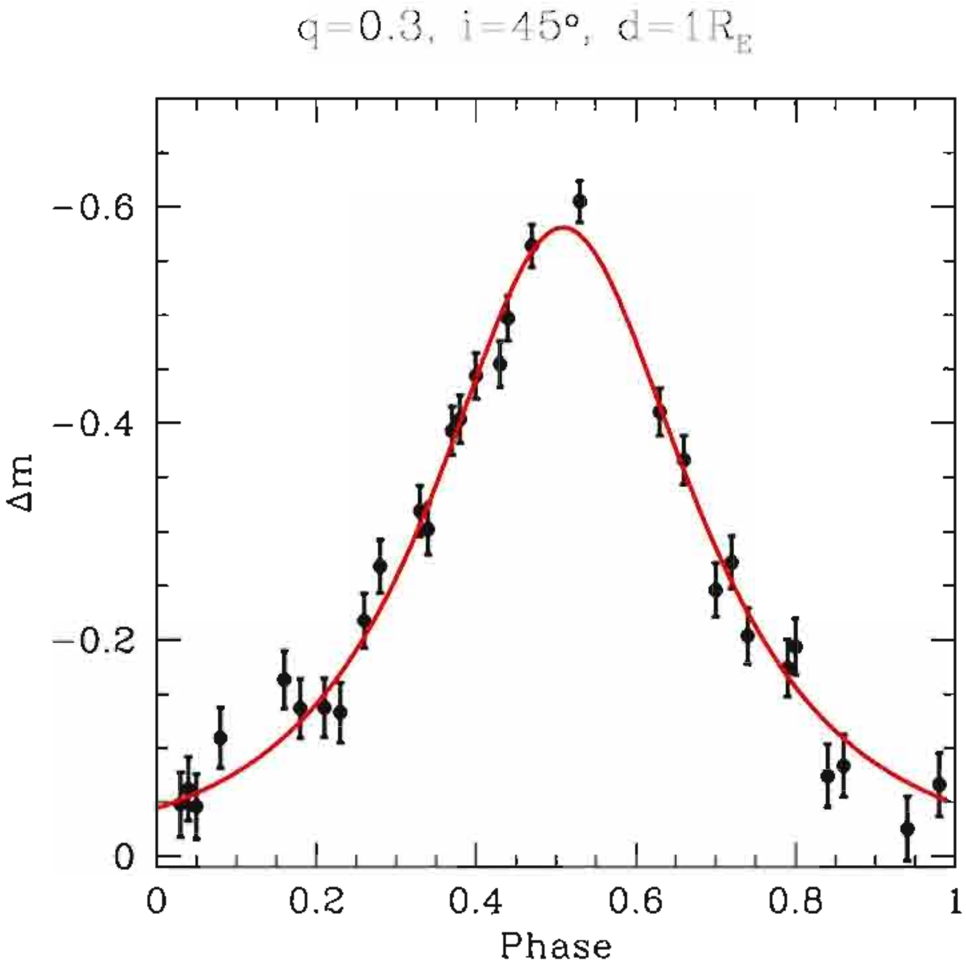
$$\chi^2 / d.o.f. = \frac{\sum (x(t)_{obs} - x(t)_{theor})^2}{n_{data} - n_{parameters} + 1}$$

(close to 1 for a good fit)

