Internal structure and rotation of the Sun from different observations

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THE FIRST SUMMER SCHOOL
IN ASTRONOMY AND GEOPHYSICS
BELGRADE, SERBIA, 06.08. - 10.08. 2007

ABSTRACT

In the lecture the basic information about internal structure of the Sun will be presented as well as its dynamics such as differential rotation and solar activity will be discussed from theoretical and observational points of view.

The study of the internal structure of the Sun can be performed by two direct methods: neutrino astronomy and helioseismology. The review of the main results about the distributions of the density, pressure, temperature, chemical composition inside the Sun, nuclear energy generation rate in the core will be given to provide the necessary base for the understanding of modern state of our knowledge about the nearest star providing the life on the Earth.

Variability of the Sun will be discussed. Solar activity cyclicity and solar dynamo models will be shortly presented as they are seen in different observational data.

Differential rotation as a function of latitude and solar depth was studied by helioseismological methods and by other methods.

Differential rotational rate of the magnetic field and its temporal dependence has been evidenced at different latitudes through activity cycles. The velocity of the meridional flows of the magnetic field was calculated.

The rotation of the plasma will be compared to the rotation rate of the large scale magnetic structures on the solar surface as well as in the bottom of the convective envelope.

Prospects of future research of solar dynamics will be discussed.

Experiments and materials

- Experiments:
- WSO, Magnetic field
- SOHO (MDI, GOLF)
- GONG, BISON, IRIS
- Homestake, Gallex
- GNO, Kamiokande
- SuperKamiokande
- SAGE, SNO, Borexino

Materials:

Cortesy of Prof. S.Solanki,
J.Christensen-Dalsgaard,
M.Tompson, S.Vorontsov,
A.Kosovichev, R.Howe,
V.Gavryusev, R.Komm,
C.P. Burgess, etc.

Structure of lectures I

THE DEPARTMENT OF ASTRONOMY, UNIVERSITY OF BELGRADE ANNOUNCES

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The Department of Astronomy, University of Belgrade is pleased to announce the 2007 Summer School in Astronomy and Geophysics. Four hours of lectures will be given each day, with evening workshops. The certification of satisfactory completion of the school will be given (without formal courses credits).

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APPLY WITH: 1. Your name, address, 2. Your year and stage of academic studies, 3. Your reasons to attend this school (no more than 100 words in English), 4. Recommendation letters of two persons, which have to be sent by email (see address below), 5. Mail separately university transcript listing grades achieved in all courses to date. PLEASE communicate by email items 1 to 4, and item 5 via mail. Write "Summer School" on all correspondence.

TOPICS:
Solar physics,
Astrobiology,
Earth rotation,
Geophysics

DEADLINE: Apply by 31.05.2007. to:

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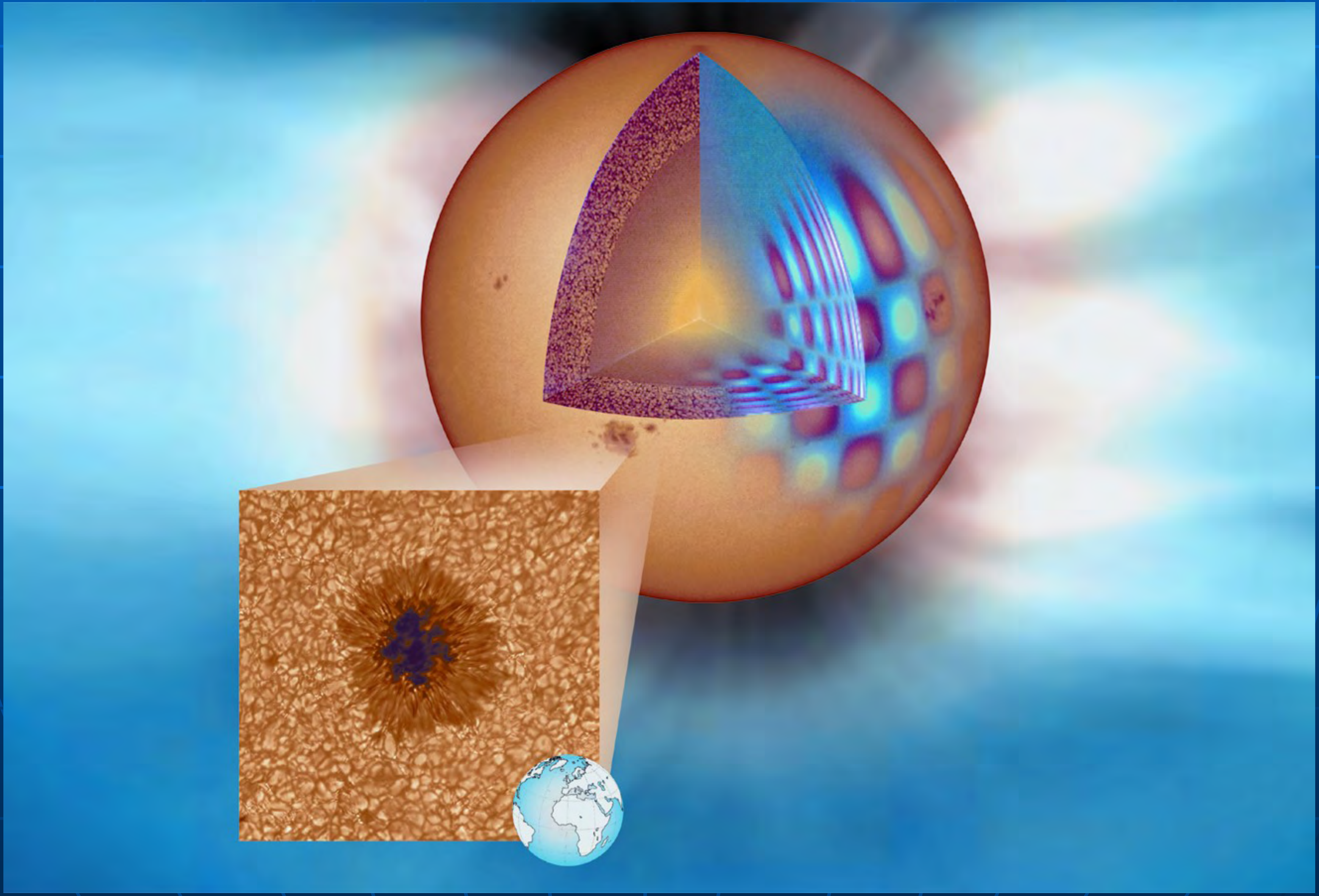
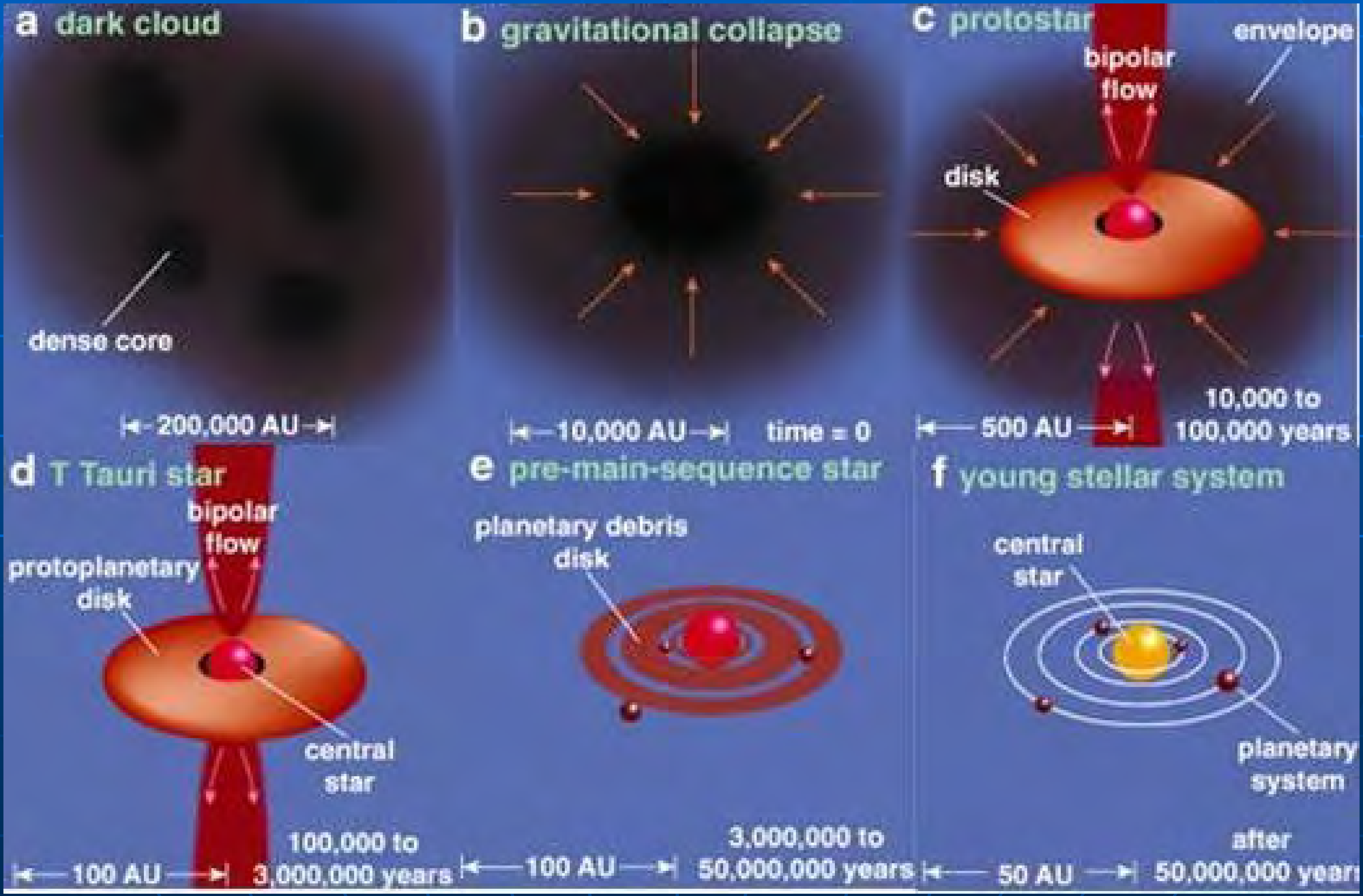
- Introduction and overview
- Core and interior: energy generation and standard solar model
- Convection zone
- Neutrino astronomy results
- Solar oscillations
- Helioseismology results
- Solar rotation