30 out of 100 data points

$$\sigma_0 = 0.03 mag$$

$$\sigma_{min} = 0.018 mag$$



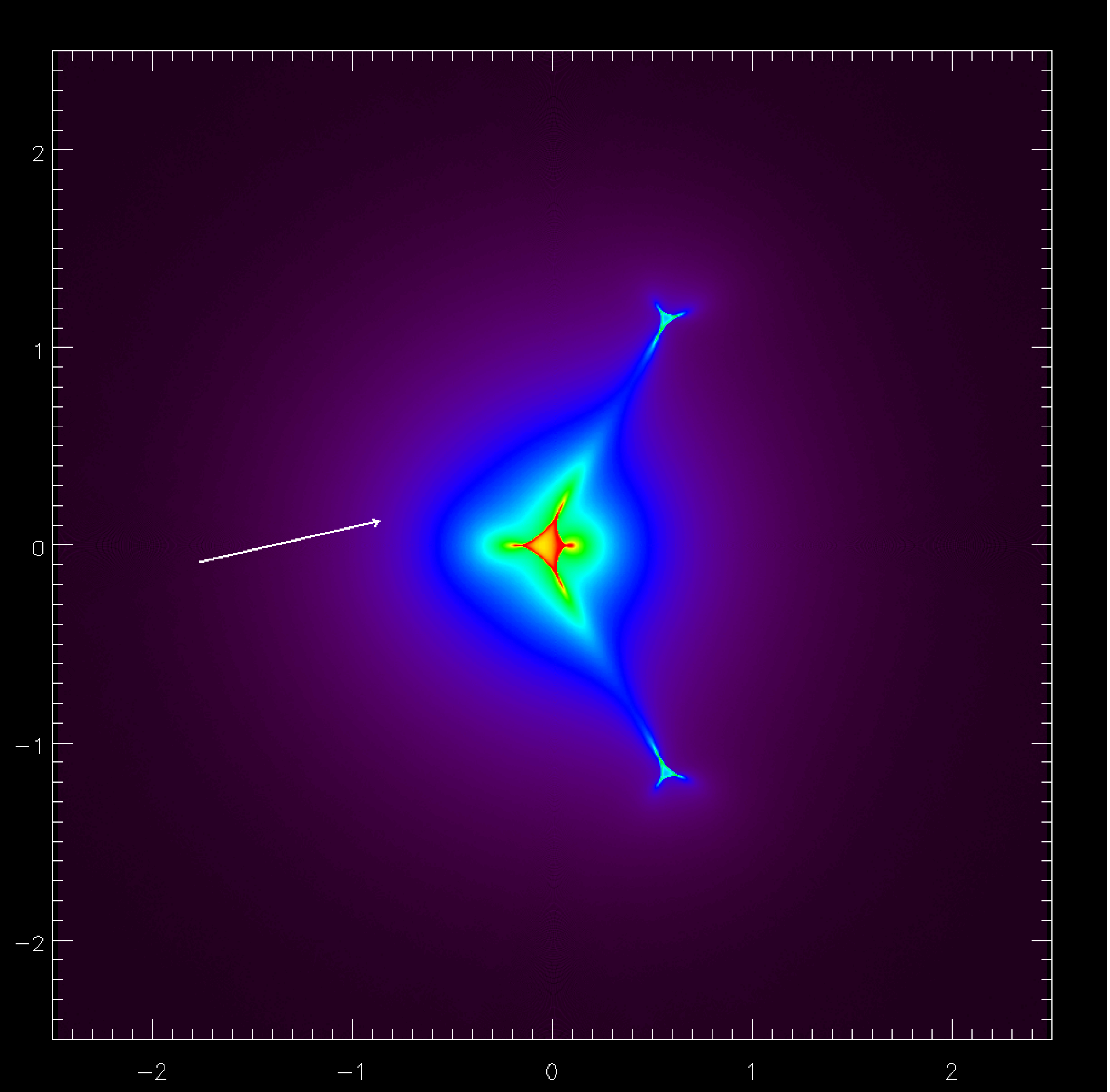
$$A_{max} = 1.71$$

$$u_0 = 0.68$$

$$t_0 = 0.23P$$

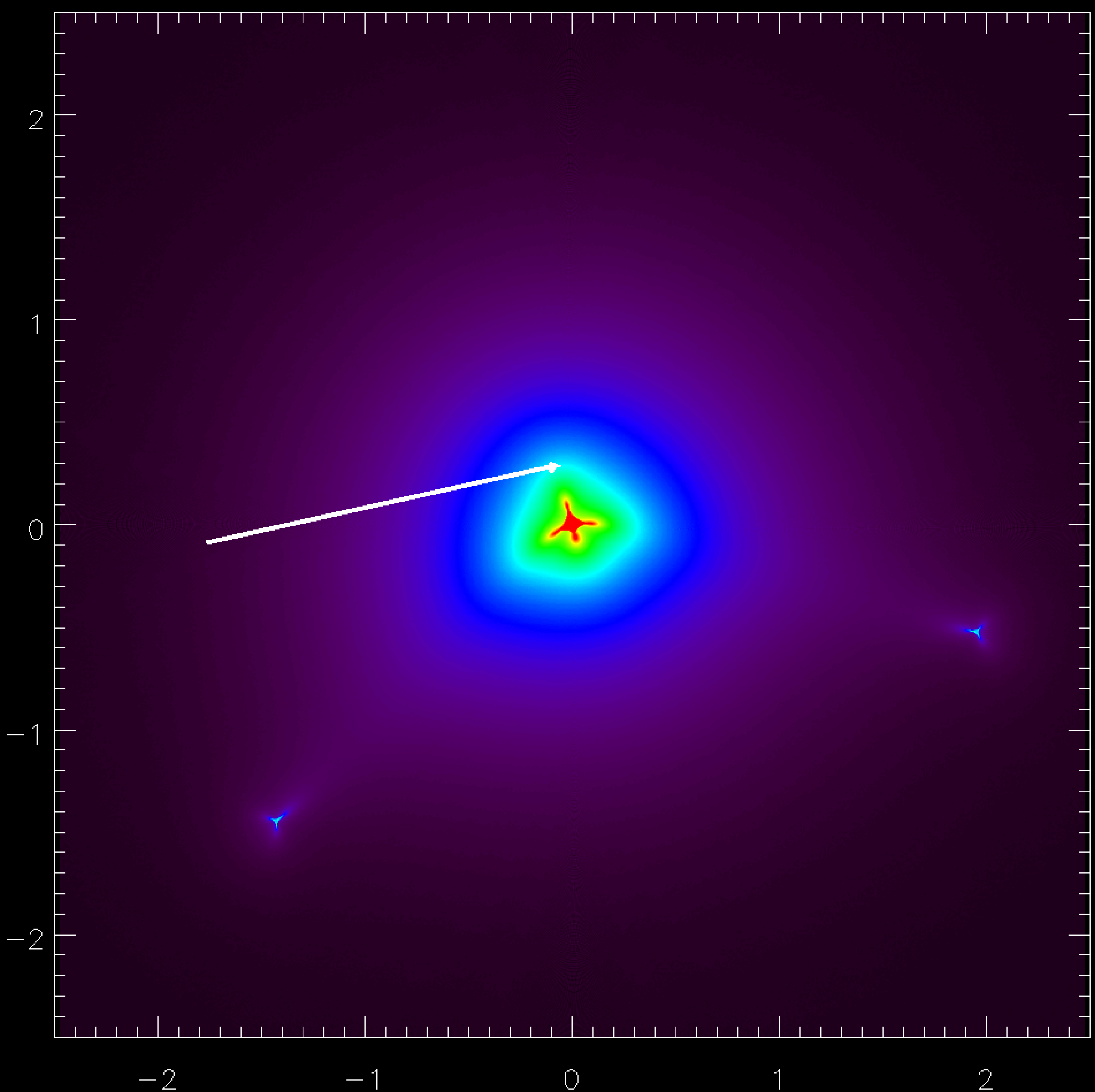
$$t_{max} = 0.5P$$

$$\chi^2 / d.o.f. = 1.16$$

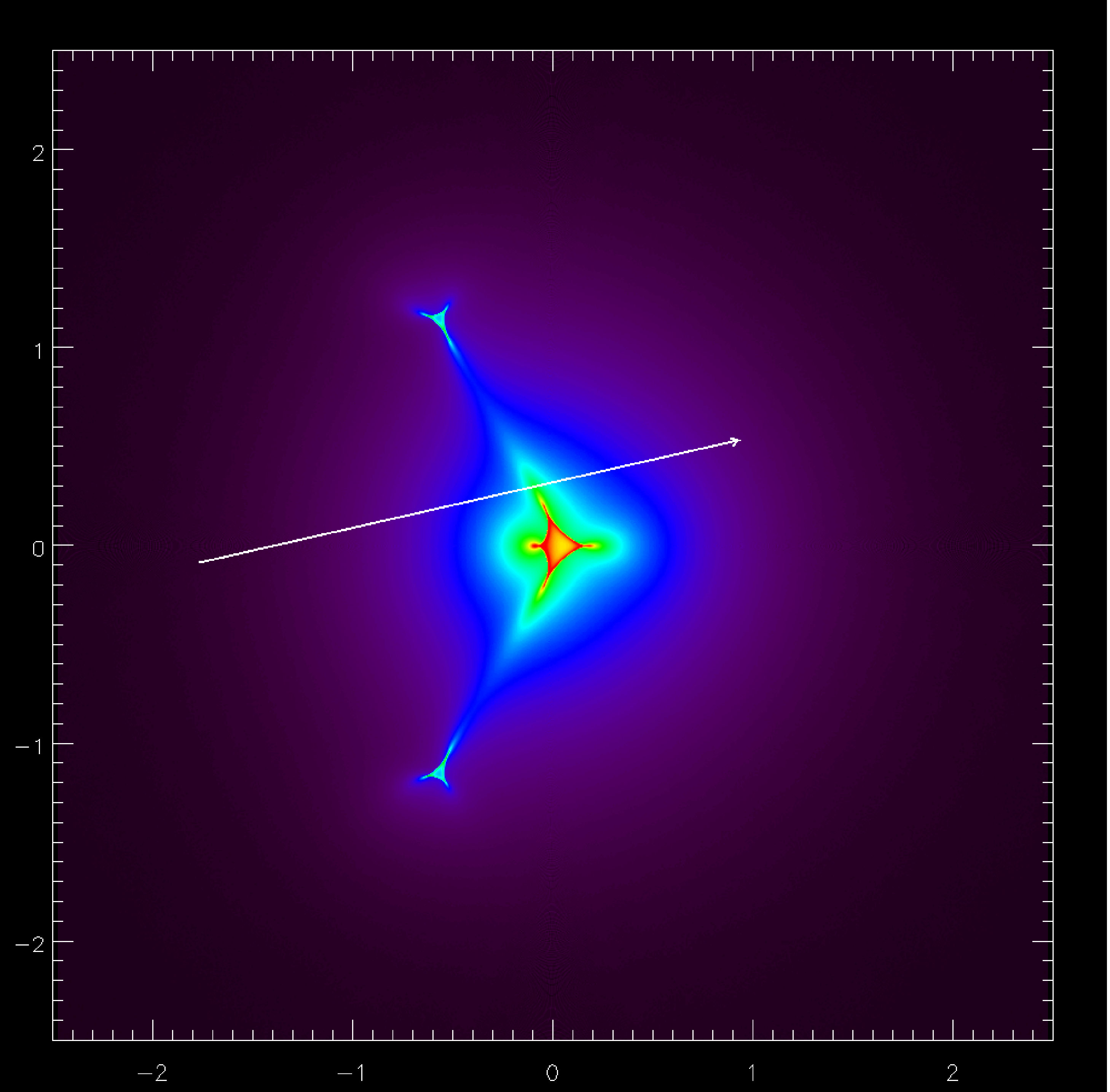


q=0.3
i=45
d=0.6

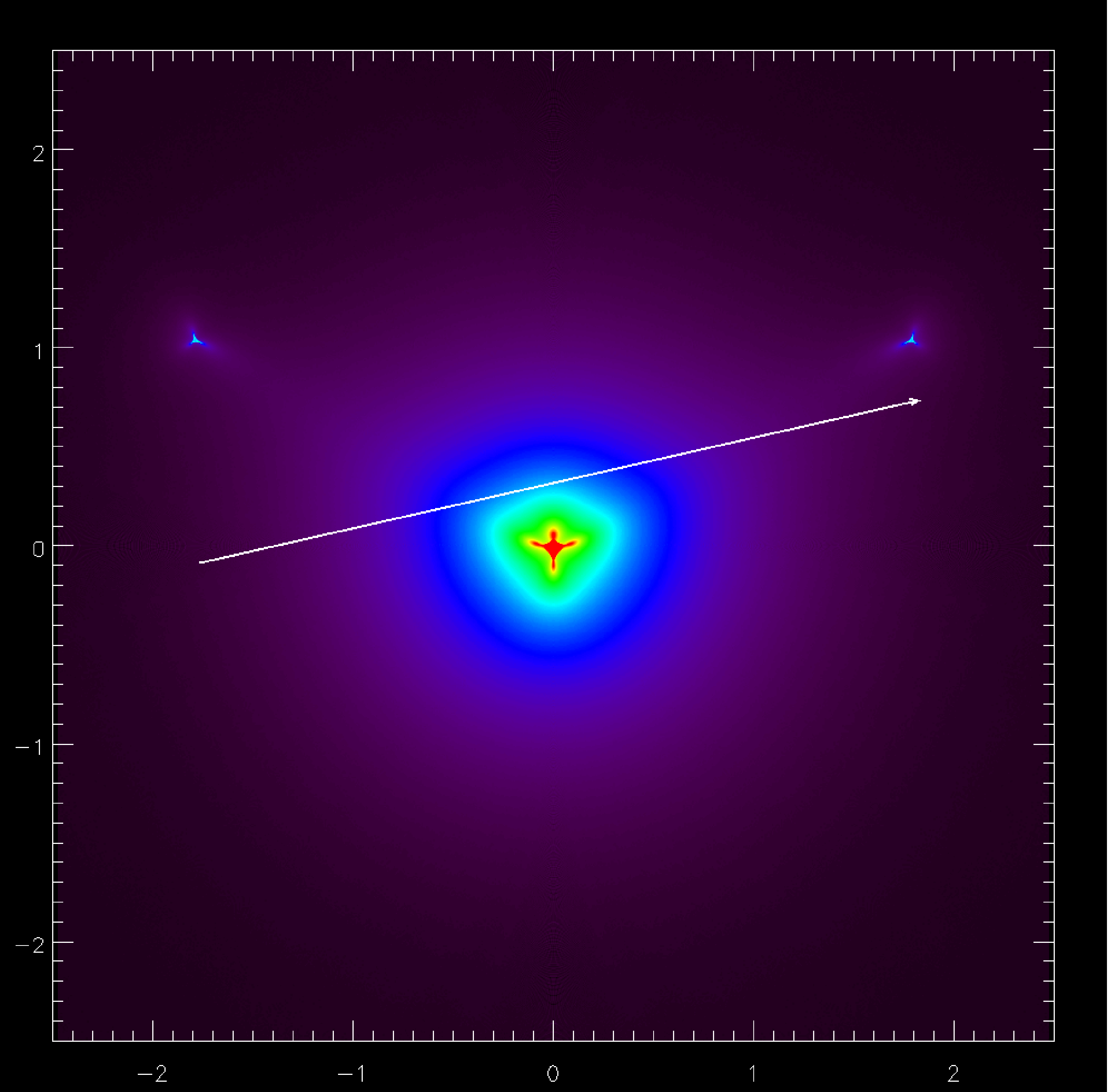
Phase:
0.25



Phase:
0.47

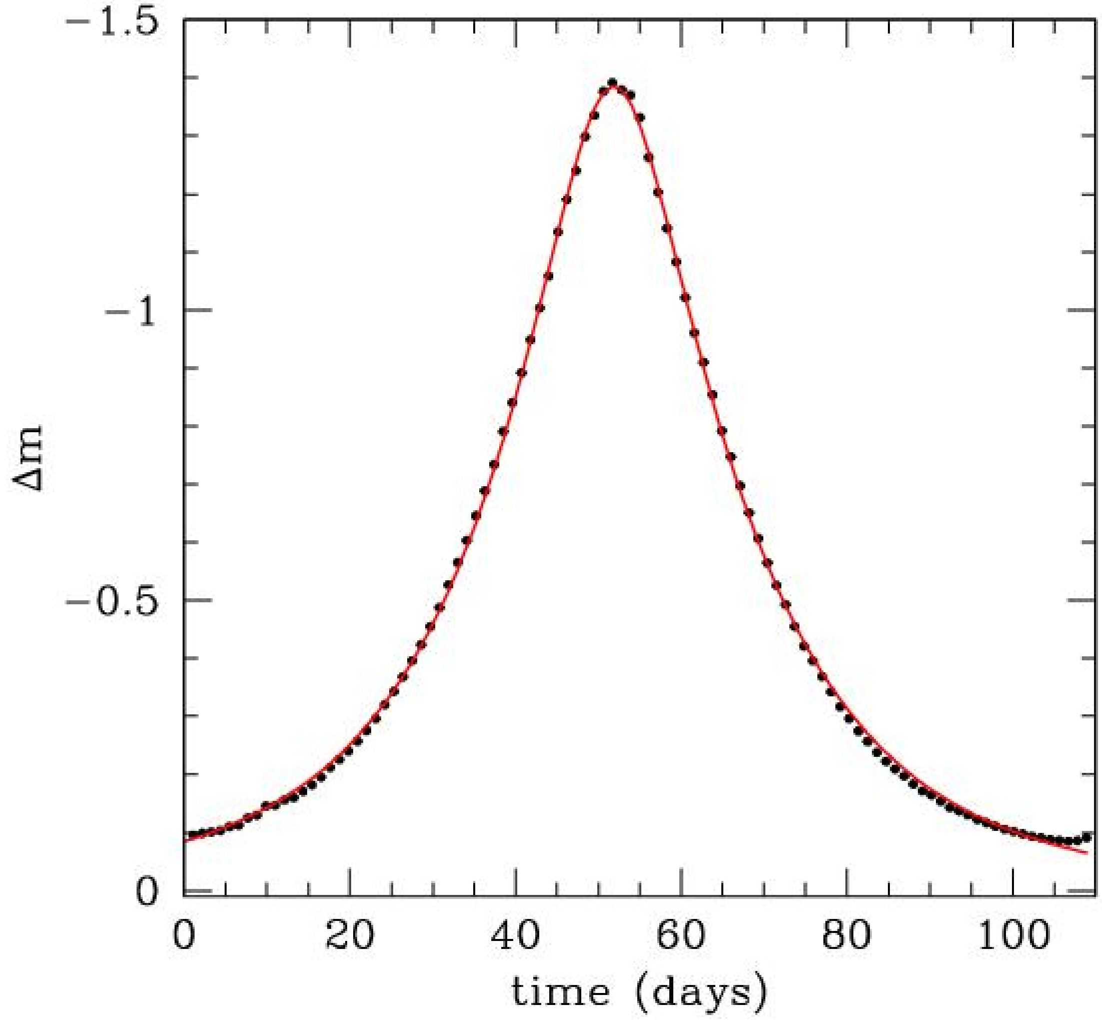


Phase:
0.75

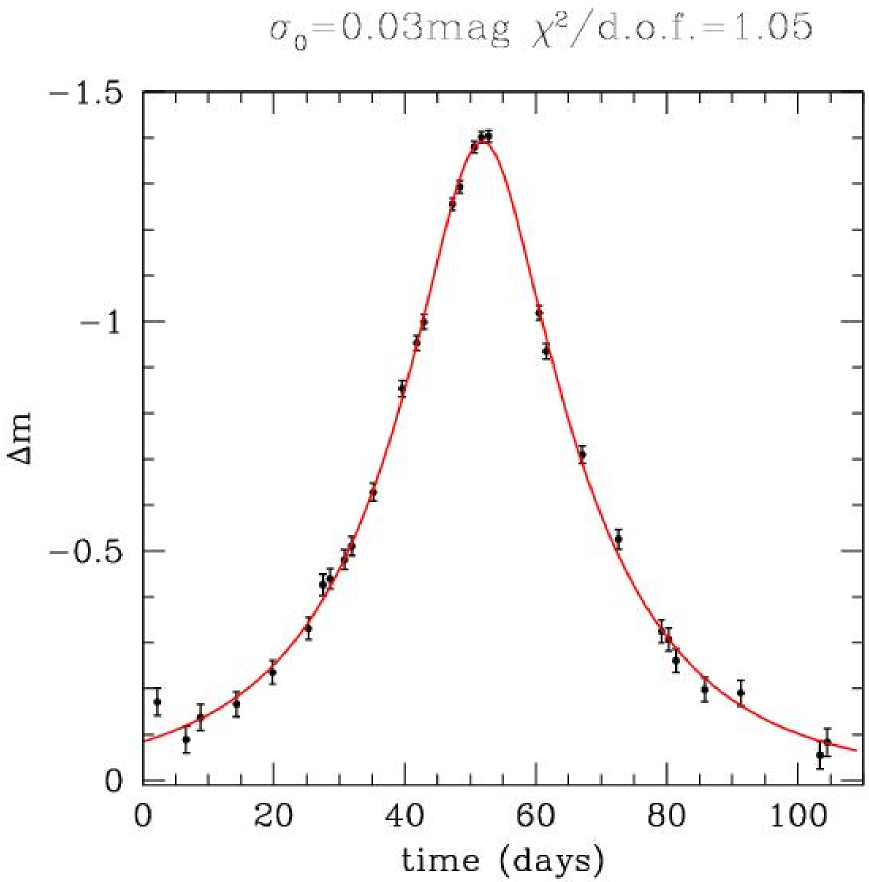


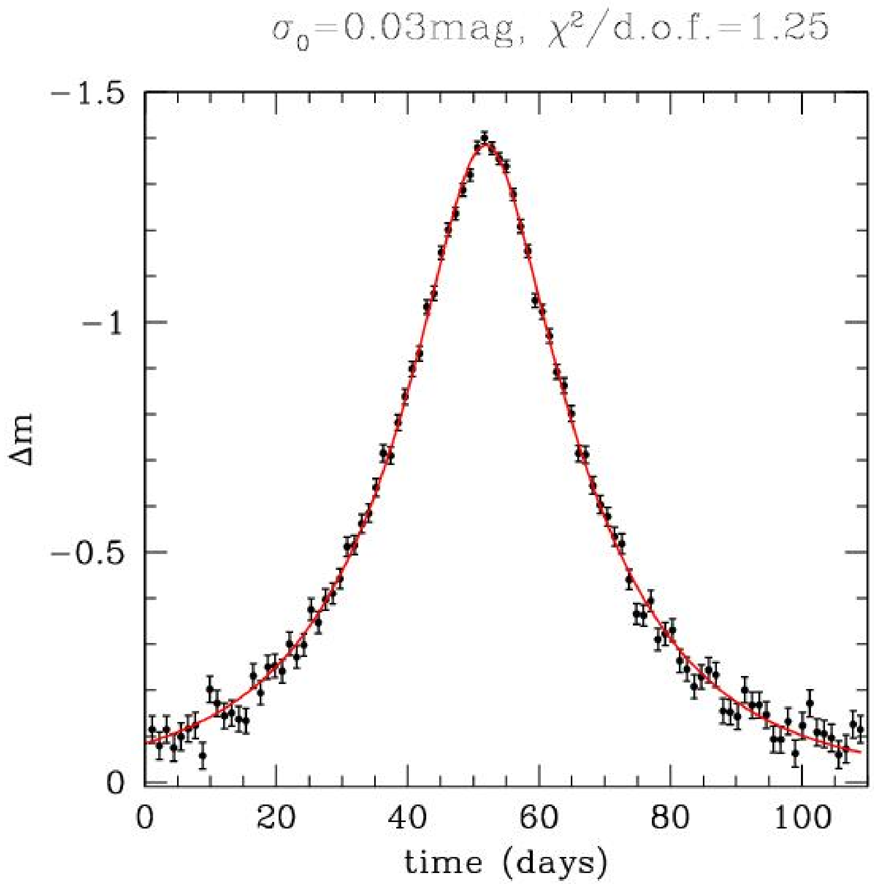
Phase:
1.00

Orbiting binary (P=100 days) with separation of 0.6 R_E



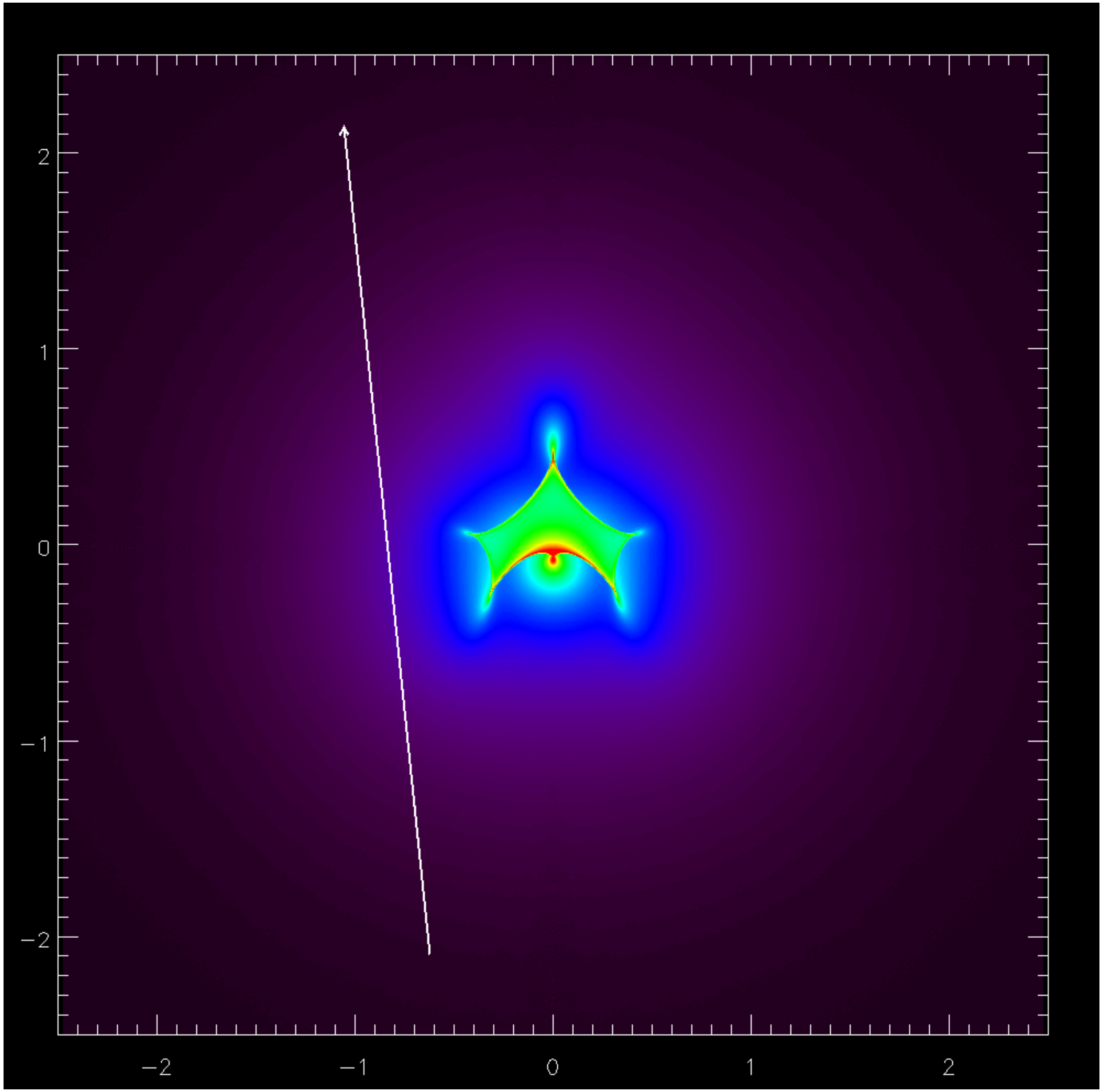
Orbiting binary (P=100 days) with separation of 0.6 R_E



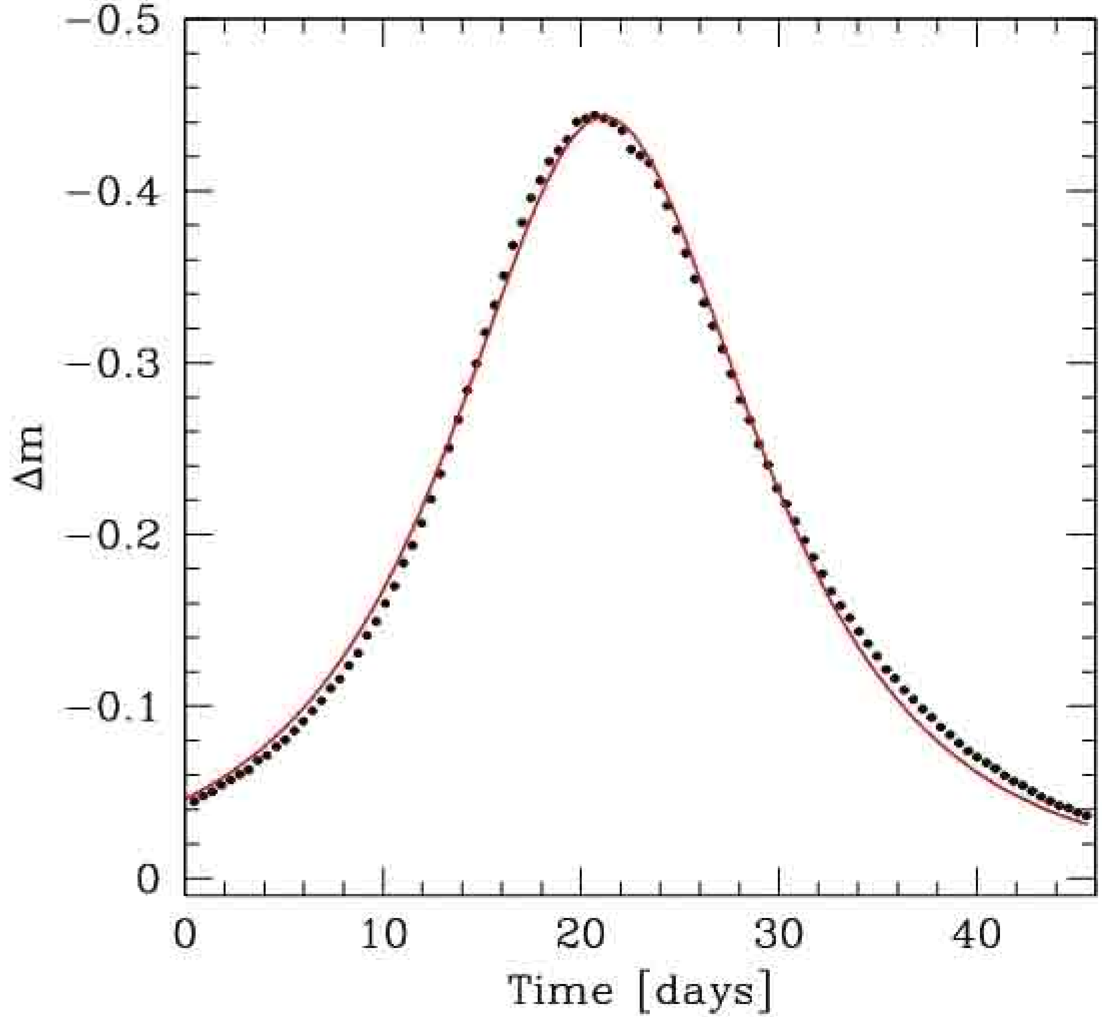


Misinterpretation even with High quality data is possible!

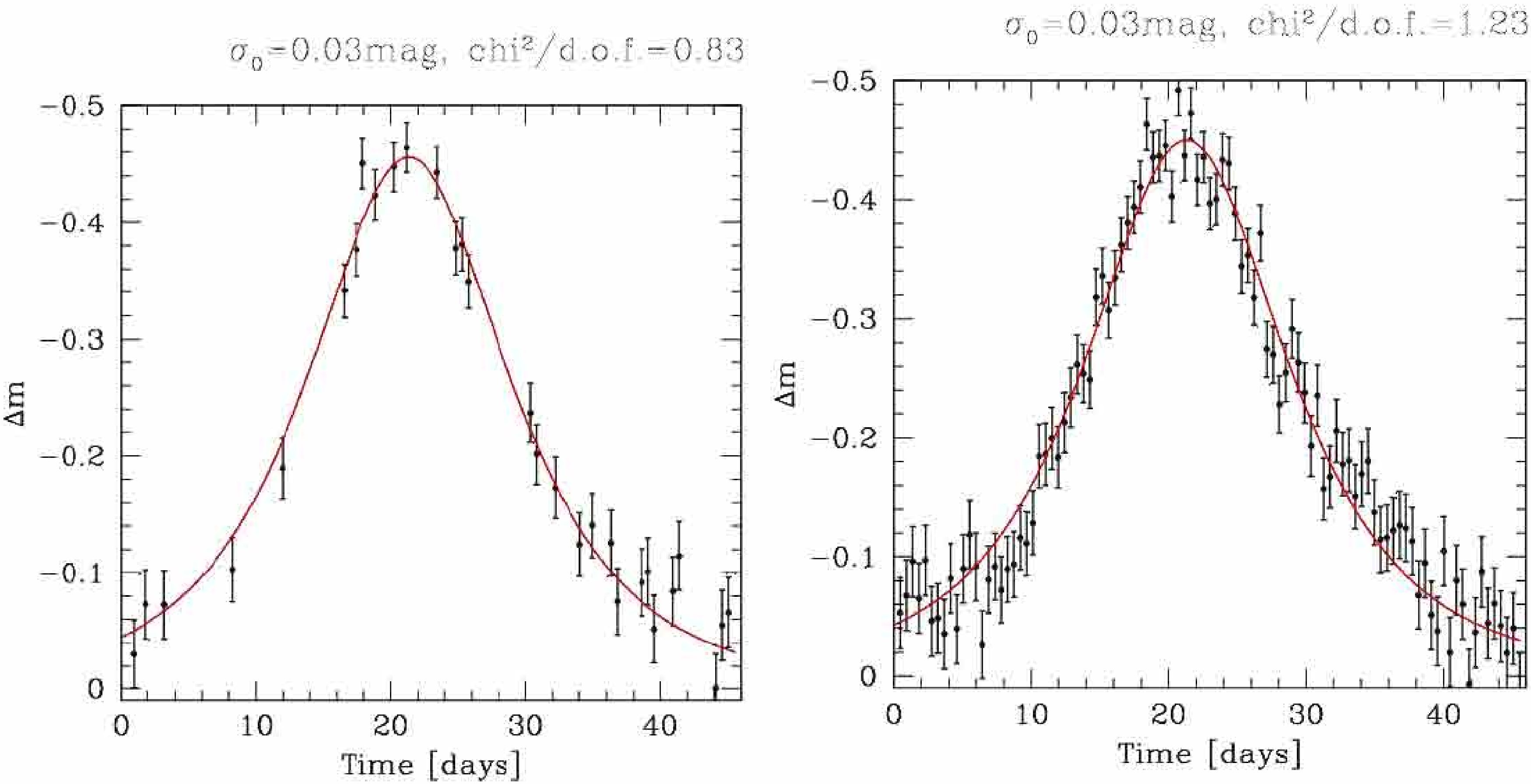
Static approximation



q=0.1
d=1



−0.5
−0.4
−0.3
−0.2
−0.1
0
Δm
0
10
20
30
40
Time [days]

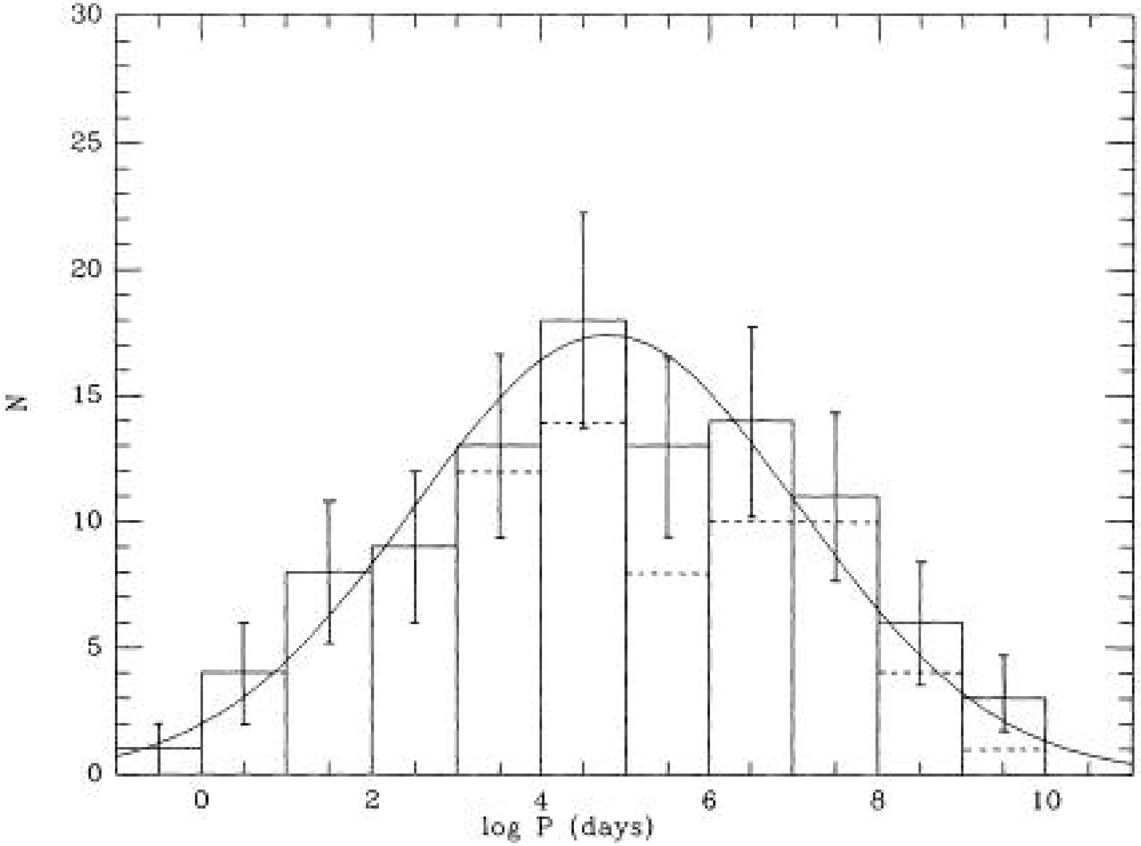


σ_0=0.03mag, chi²/d.o.f.=0.83
σ_0=0.03mag, chi²/d.o.f.=1.23
Δm
Time [days]
Δm
Time [days]

Periods of Binary Systems

- A large fraction of the Galactic stars are in binary systems
- Gaussian-like distribution in ***log P*** with **maximum** around 10^4 days
- (between 1 day and 10^{10} days)
- **Long period binaries** (P > 100 days): more binaries with **high mass ratios**

Distribution of binary periods



Duquenney & Mayor (1991)

Time scales

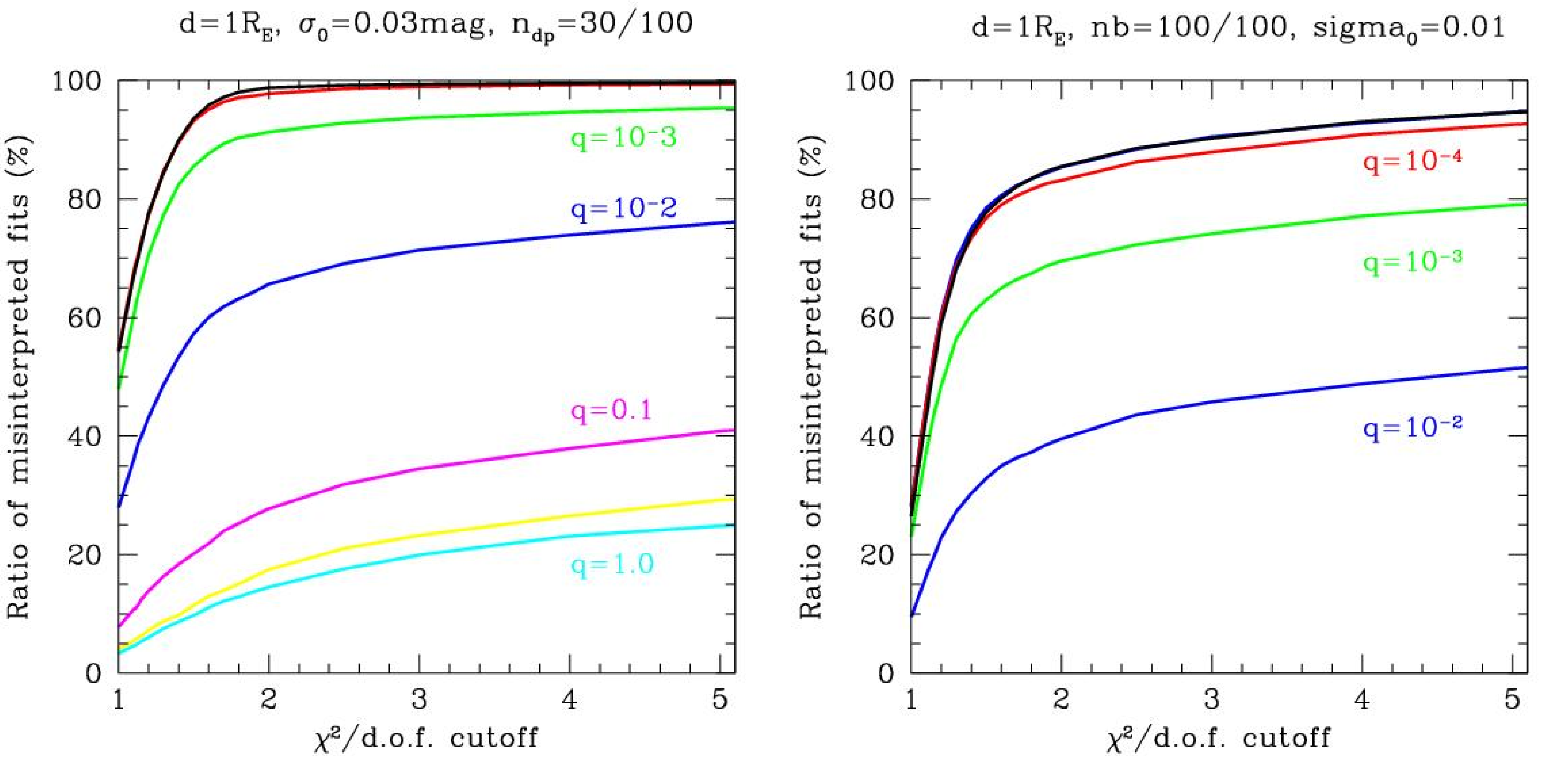
- **Orbiting binary**
- Separation ~ A.U.
- Period *P* ~months/years
- Event duration ~ *P*
- Long lasting events

- **Static approximation**
- Short Einstein crossing times (~ days/weeks)
- Much more common