The Sun: a brief overview



Early life

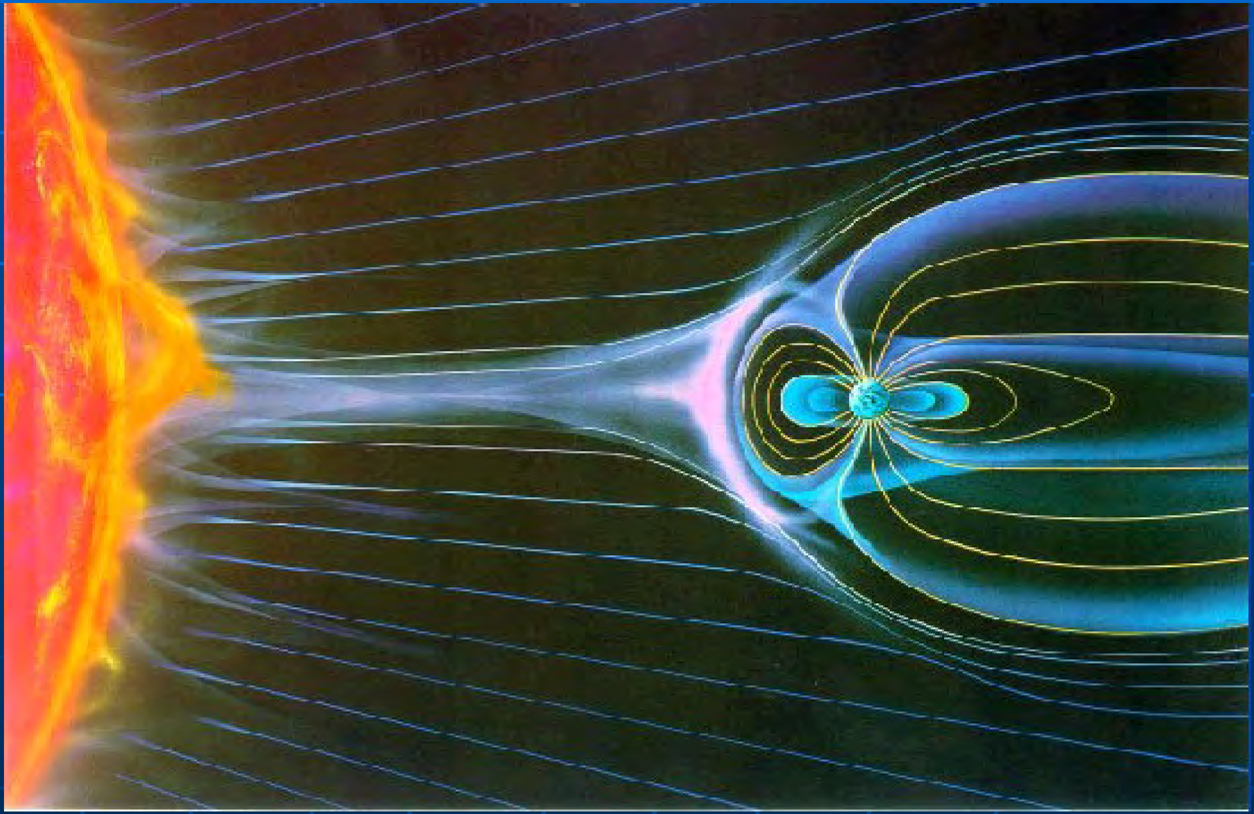

a dark cloud
dense core
200,000 AU
b gravitational collapse
10,000 AU
time = 0
c protostar
bipolar flow
envelope
disk
500 AU
10,000 to 100,000 years
d T Tauri star
bipolar flow
protoplanetary disk
central star
100 AU
100,000 to 3,000,000 years
e pre-main-sequence star
planetary debris disk
100 AU
3,000,000 to 50,000,000 years
f young stellar system
central star
planetary system
50 AU
after 50,000,000 years

The Sun, our star

- The Sun is a normal star: middle aged (4.5 Gyr) main sequence star of spectral type G2
- The Sun is a special star: it is the only star on which we can resolve the spatial scales on which fundamental processes take place.
- The Sun is a special star: it provides almost all the energy to the Earth
- The Sun is a special star: it provides us with a unique laboratory in which to learn about various branches of physics.

Sun, Earth and planets

- Solar output affects the magnetospheres and atmospheres of planets
- Solar energy is
- responsible for
- providing a
- habitable
- Environment
- on Earth and
- liquid water
- Mars...