Statistic

- Percentage of fits with small χ^2
- **100 magnification patterns** for each binary system
- n_{dp} data points (out of 100)
- **10000 light curves** per realization

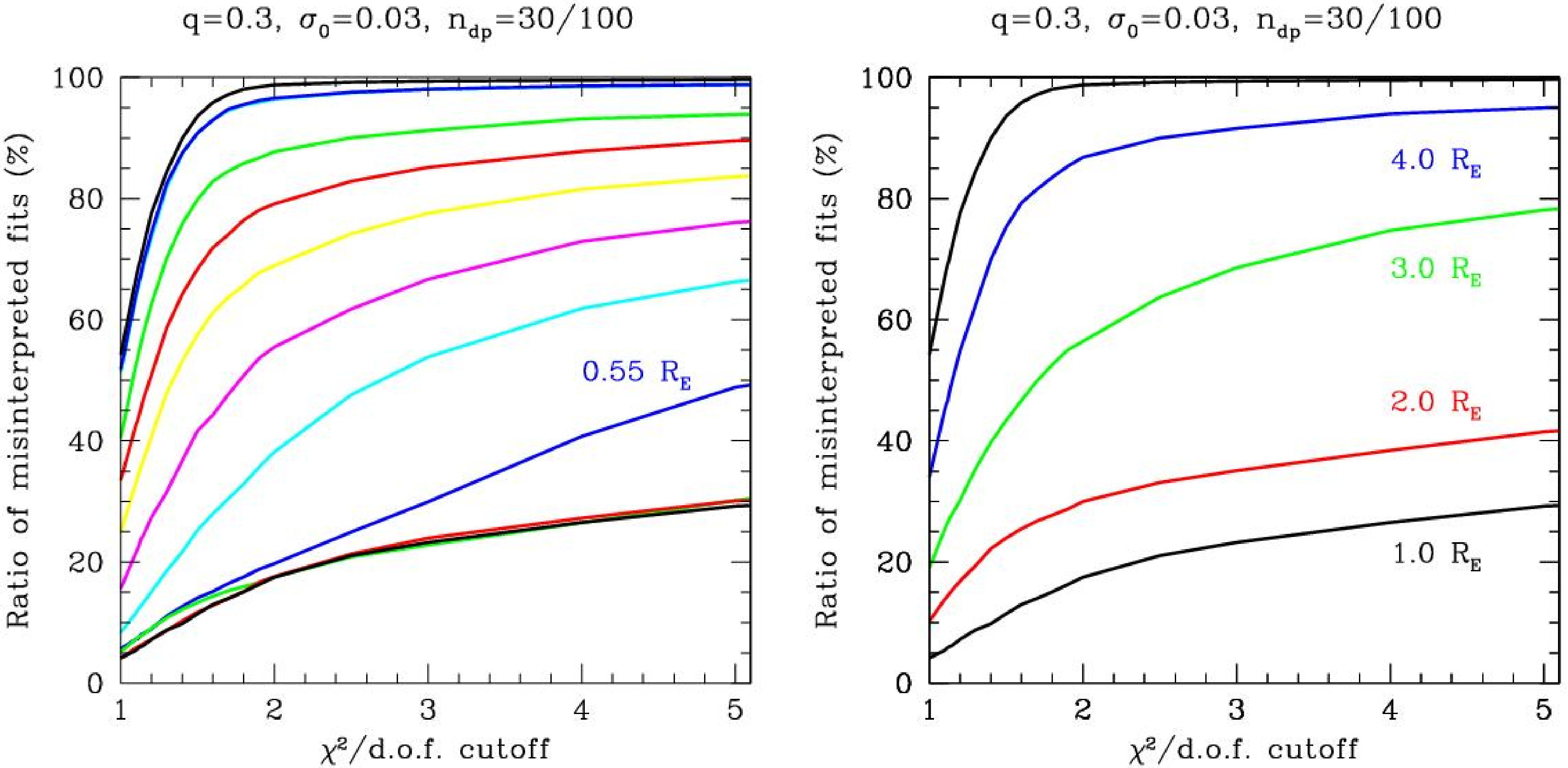
Binary mass ratio *q*



Static approximation
(black line: single lens)

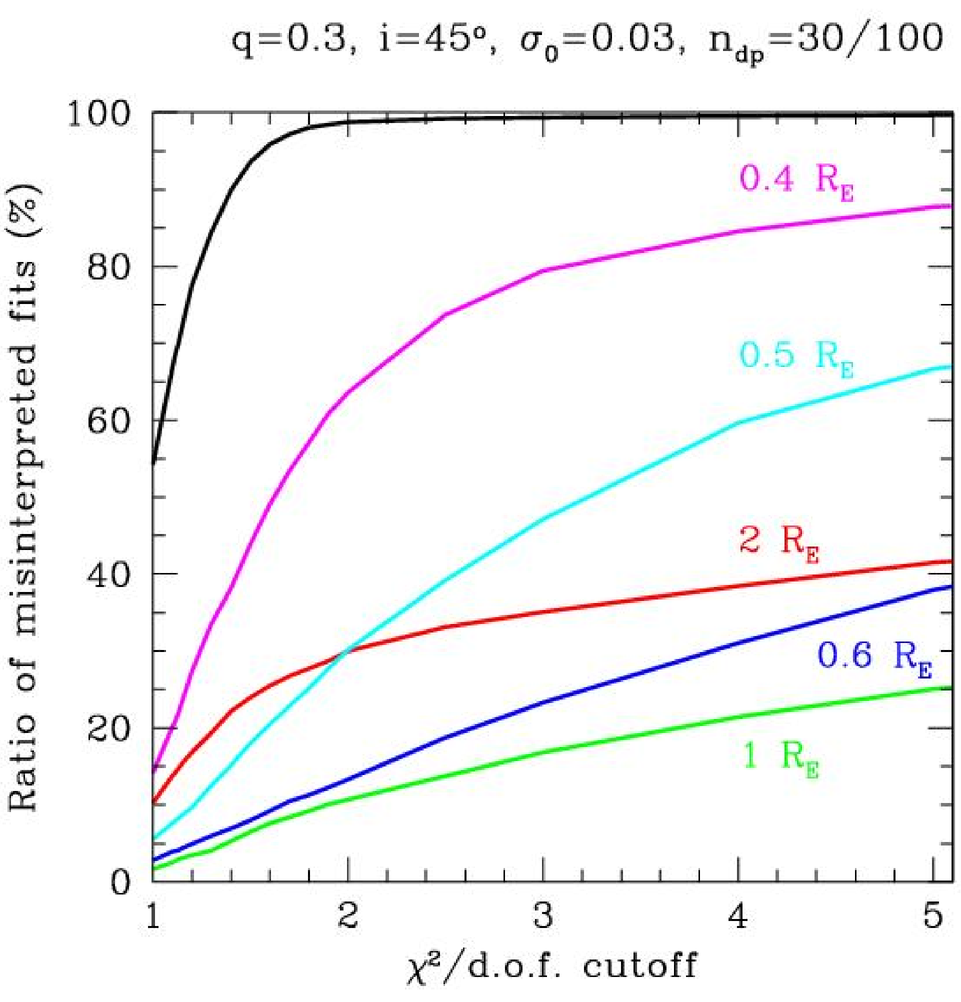
Easier to detect stellar binary lenses than planetary binaries!

Separation of the binary components *d*



(static approximation)

Separation of the binary components *d*

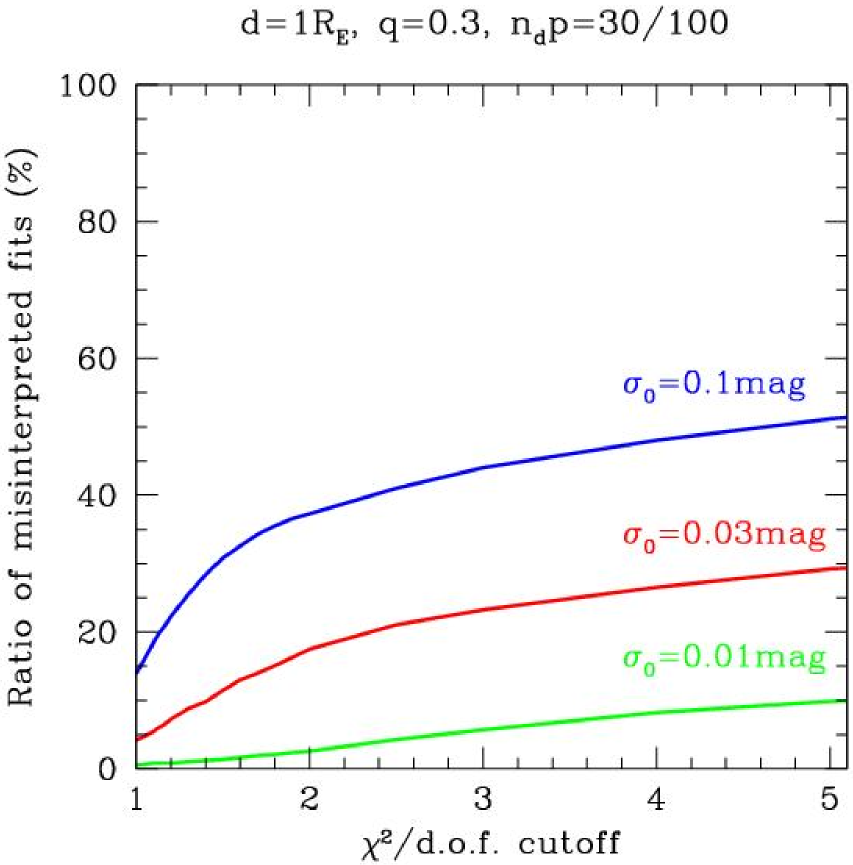


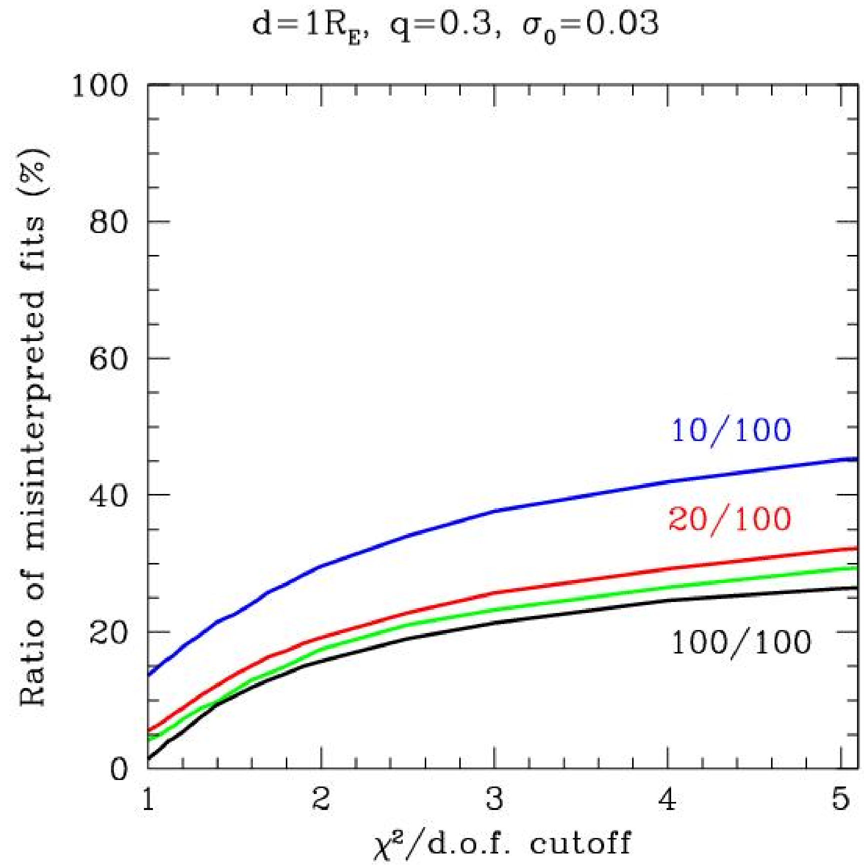
(orbiting binary)

Microlensing is the most sensitive to separations around 1 Einstein radius!

Size of errorbars

Number of data points (data sampling)





Better data quality increases the detection!

Summary of part I

- **Statistical analysis**
- **Significant chance** for misinterpretation of a binary lens by a single lens
- Higher probability for misinterpretation for **short lasting events**
- **Separation** and **mass ratio** play an important role:
- Minimum around **1 Einstein radius**
- Probability for **planet detection**

Binary source - single lens model (BS-SL)

- Superposition of two light curves:

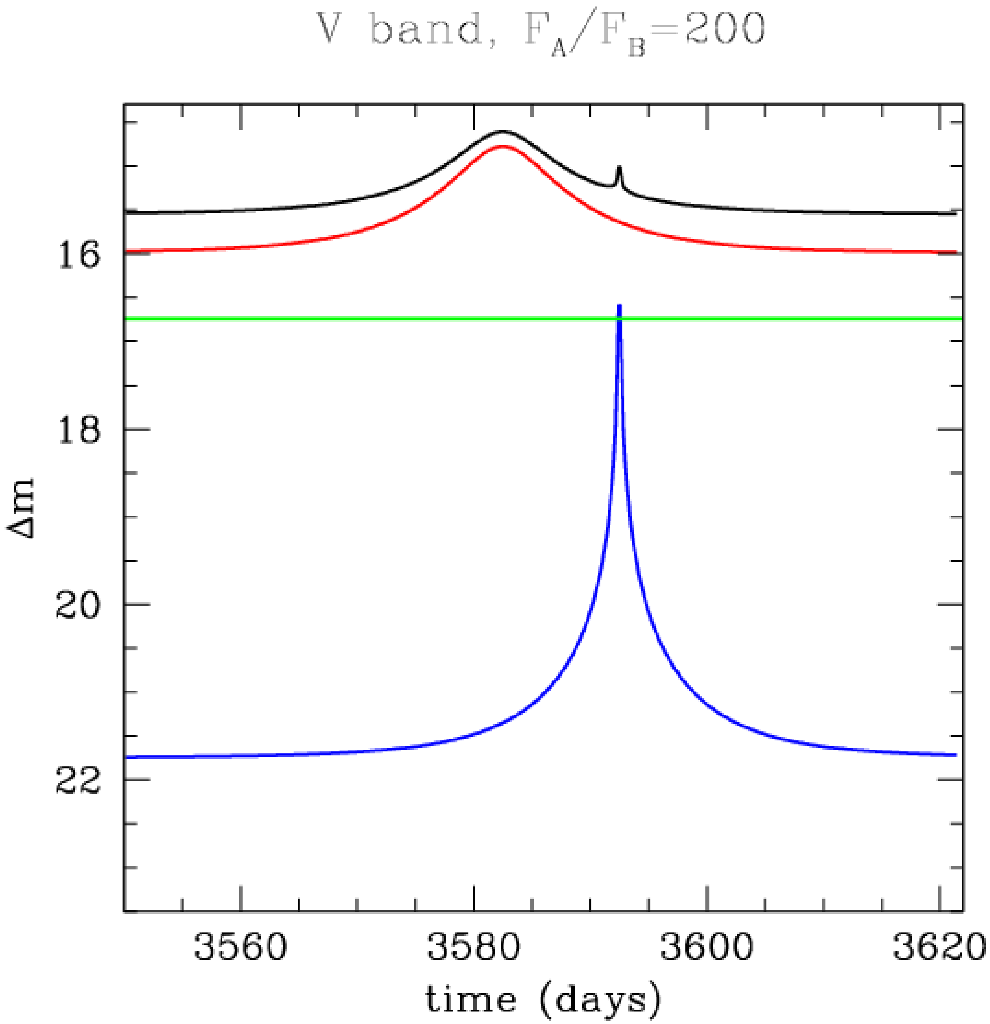
$$F^{i}(t) = F_{A}^{i} A_{A}(t) + F_{B}^{i} A_{B}(t) + F_{blend}^{i}$$

- observing site *i*
- Optimizing code **BISCO** (Binary Source Code for Optimization, Dominis 2005) uses the **genetic algorithm PIKAIA** (Charbonneau 1995) to find the best solution:

$$\{t_{E}, u_{0}(A), u_{0}(B), t_{0}(A), t_{0}(B), F_{A}/F_{B}\}$$

$$j_{par} = (2n_{os} + 1) n_{pb} + 5$$

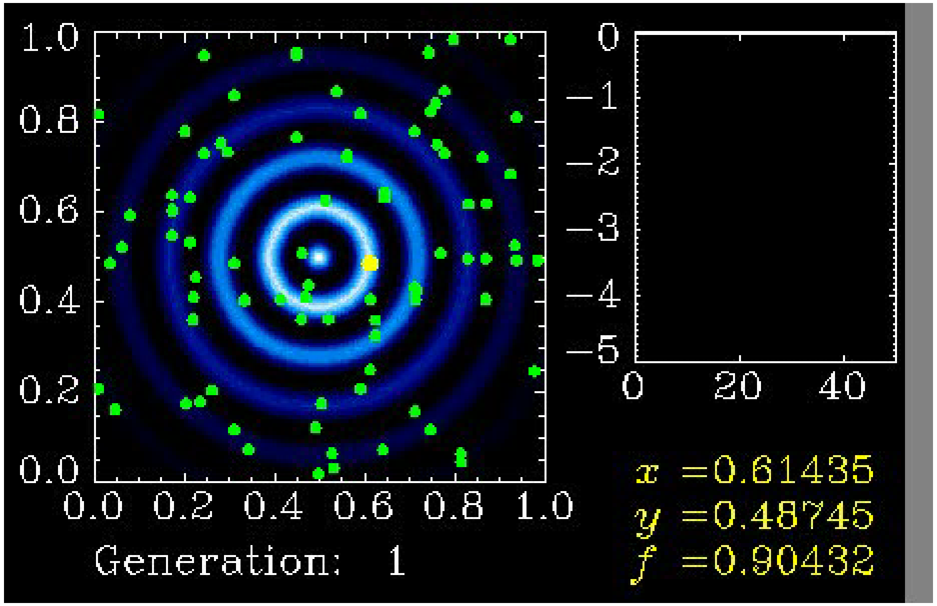
Binary source light curve (in magnitudes)

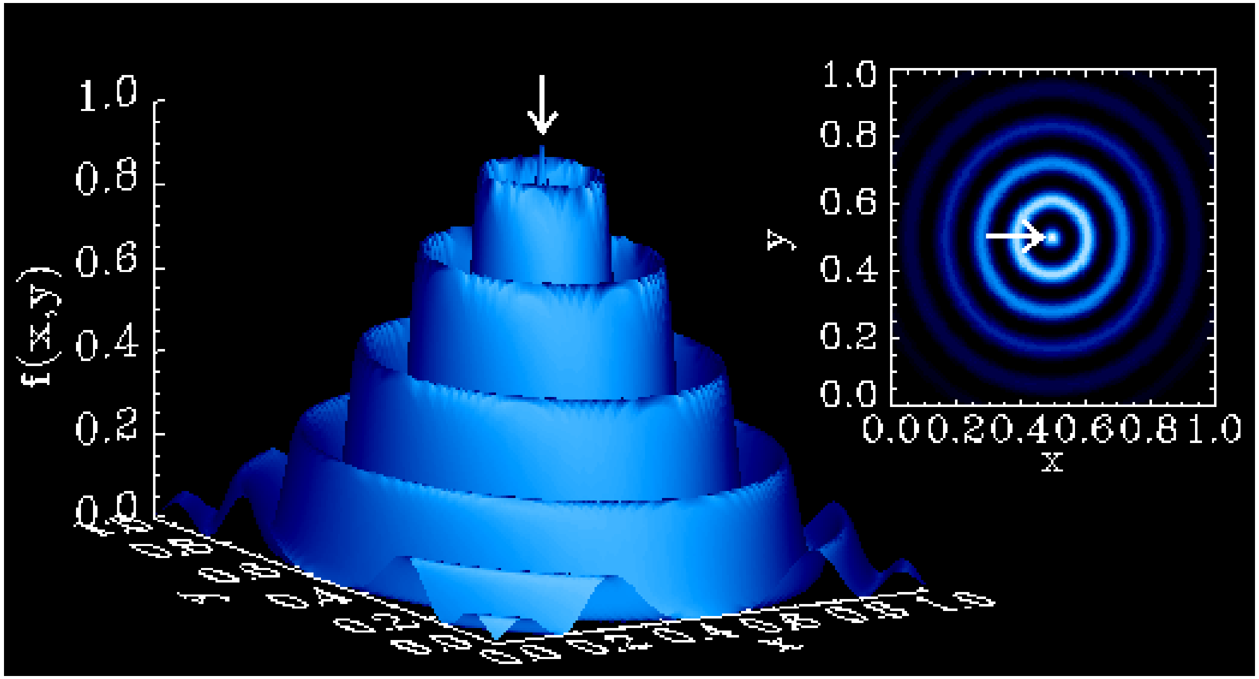


Genetic algorithm

- **Numeric optimization** of an **inverse problem** (one light curve => set of physical parameters)
- Uses **natural selection** (selection of parents, mutation, crossing, evolution)
- Useful for complicated parameter spaces with **many local minima**

- GENETIC ALGORITHM (numeric optimization)
- evolution of a random initial population

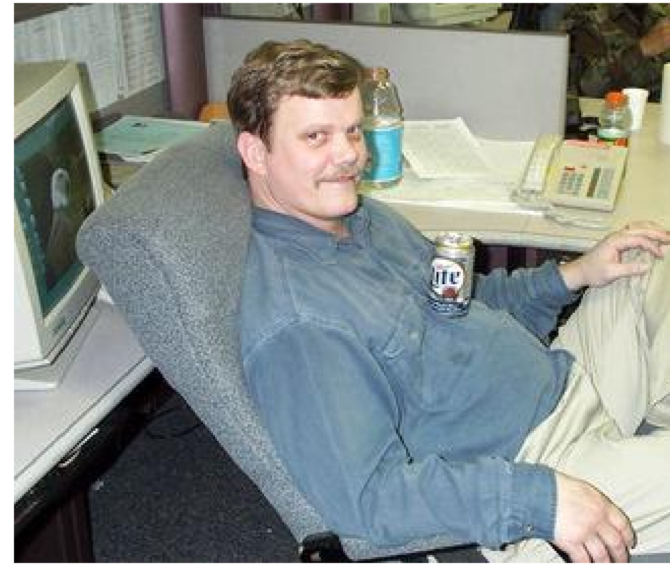




?

?

?

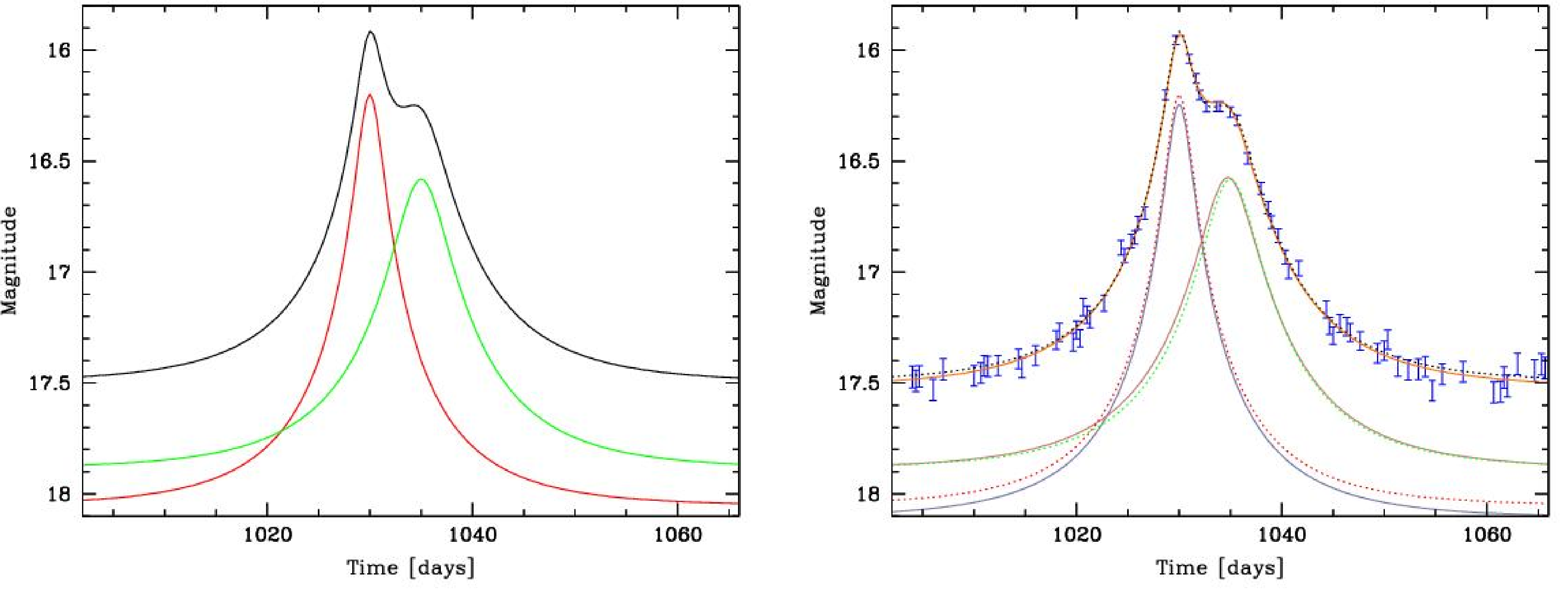


FITNESS – selection criterion (probability for crossing and survival)

$$\chi^2 / d.o.f.$$

(goodness of fit, sum of squared residuals)

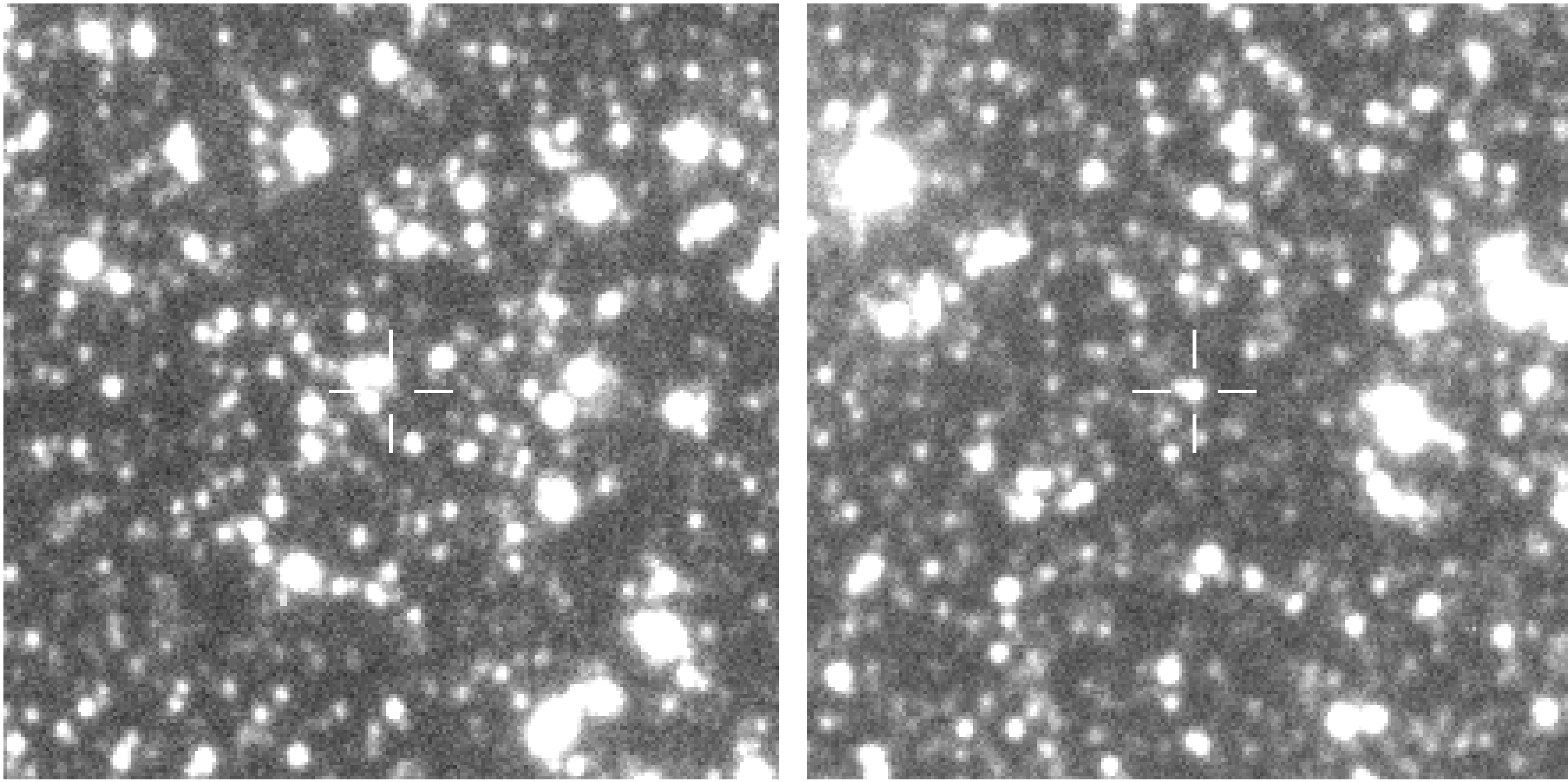
A simulated binary source – single lens microlensing event



BISCO is capable of reconstructing the event parameters from simulated data

OGLE-2003-BLG-222 (I=19.9)

OGLE-2004-BLG-347 (I=17.5)