The Sun: Overview

The Sun: a few numbers

- Mass = $1.99\ 10^{30}$ kg (= 1 $M_\odot$)
- Average density = 1.4 g/cm^3
- Luminosity = $3.84\ 10^{26}$ W (= 1 $L_\odot$)
- Effective temperature = 5777 K (G2 V)
- Core temperature = $15\ 10^6$ K
- Surface gravitational acceleration g = 274 m/s^2
- Age = $4.55\ 10^9$ years (from meteorite isotopes)
- Radius = $6.96\ 10^5$ km
- Distance = 1 AU = 1.496 (+/-0.025) 10^8 km
- 1 arc sec = 722±12 km on solar surface (elliptical Earth orbit)
- Rotation period = 27 days at equator (sidereal, i.e. as seen from Earth; Carrington rotation)

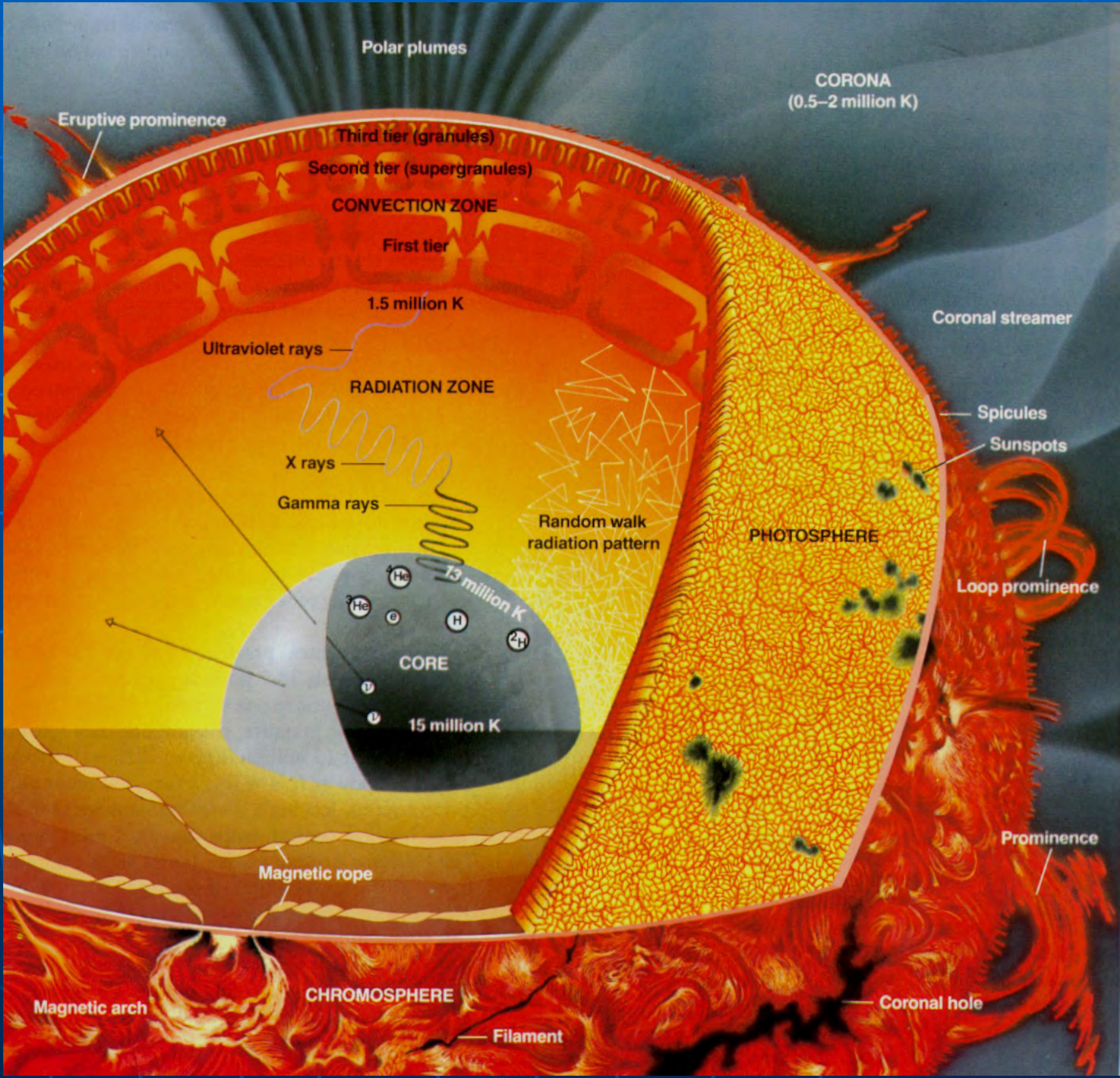
The Sun’s Structure

Solar interior:

- Everything below the Sun’s (optical) surface
- Divided into hydrogen-burning core, radiative and convective zones

Solar atmosphere:

- Directly observable part of the Sun.
- Divided into photosphere, chromosphere, corona, heliosphere



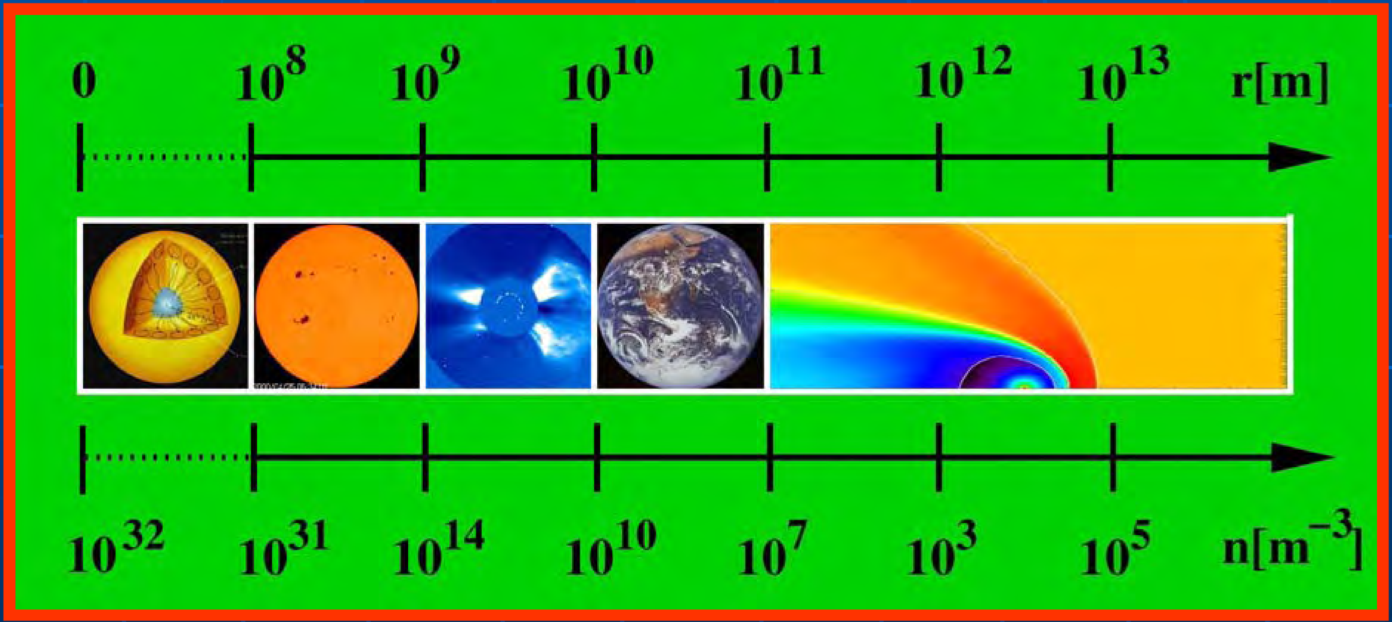
The solar surface

- Since solar material does not exhibit a phase transition (e.g. from solid or liquid to gaseous as for the Earth), a standard way to define the solar surface is through its radiation.
- The photons travelling from the core outwards make a random walk, since they are repeatedly absorbed and reemitted. The mean free path increases rapidly with radial distance from the solar core (as the density and opacity decrease).
- A point is reached where the average mean free path becomes so large that the photons escape from the Sun. This point is defined as the solar surface. It corresponds to optical depth $\tau = 1$. Its height depends on λ.
- Often $\tau = 1$ at λ=5000 Å is used as a standard for the solar surface.