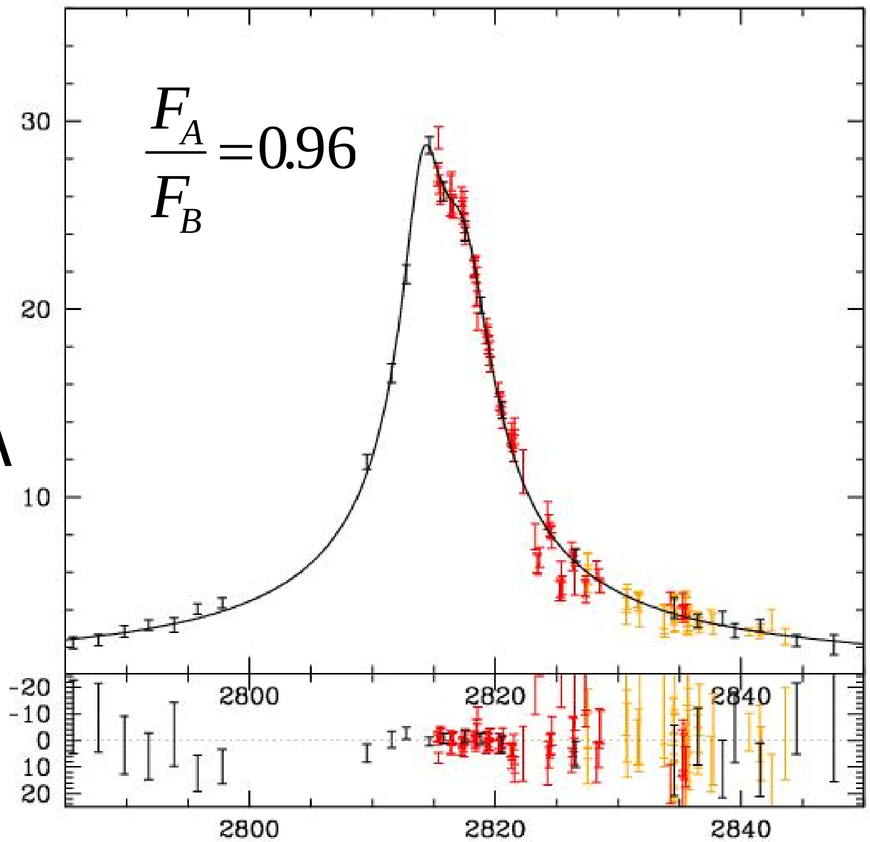


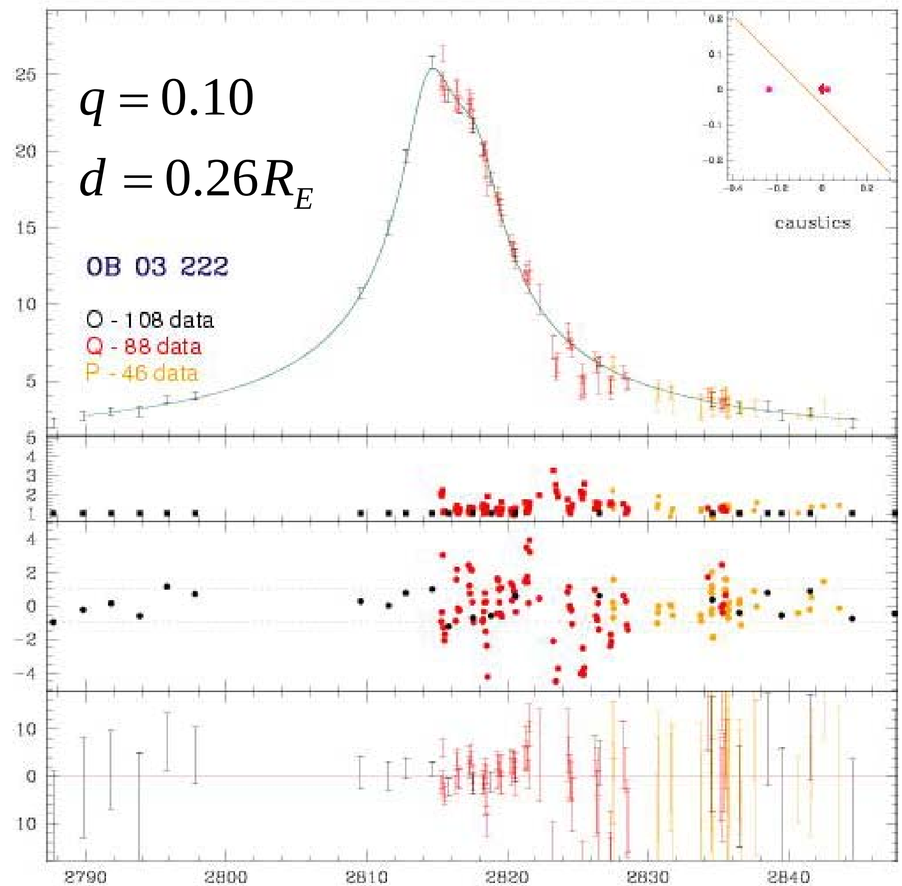
1‘x1‘

OGLE-2003-BLG-222

Binary **source** fit, GA (Dominis)   Binary **lens** fit, Powell (Cassan)

A





Time (days)

Degeneracy!

OB-03-222 Binary **source** model

BINARY SOURCE MODEL:
Einstein ring radius crossing time: $t_E = (68.4 \pm 1.1) days$
Closest approach of the A component: $u_0(A) = (0.0251 \pm 0.0012) R_E$
Closest approach of the B component: $u_0(B) = (0.0325 \pm 0.0007) R_E$
Time of closest approach (A): $t_0(A) = (2814.13 \pm 0.06) JD - 245000$
Time of closest approach (B): $t_0(B) = (2817.52 \pm 0.05) JD - 24500$
Flux ratio $F_A(I)/F_B(I)$: $fr(I) = (0.965 \pm 0.094)$
Baseline magnitudes in [mag] and blending factors for each site:
$m_{base}(OGLE) = 19.94$, $g(OGLE) = 0.926$
$m_{base}(Danish) = 20.65$, $g(Danish) = 1.135$
$m_{base}(SAAO) = 21.01$, $g(SAAO) = 0.001$
$\chi^2/d.o.f. = 454/235$

OB-03-222 Binary **lens** model

BINARY LENS MODEL :

Binary separation: $(d = 0.258^{+0.002}_{-0.026}) R_E$

Mass ratio: $q = (0.1022^{+0.0748}_{-0.0007})$

Einstein ring radius crossing time: $t_E = (68.1^{+0.3}_{-7.6}) days$

Closest approach: $u_0 = (0.036^{+0.002}_{-0.0009}) R_E$

Time of closest approach: $t_0 = (2815.64^{+0.01}_{-0.02}) JD - 245000$

$\Theta = (2.574^{0.001}_{0.01}) rad$

Baseline magnitudes in [mag] and blending factors for each site:

$m_{base}(OGLE) = 19.95$, $g(OGLE) = 0.694$

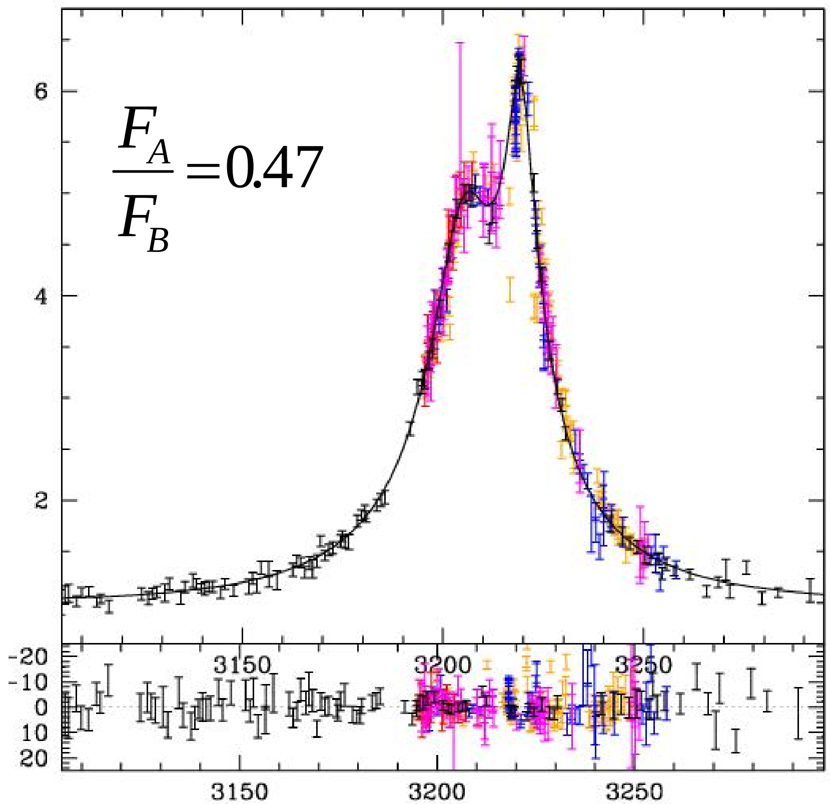
$m_{base}(Danish) = 20.68$, $g(Danish) = 0.926$

$m_{base}(SAAO) = 20.89$, $g(SAAO) = 0.000$

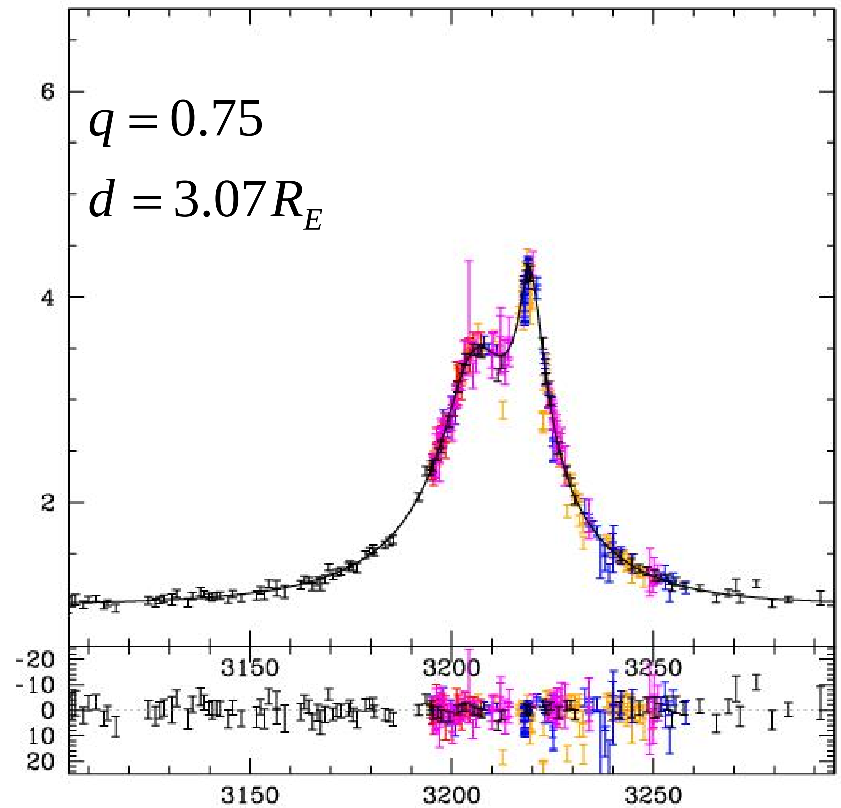
$\chi^2/d.o.f. = 487/235$

OGLE-2004-BLG-347

Binary source fit (Dominis)



Binary lens fit (Cassan)



Magnification

Time (days)

Degeneracy!

OB-04-347 Binary **source** model

BINARY SOURCE MODEL:

Einstein ring radius crossing time: $t_E = (45.7 \pm 2.2) days$

Closest approach of the A component: $u_0(A) = (0.081 \pm 0.005) R_E$

Closest approach of the B component: $u_0(B) = (0.182 \pm 0.013) R_E$

Time of closest approach (A): $t_0(A) = (3219.63 \pm 0.06) JD - 245000$

Time of closest approach (B): $t_0(B) = (3205.45 \pm 0.16) JD - 245000$

Flux ratio $F_A(I)/F_B(I)$: $fr(I) = (0.468 \pm 0.036)$

Baseline magnitudes in [mag] and blending factors for each site:

$m_{base}(OGLE) = 17.48$, $g(OGLE) = 1.422$

$m_{base}(Danish) = 16.92$, $g(Danish) = 1.494$

$m_{base}(SAAO) = 17.66$, $g(SAAO) = 2.078$

$m_{base}(Hobart) = 17.99$, $g(Hobart) = 1.605$

$m_{base}(Perth) = 17.49$, $g(Perth) = 1.510$

$\chi^2/d.o.f. = 1221/552$

OB-04-347 Binary **lens** model

BINARY LENS MODEL:

Binary separation: $(d = 3.0717^{+0.001}_{-0.0009})R_E$

Mass ratio: $q = (0.74969^{+0.00003}_{-0.0001})$

Einstein ring radius crossing time: $t_E = (50.082^{+0.006}_{-0.004})days$

Closest approach: $u_0 = (0.5902^{+0.0002}_{-0.0002})R_E$

Time of closest approach: $t_0 = (3254.58 \pm^{+0.001}_{-0.0})JD - 24500$

$\Theta = (2.42171^{+0.00005}_{-0.00002})rad$

Baseline magnitudes in [mag] and blending factors for each site:

$m_{base}(OGLE) = 17.47,\ g(OGLE) = 0.510$

$m_{base}(Danish) = 16.90,\ g(Danish) = 0.593$

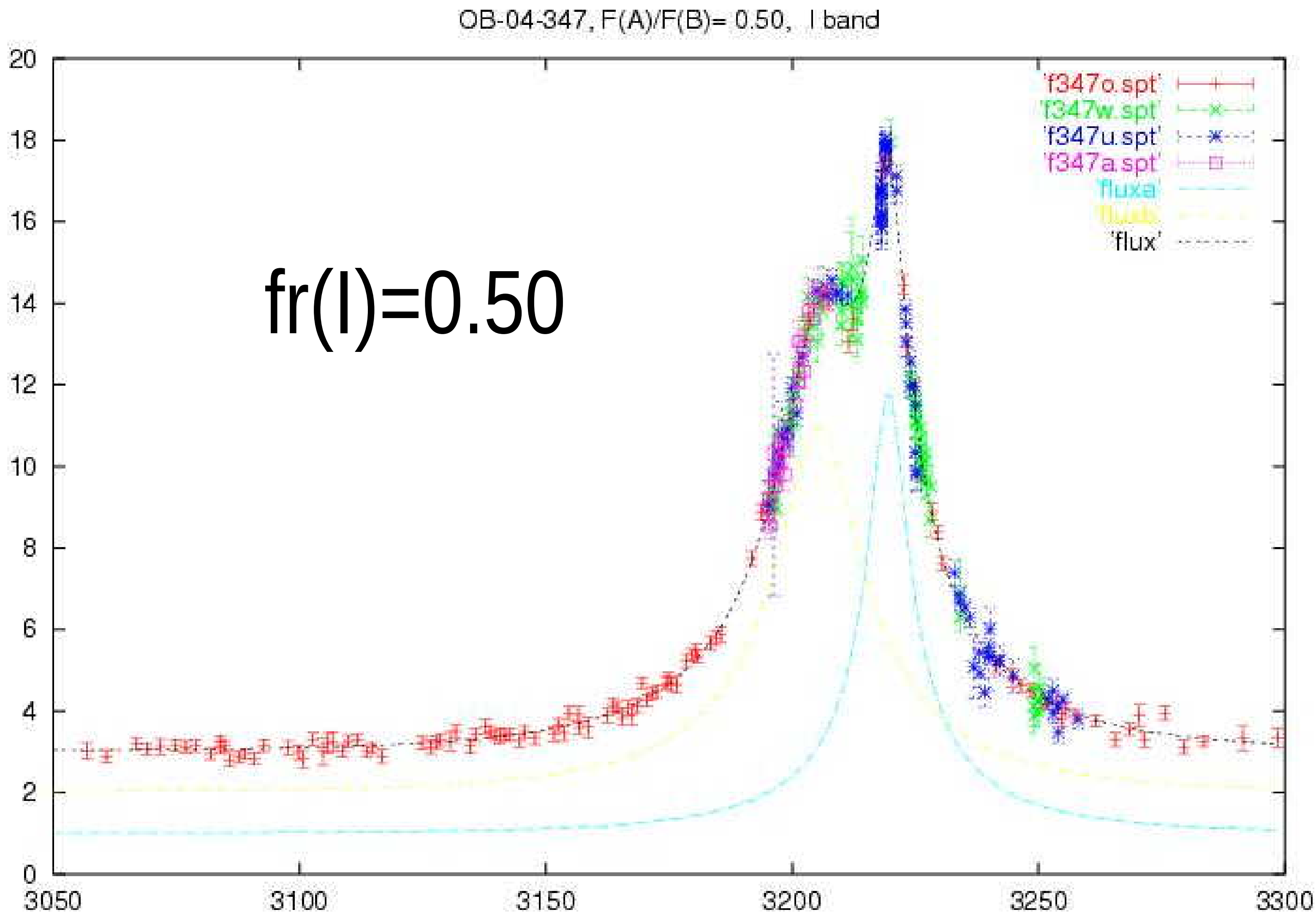
$m_{base}(SAAO) = 17.64,\ g(SAAO) = 0.943$