Solar physics in relation to other branches of physics

Wide range of physical parameters

- The Sun presents a wide variety of physical phenomena and processes, between solar core and corona.
- E.g. Gas density varies by ≈ 30 orders of magnitude, temperature by 4 orders, relevant time scales from 10^{-10} sec to 10 Gyr



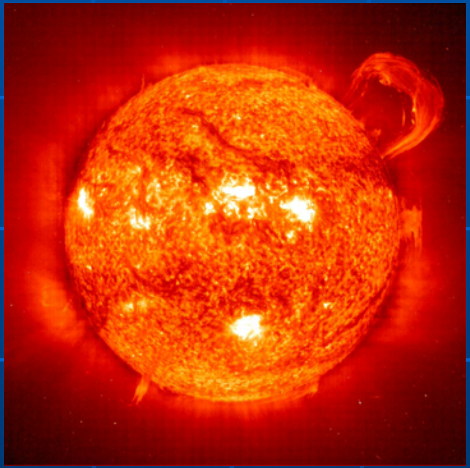
→ Different observational and theoretical techniques needed to study different parts of Sun, e.g. helioseismology & nuclear physics for interior, polarimetry & MHD for magnetism, etc.

Solar Physics in Relation to Other Fields

Sun-Earth relations: climate, space weather

Turbulence, dynamos

Cosmic rays, local interstellar medium



Plasma physics

Solar planets, extrasolar planets

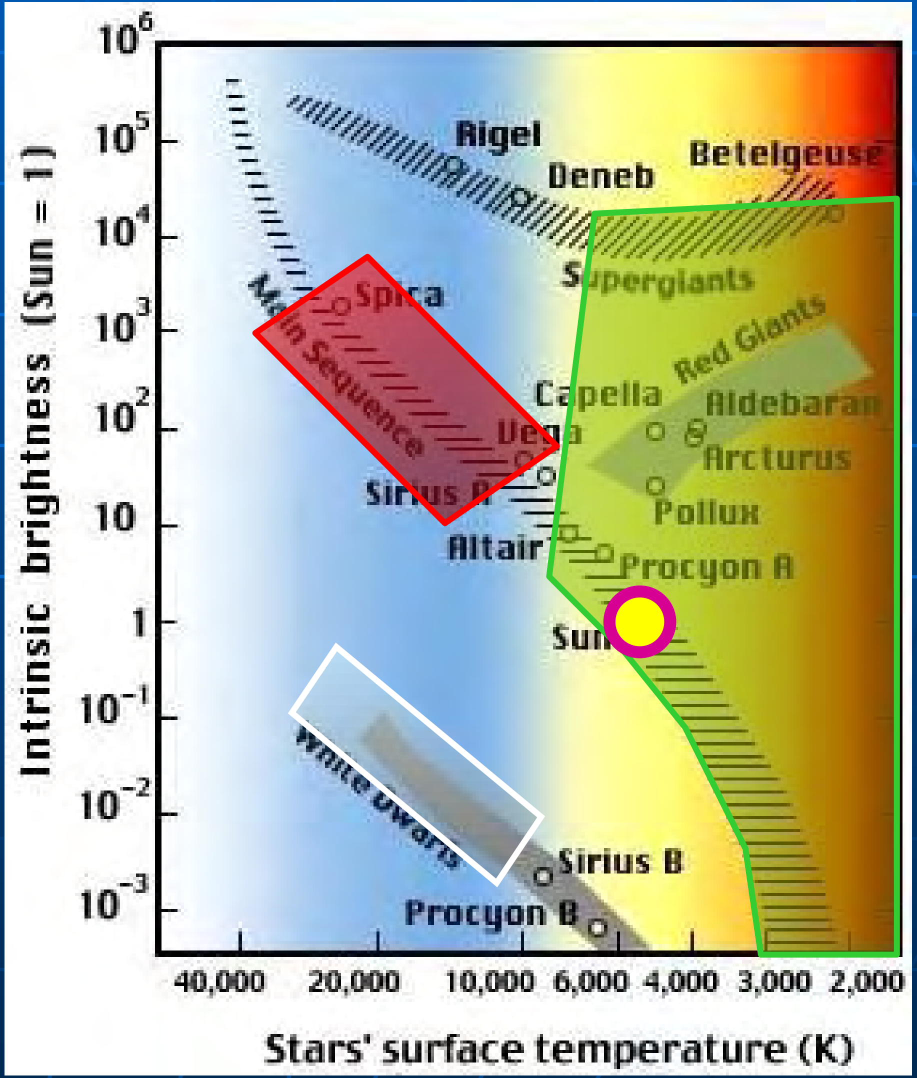
Atomic/molecular physics

Cool stars: activity, structure & evolution

Fundamental physics: Neutrinos, Gravitation

Which stars have magnetic fields or show magnetic activity?

- Best studied star: Sun
- F, G, K & M stars (outer convection zones) show magnetic activity & have <B> fields of G-kG.
- Early type stars: Ap, Bp, (kG-100kG), Be (100G)
- White dwarfs have B ≈ kG-10^9 G, no activity
- Not on diagram: pulsars



Solar Tests of Gravitational Physics

- Curved light path in solar gravitational field ➔ Test of General Relativity
- Red shift of solar spectral lines ➔ Test of EEP
- Oblate shape of Sun ➔ Quadrupol moment of solar gravitational field: Test of Brans-Dicke theory
- Comparison of solar evolution models with observations ➔ Limits on evolution of fundamental constants
- Polarization of solar spectral lines: ➔ Tests of equivalence principle & alternative theories of gravity

The Sun and particle physics

- The fact that the rate of neutrinos measured by the Homestake ^{37}Cl detector is only 1/3 of that predicted by standard solar models was for > 30 years one of the major unsolved problems of physics.
- Possible resolutions:
  - Standard solar model is wrong
  - Neutrino physics is incomplete
- Recent findings from SNO and Superkamiokande: Problem lies with the neutrino physics

➔ Standard model of particle physics needs to be revised

- Nobel prize 2002 for R. Davies for discovery of the solar neutrino problem.

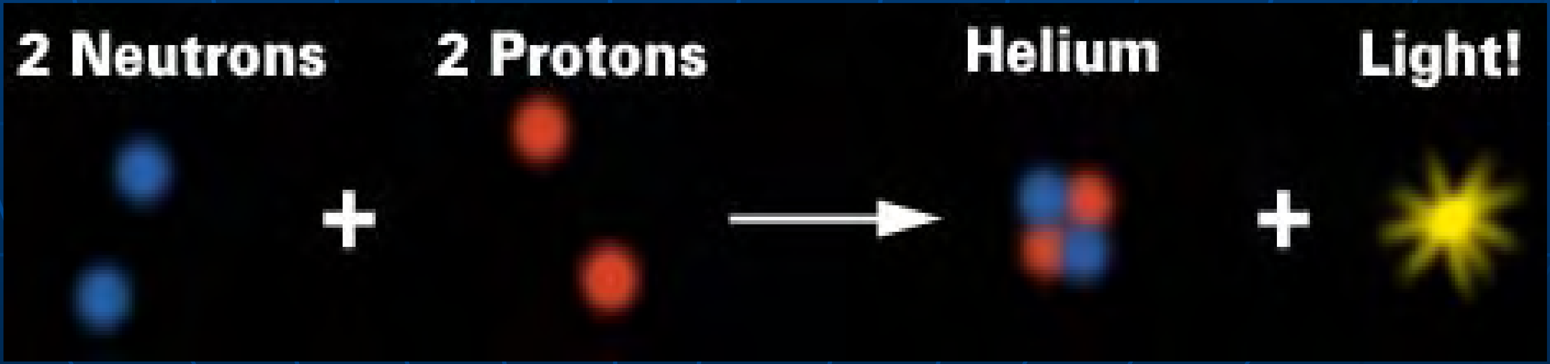
The solar interior

Model of the Sun,

cortesy of J.Christensen Dalsgaard, 2006

The Sun’s core

- In the Sun’s core mass is turned into energy.
- Nuclear reactions burn $7x10^{11}$ kg/s of hydrogen into helium.
- Inside the core the particle density and temperature are so high, that individual protons ram into each other at sufficient speed to overcome the Coulomb barrier, forming heavier He atoms and releasing energy



Nuclear reactions in cores of stars

- Sun gains practically all its energy from the reaction

$$4p \rightarrow \alpha + 2e^{+} + 2\nu = {}^{4}He + 2e^{+} + 2\nu$$

- Two basic routes
  - p-p chain: yields about 99% of energy in Sun
  - CNO cycle : 1% of energy released in present day Sun (but dominant form of energy release in hotter stars)
- Both chains yield a total energy Q of 26.7 MeV, mainly in the form of γ-radiation Q_γ (which is absorbed and heats the gas) and neutrinos Q_ν (which escapes from the Sun).