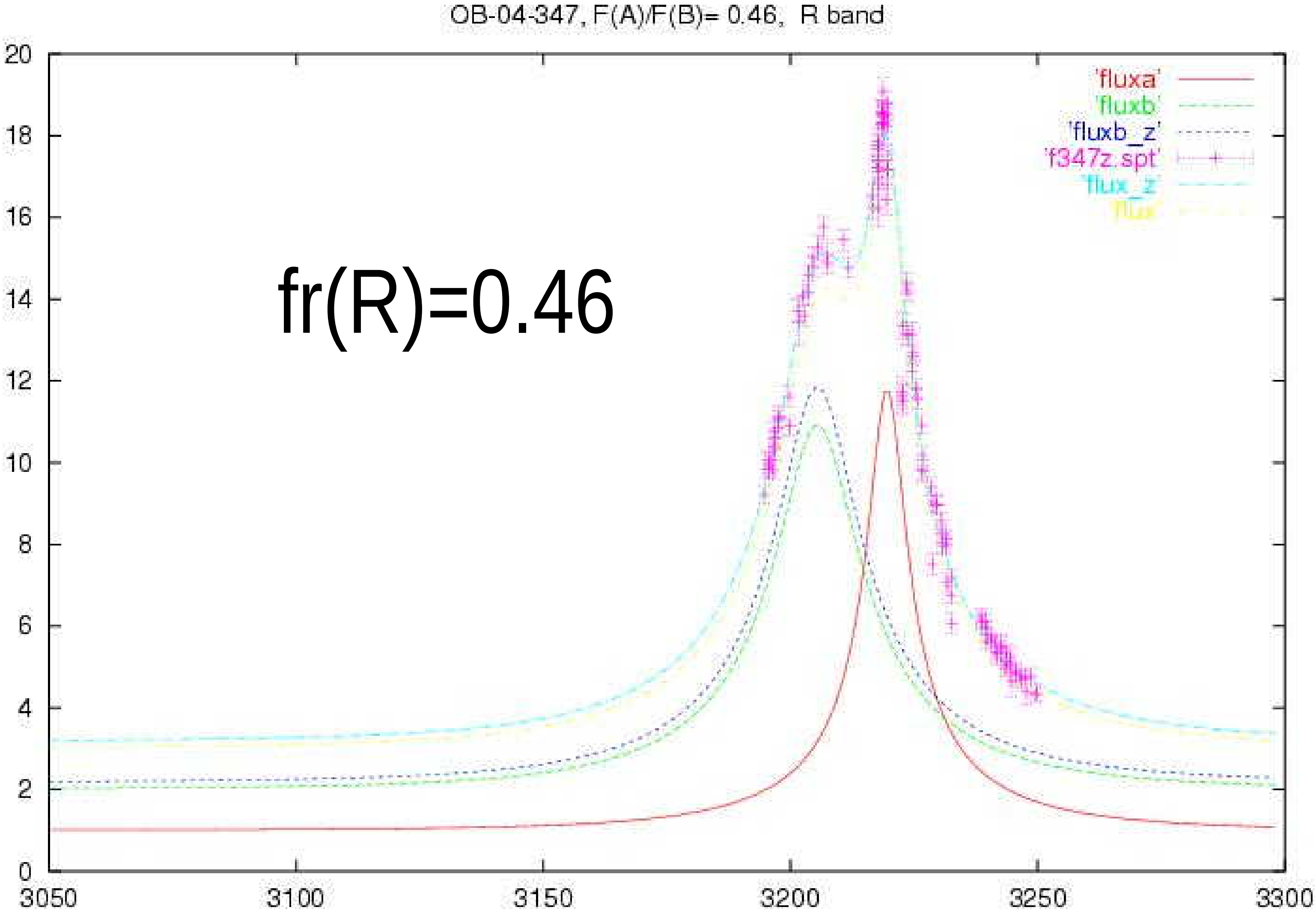
$m_{base}(Hobart) = 17.97,\ g(Hobart) = 0.683$

$m_{base}(Perth) = 17.47,\ g(Perth) = 0.596$

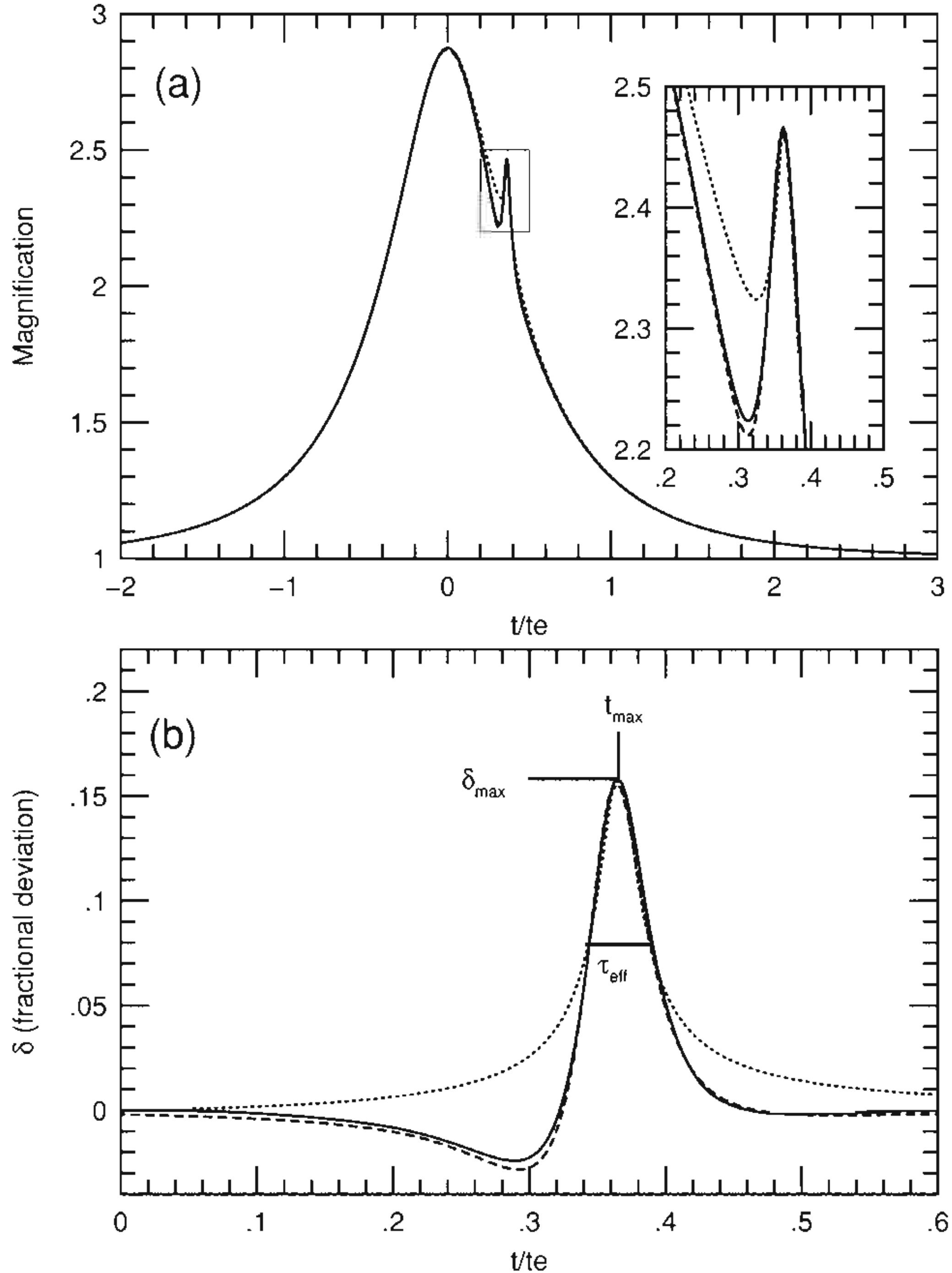
$\chi^2/d.o.f. = 1171/552$

Flux ratio method applied on OB-04-347



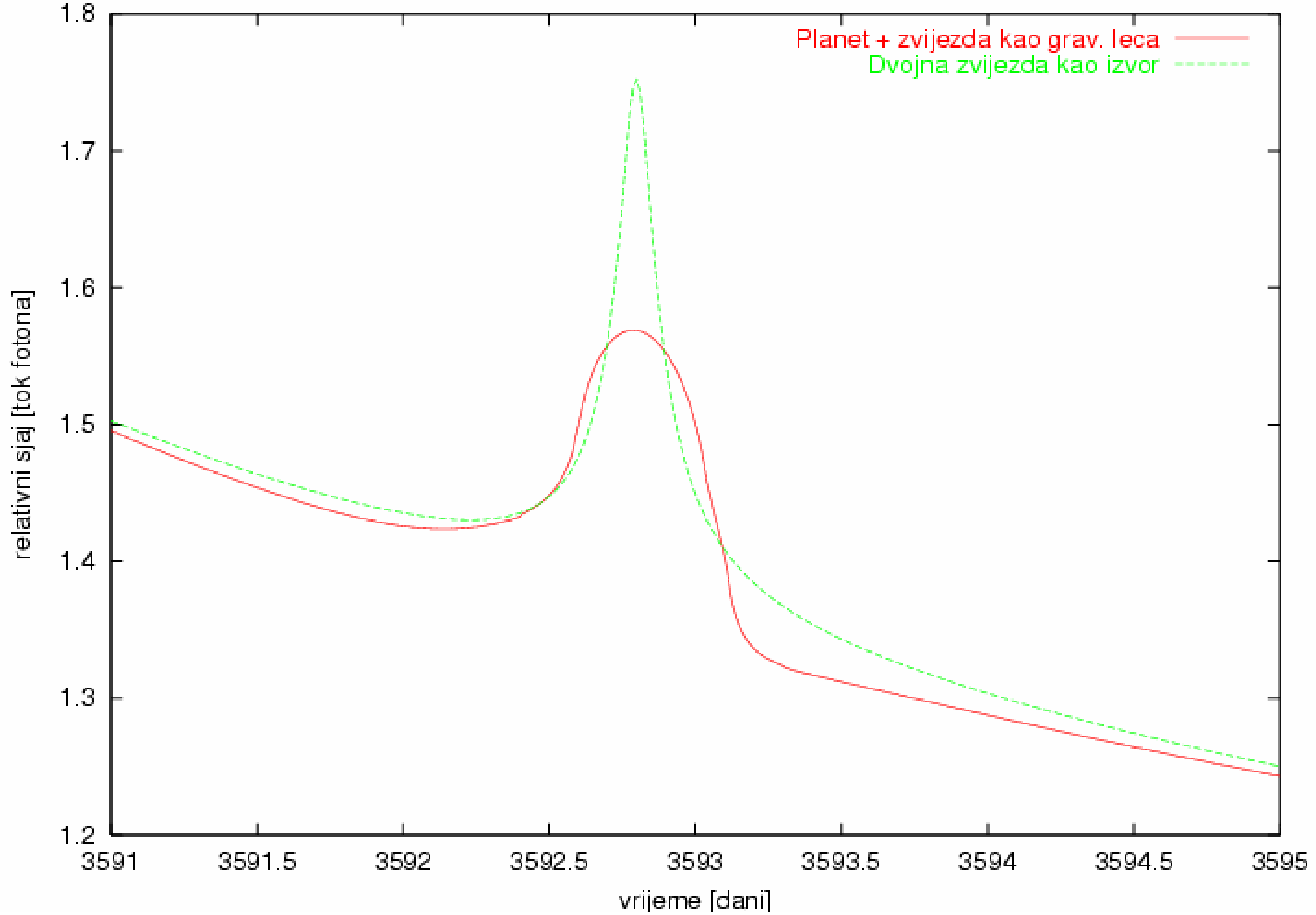


OB-04-347, F(A)/F(B)= 0.46, R band
'fluxa'
'fluxb'
'fluxb_z'
'f347z.spt'
'flux_z'
'flux'
fr(R)=0.46
20
18
16
14
12
10
8
6
4
2
0
3050
3100
3150
3200
3250
3300



(a)
Magnification
t/te
(b)
δ (fractional deviation)
t_{max}
δ_{max}
τ_{eff}
t/te

Ambiguity in the light curve solution



- Is there a **limit** on chi-squared-per-degree-of-freedom difference to decide about the model to be accepted?

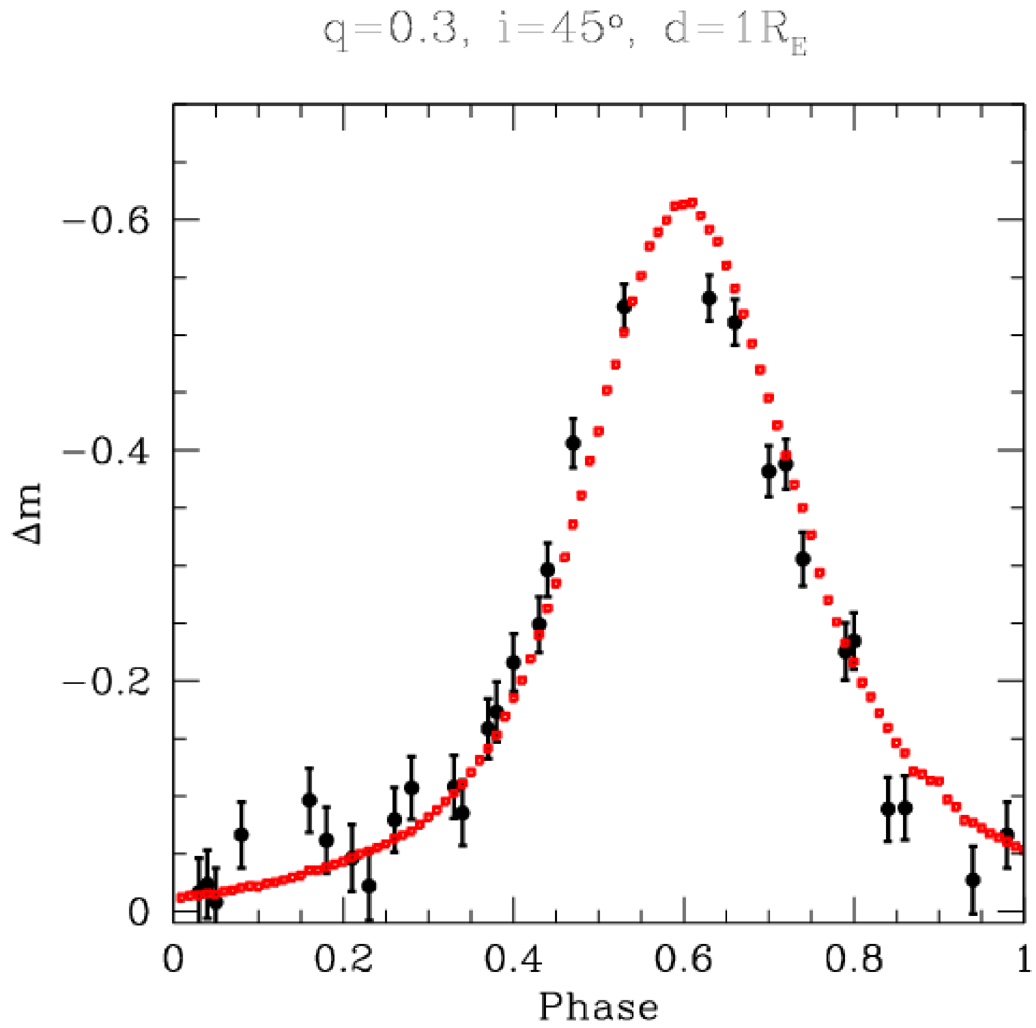
$$\Delta\chi^2 / d.o.f.(OB-03-222) = 14\%$$