Nucle

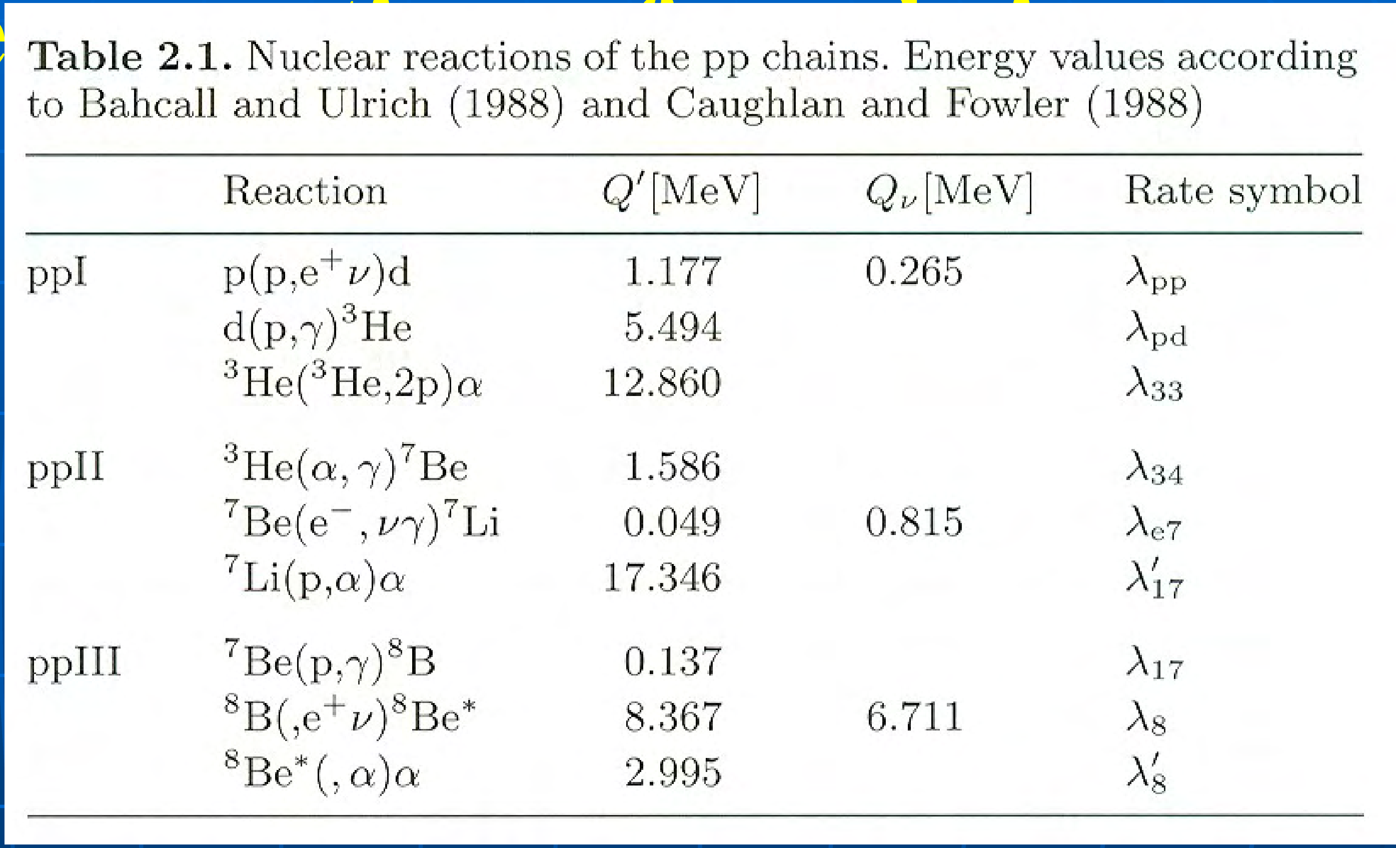


Table 2.1. Nuclear reactions of the pp chains. Energy values according to Bahcall and Ulrich (1988) and Caughlan and Fowler (1988)

	Reaction	Q'[MeV]	Q_ν[MeV]	Rate symbol
ppI	$p(p,e^+\nu)d$	1.177	0.265	λ_{pp}
	$d(p,\gamma)^3He$	5.494		λ_{pd}
	$^3He(^3He,2p)\alpha$	12.860		λ_{33}
ppII	$^3He(\alpha,\gamma)^7Be$	1.586		λ_{34}
	$^7Be(e^-,\nu\gamma)^7Li$	0.049	0.815	λ_{e7}
	$^7Li(p,\alpha)\alpha$	17.346		λ'_{17}
ppIII	$^7Be(p,\gamma)^8B$	0.137		λ_{17}
	$^8B(,e^+\nu)^8Be^*$	8.367	6.711	λ_8
	$^8Be^*(,\alpha)\alpha$	2.995		λ'_8

- p=proton
- d=deuterium
- α=Helium
- γ=radiation
- ν=neutrino
- 2nd reaction replaces step 3 of 1st reaction
- 3rd reaction replaces steps 2+3 of 2nd reaction
- Branching ratios:
  - 1st vs. 2nd + 3rd 87 : 13
  - 2nd vs. 3rd → 13 : 0.015

Nuclear reactions of CNO-cycle

- C, N and O act only as catalysts: Basically the same things happens as with proton chain.

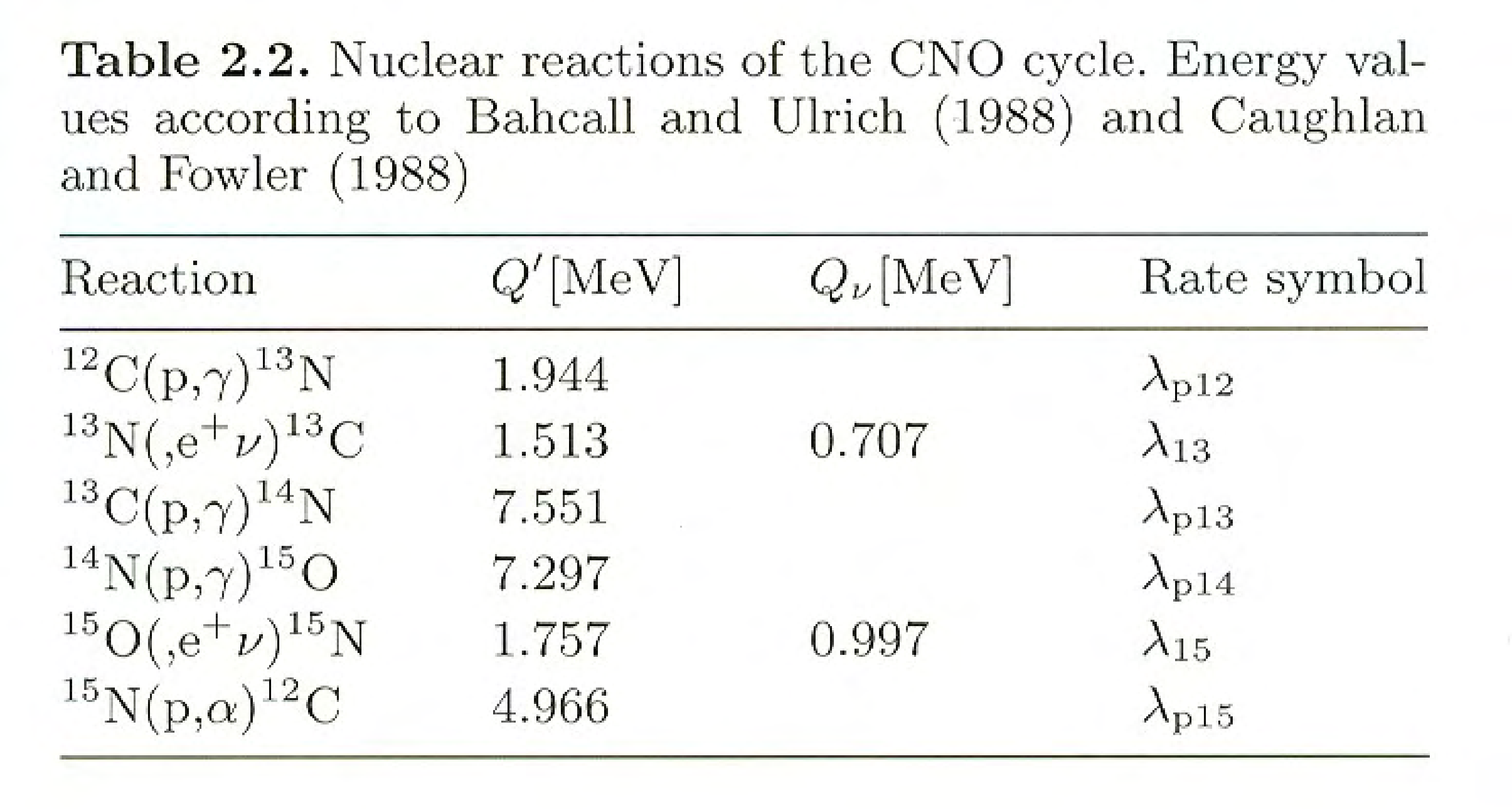
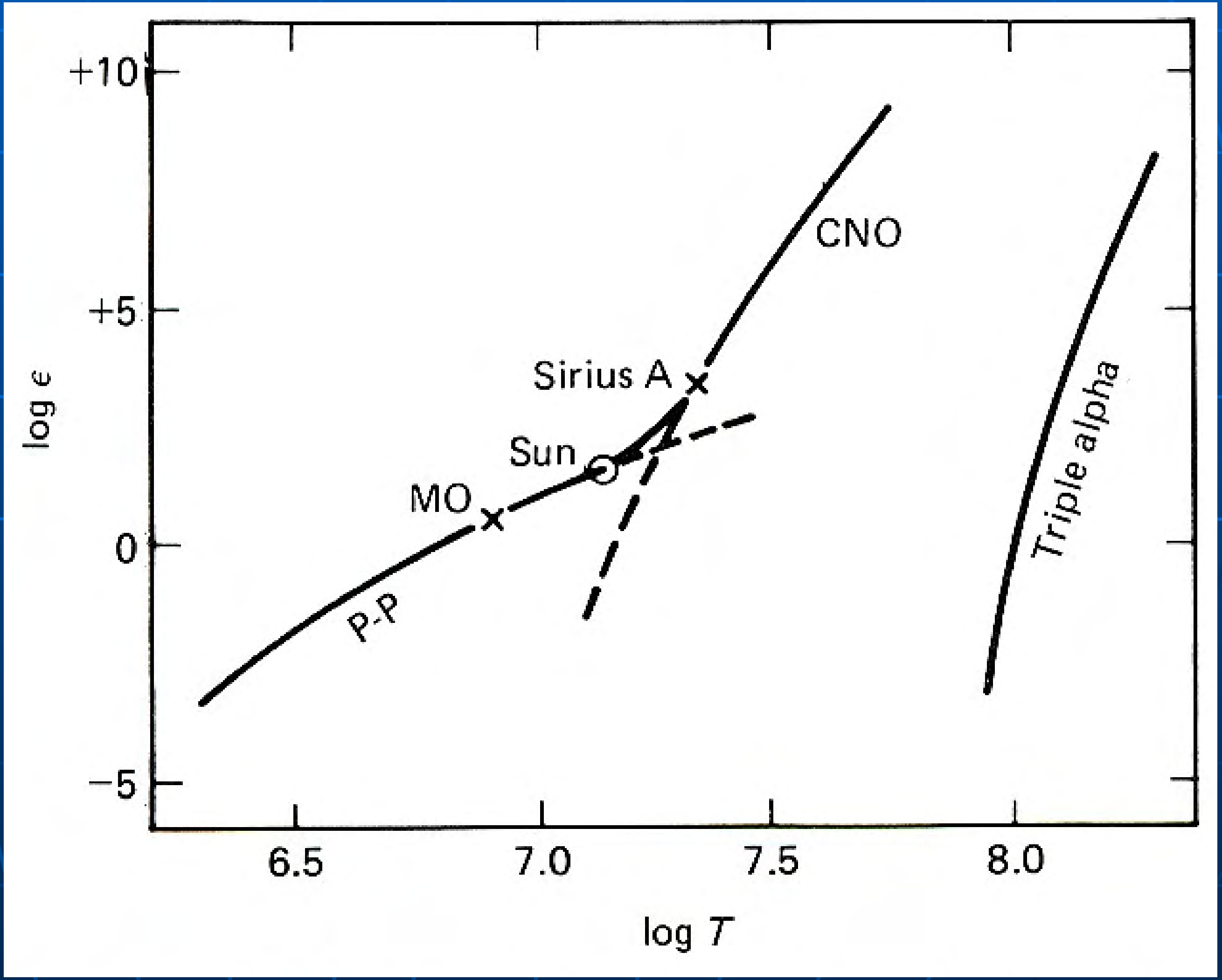


Table 2.2. Nuclear reactions of the CNO cycle. Energy values according to Bahcall and Ulrich (1988) and Caughlan and Fowler (1988)

Reaction	Q' [MeV]	Q_ν [MeV]	Rate symbol
$^{12}C(p,\gamma)^{13}N$	1.944		λ_{p12}
$^{13}N(,e^+\nu)^{13}C$	1.513	0.707	λ_{13}
$^{13}C(p,\gamma)^{14}N$	7.551		λ_{p13}
$^{14}N(p,\gamma)^{15}O$	7.297		λ_{p14}
$^{15}O(,e^+\nu)^{15}N$	1.757	0.997	λ_{15}
$^{15}N(p,\alpha)^{12}C$	4.966		λ_{p15}

Temperature dependence of pp-chain and CNO cycle

- p-p chain in cool main-sequence stars
- CNO cycle in hot main-sequence stars
- Triple alpha process in red giants: 3He → C



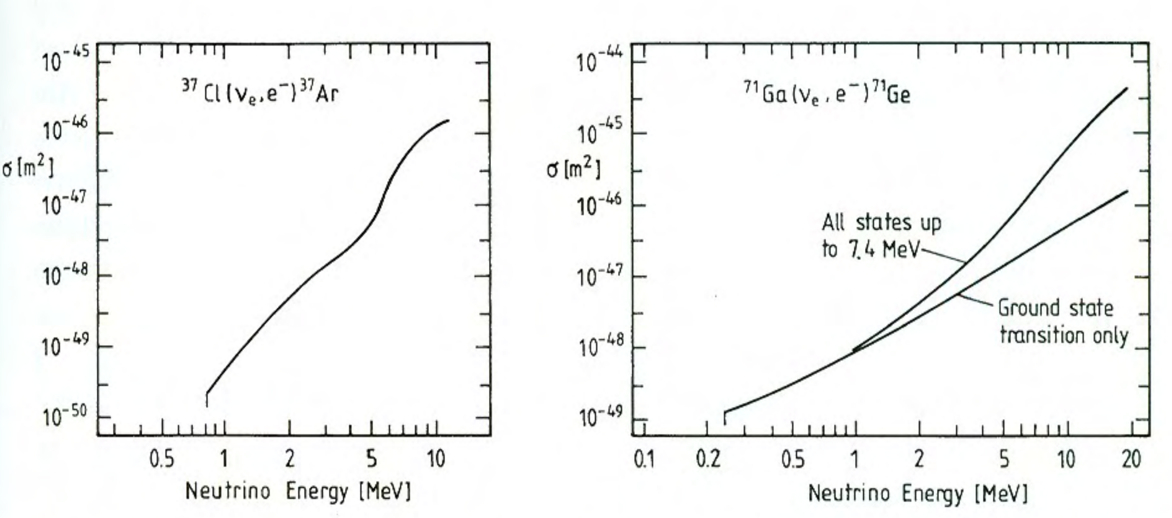
Solar neutrinos II

- Since 1968 the Homestake ^{37}Cl experiment has given a value of 2.1 ± 0.3 snu (1snu =1 ν / 10^{36} target atoms)
- Standard solar models predict: 7±2 snu

➔Solar Neutrino Problem!