$$\Delta\chi^2 / d.o.f.(OB-04-347) = 1.7\%$$

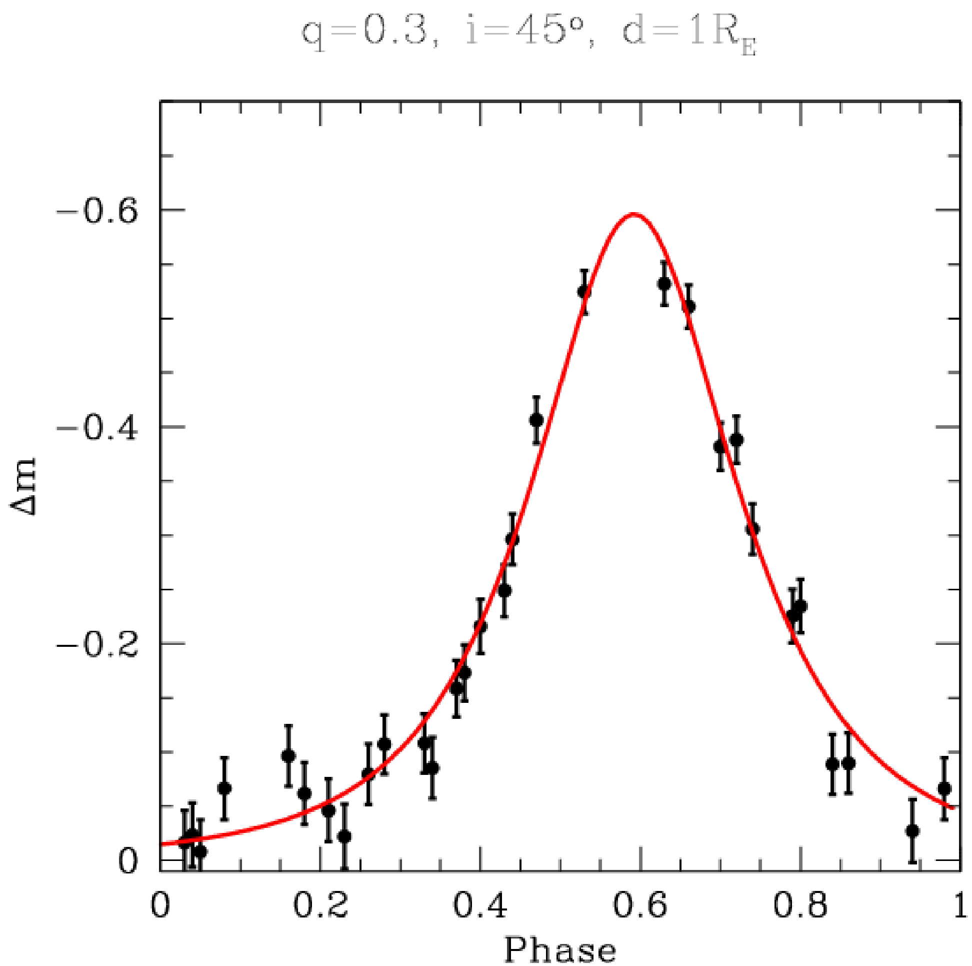
$$\Delta\chi^2 / d.o.f.(OB-05-390) = 7.3\%$$

- Or do we need a merit of fit adjusted to the relative „sizes“ of the two peaks?

Synthetic „realistic“ light curve (black) (*Gaussian noise, irregular data sampling*)



Best fit (red line) to the synthetic data (black)



$$\sigma_0 = 0.03 mag$$

$$\sigma_{\min} = 0.018 mag$$

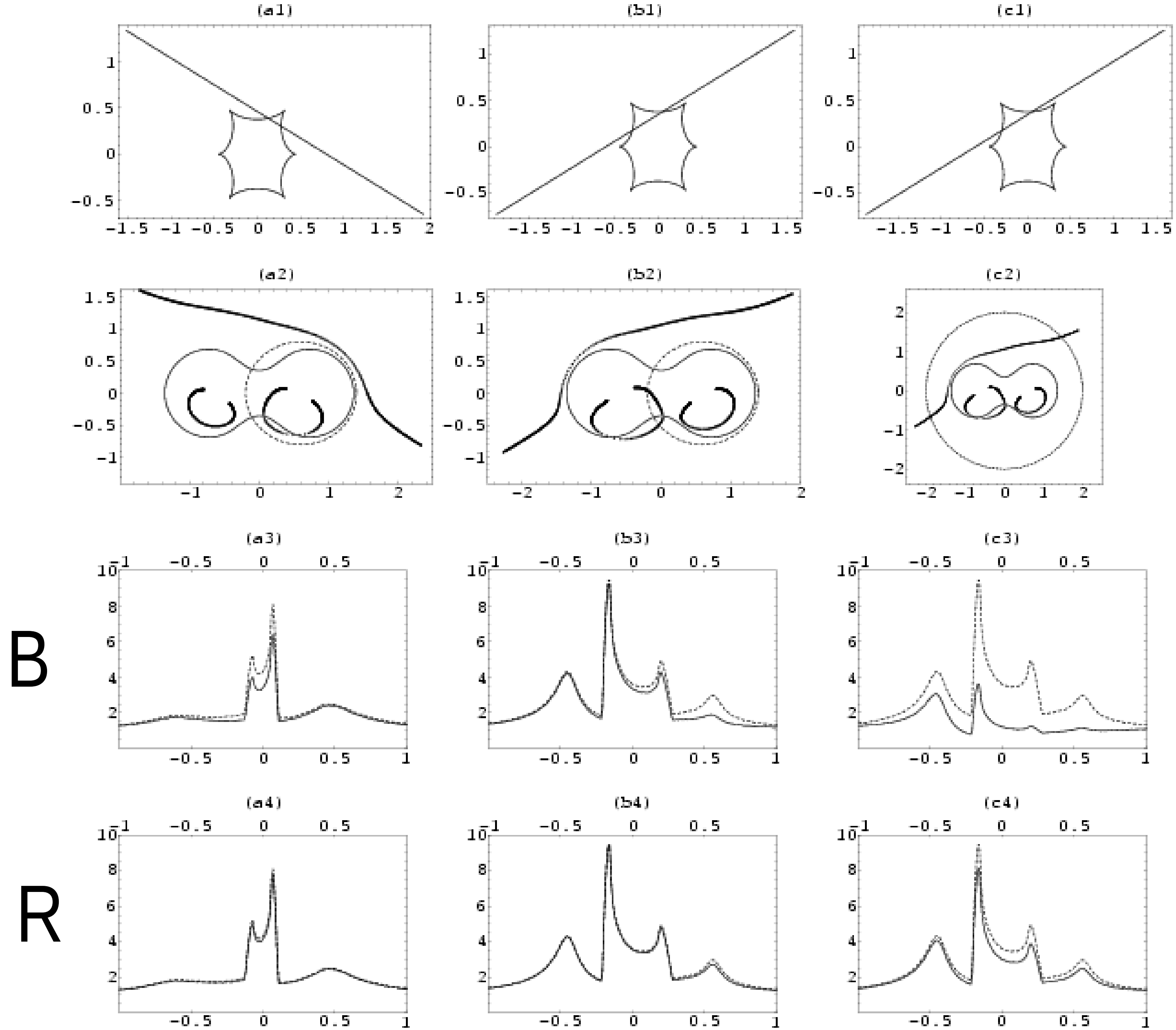
30/100 data points

$$A_{\max} = 1.73$$

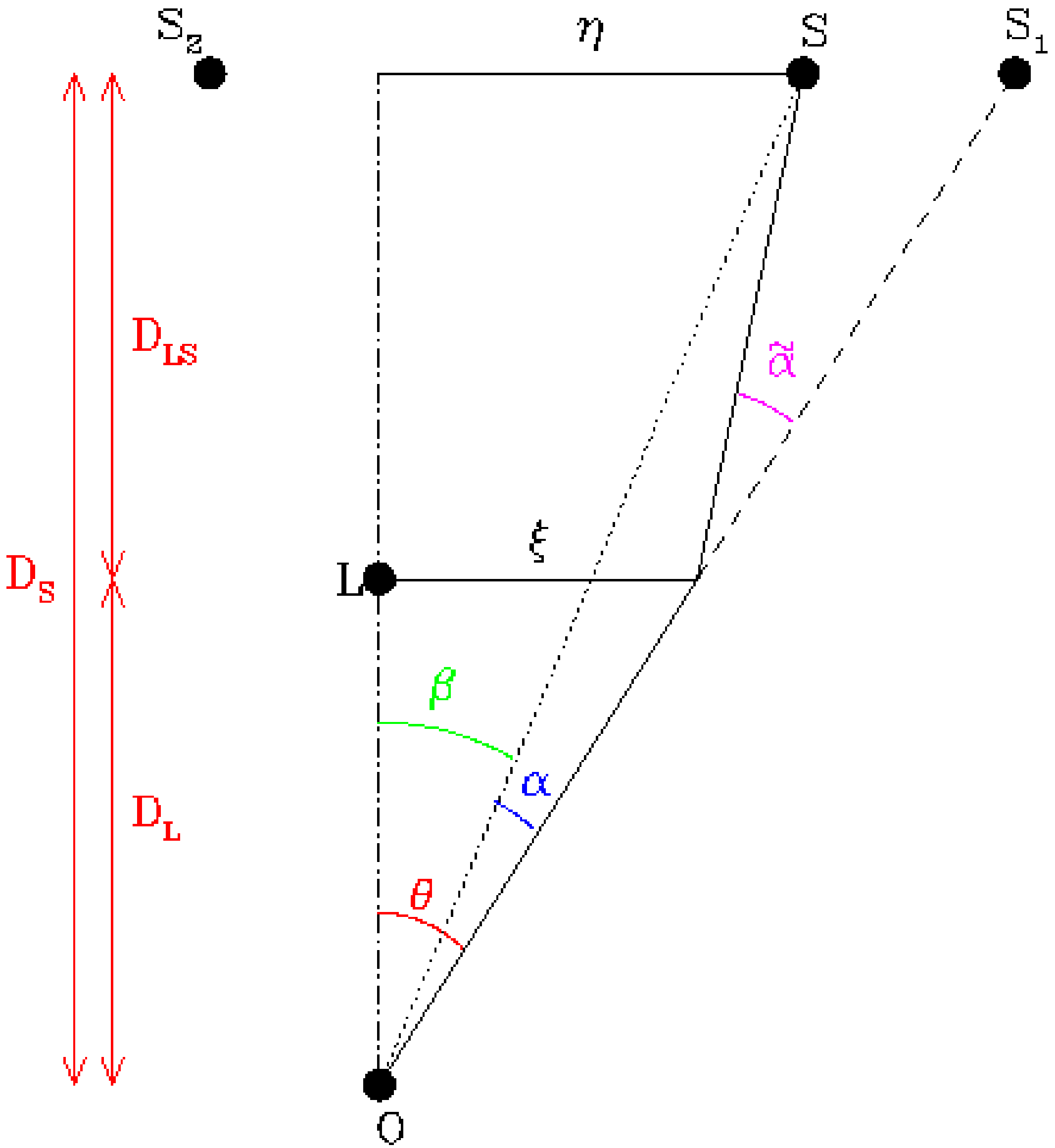
$$u_0 = 0.67$$

$$t_0 = 0.18P$$

$$\chi^2 / d.o.f = 1.32$$


(a1)
(b1)
(c1)
(a2)
(b2)
(c2)
(a3)
(b3)
(c3)
B
(a4)
(b4)
(c4)
R

Single Point Mass Lens



Einstein radius:

$$R_E = \sqrt{\frac{4GM_{tot}D_{LS}}{c^2 D_L D_S}}$$

Microlensing: the source and the images cannot be resolved

$$\phi = -\frac{GM}{r}$$

Gravitational potential

$$\vec{\alpha}' = \frac{4GM}{c^2 u^2}\vec{u}$$

u – angular distance of the light ray from the mass

$$\vec{\theta} D_S = \vec{\beta} D_S + \vec{\alpha}' D_{LS}$$

$$\vec{\alpha} = \frac{D_{LS}}{D_S}\vec{\alpha}'$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

Lens equation

for *n* point masses:

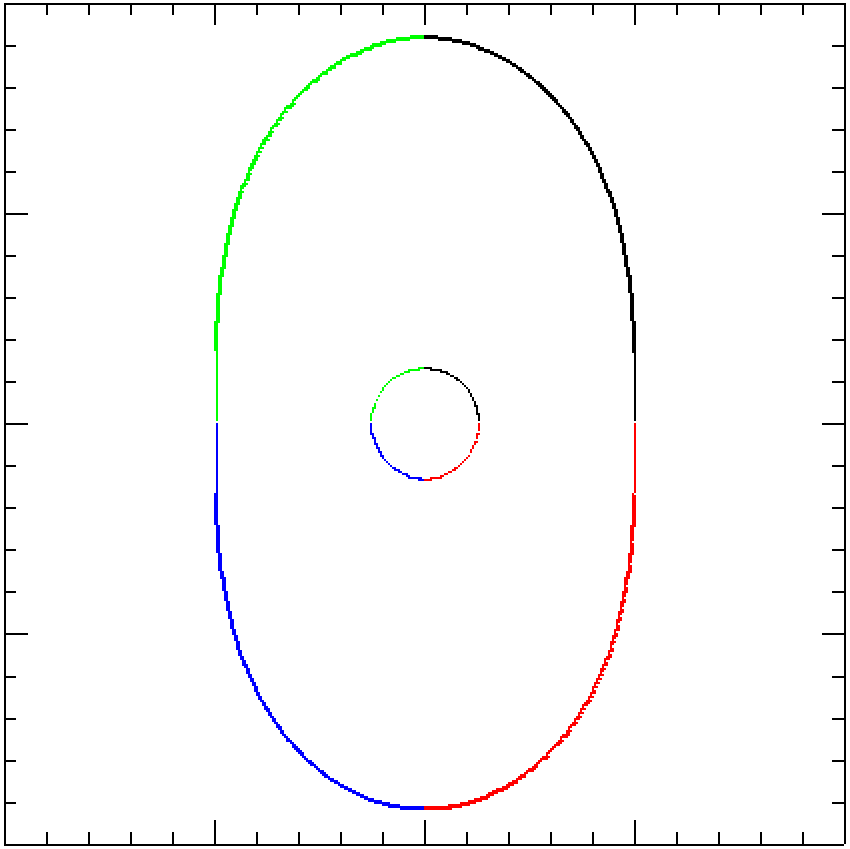
$$\vec{\alpha}(\vec{x}) = \frac{4G}{c^2} \sum_{i}^{n} m_i \frac{\vec{x} - \vec{x}_i}{|\vec{x} - \vec{x}_i|^2}$$

$$\vec{x} = \frac{\vec{\theta}}{\theta_{\mathrm{E}}},$$

$$\vec{y} = \frac{\vec{\beta}}{\theta_{\mathrm{E}}}$$

$$\vec{y} = \vec{x} - \sum_{i}^{n} m_i \frac{\vec{x} - \vec{x}_i}{|\vec{x} - \vec{x}_i|^2}$$

Critical curves



Caustics