- In 1980s & 90s water based Kamiokande and larger Superkamiokande detectors found that approximately half the rare, high energy ^{8}B ν were missing.
- ^{71}Ga experiments (GALLEX at Gran Sasso and SAGE in Russia) showed that the neutrino flux was too low, even including the $p(p,e^{+}\nu)d$ neutrinos.

Solar neutrinos III



- Sensitivity of H_2O, ^{37}Cl

➔Homestake ^{37}Cl detector and (Super-) Kamiokande see mainly high-energy ν from rare β^+-decay of ^8B.

- Branching ratios between the various chains: central for predicting exact ν-flux detectable by ^{37}Cl & H_2O
- Branching ratios depend very sensitively on T(r=0), while total ν-flux depends only linearly on luminosity.
- Even ^{71}Ga experiments sensitve largely to high energy ν.

Solar Neutrinos IV

- Possible solutions to solar neutrino problem:
  - Standard solar model is incorrect (5-10% lower temperature in core gives neutrino flux consistent with Homestake detector).
  - Neutrino physics is incomplete (i.e. the standard model of particle physics is wrong!)
  - Nuclear physics describing the pp-chain is incorrect
  - Nuclear physics describing interaction between neutrino and ^{37}Cl is incorrect (Kamiokande & ^{71}Ga showed that this wasn’t the problem)

Resolution of neutrino problem

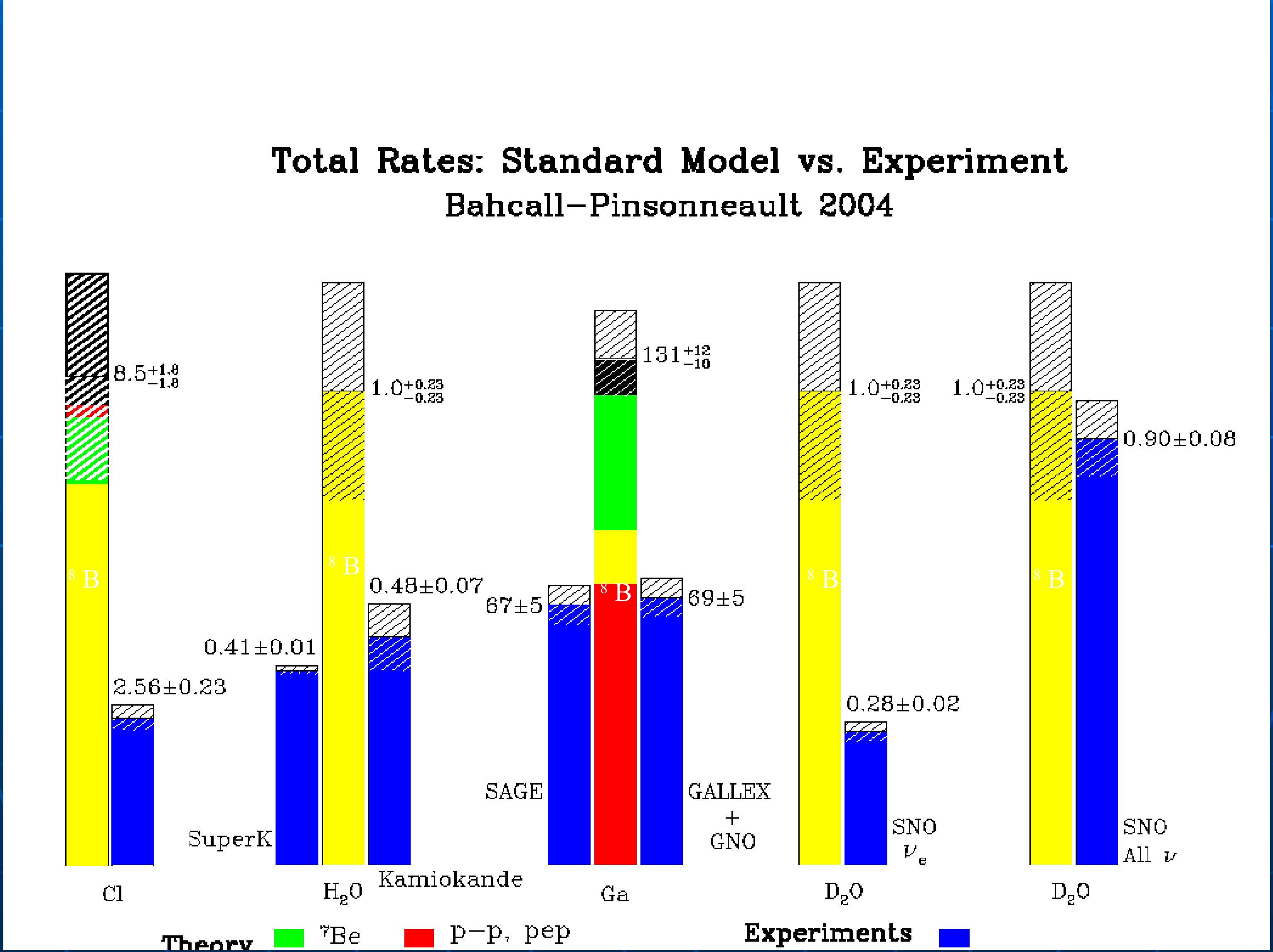
- SNO (Sudbury Neutrino Observatory) in Sudbury, Canada uses D_2O and can detect not just the electron neutrino, but also μ and τ neutrinos

➔ The neutrinos aren’t missing, e^- neutrinos produced in the Sun just convert into μ and τ neutrinos

➔ The problem lies with the neutrino physics.

- The neutrino has a small rest mass (10^{-8} m_e), which allows it to oscillate between the three flavours: e^- neutrino, μ neutrino and τ neutrino (proposed 1969 by russian theorists: Bruno Pontecorvo and Vladimir Gribov, … but nobody believed them)
- Confirmation by measuring anti-neutrinos from power plant (with Superkamiokande).

Neutrino results



Neutrino oscillations

$$\nu_{\mathrm{e}} \longleftrightarrow \nu_\mu \longleftrightarrow \nu_\tau$$

Neutrinos produced: ν_{e}

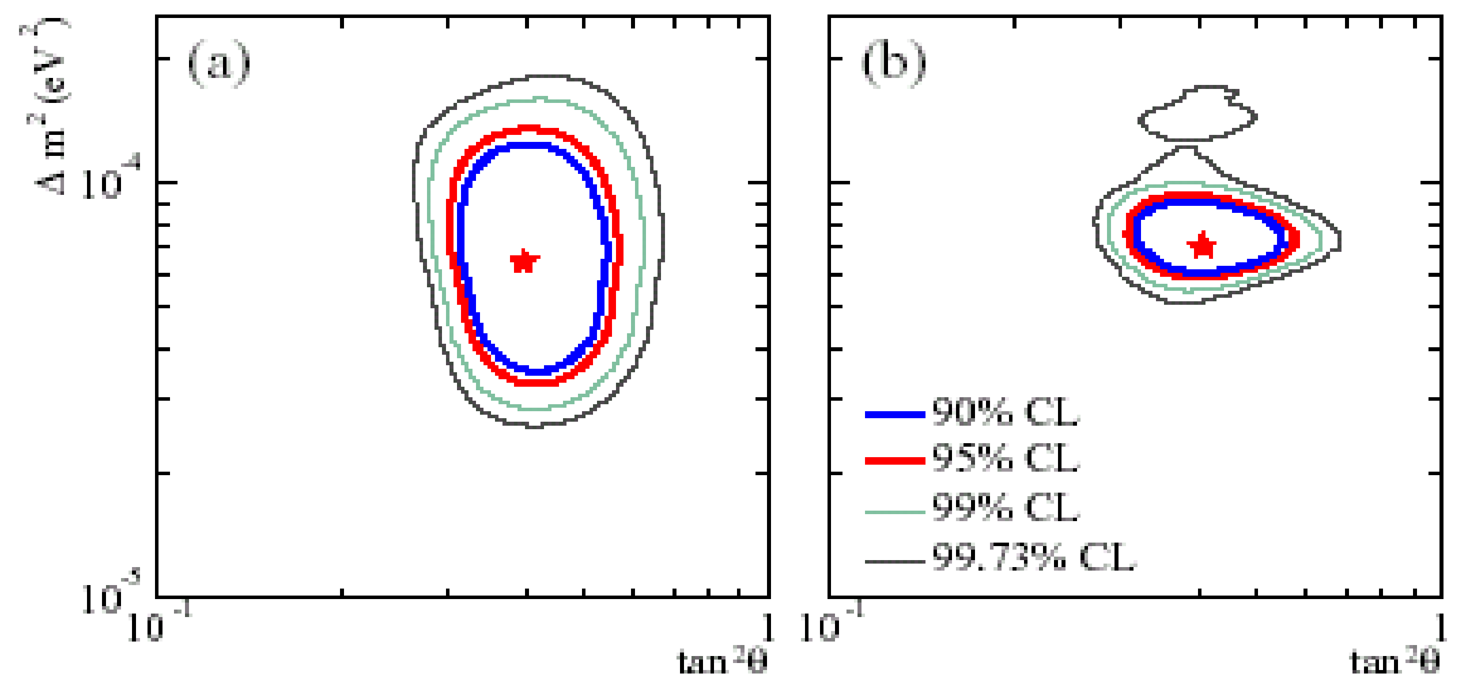
Neutrinos detected:

- $^{37}\mathrm{Cl}, {}^{71}\mathrm{Ga}$: ν_{e}
- H_2O: ν_{e} (and some ν_μ or ν_τ)

Sudbury Neutrino Observatory (heavy-water detector):

$$\begin{aligned} \nu_{\mathrm{e}} + {}^2\mathrm{D} &\to {}^1\mathrm{H} + {}^1\mathrm{H} + \mathrm{e}^- && \text{(CC)}\ , \\ \nu_x + {}^2\mathrm{D} &\to {}^1\mathrm{H} + \mathrm{n} + \nu_x && \text{(NC)}\ , \\ \nu_x + \mathrm{e}^- &\to \nu_x + \mathrm{e}^- && \text{(ES)} \end{aligned}$$

After Seasoning with Salt



Allowed oscillation parameters after SNO salt

Resolution of neutrino problem II

- Lesson learnt: neutrinos have a multiple personality problem (J. Bahcall)
- Other lesson learnt: the "dirty" and difficult solar model turned out to be correct, the clean and beautiful standard theory of particle physics turned out to be wrong, or at least incomplete (J. Bahcall)
- 2002: Raymond Davis got Nobel prize for uncovering the neutrino problem

Neutrino generation

Basic net reaction: $4\,{}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2\mathrm{e}^+ + \boxed{2\nu_\mathrm{e}}$

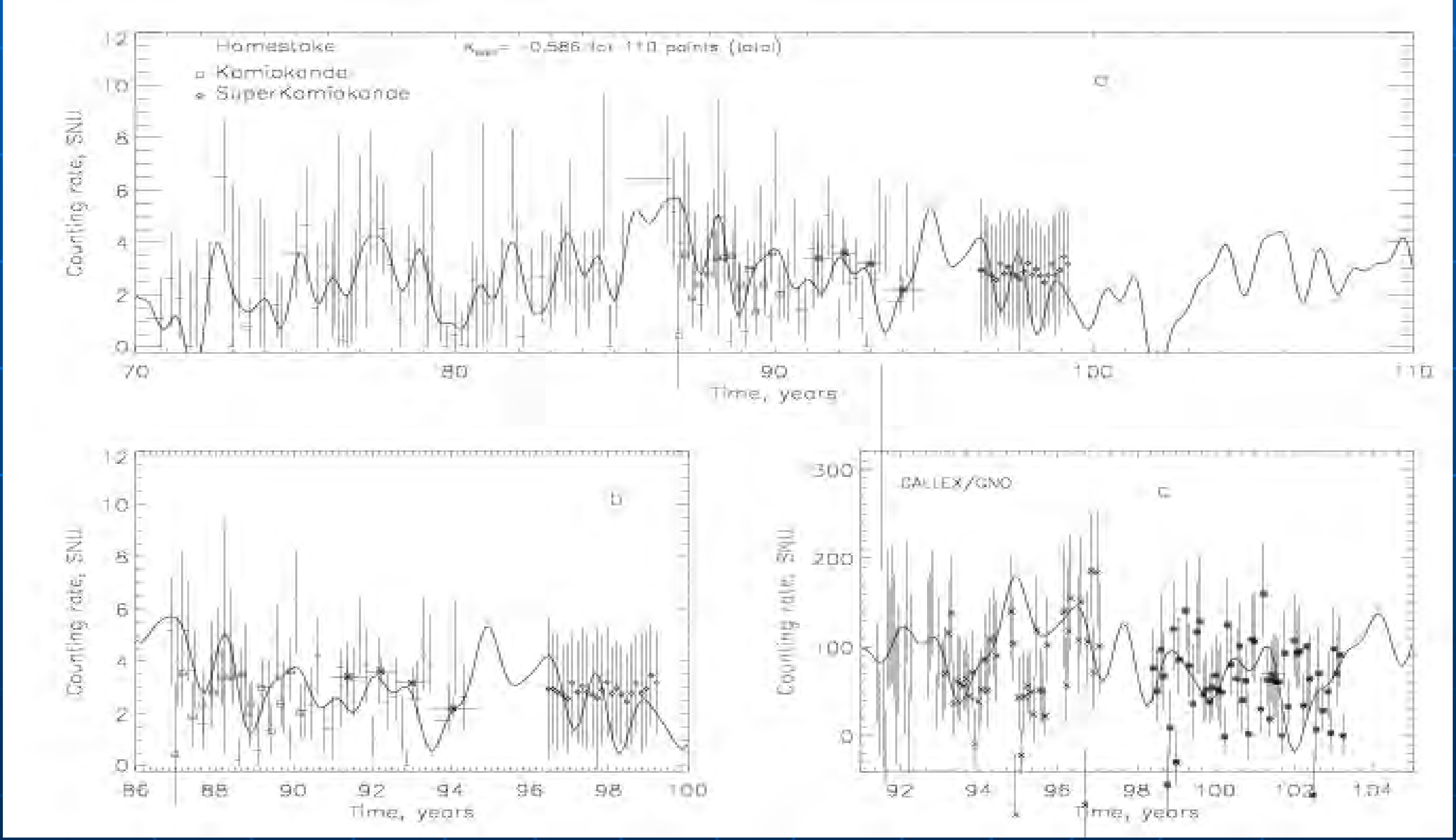
$${}^1\mathrm{H}({}^1\mathrm{H}, \mathrm{e}^+\boxed{\nu_\mathrm{e}})\,{}^2\mathrm{D}({}^1\mathrm{H}, \gamma)\,{}^3\mathrm{He}({}^3\mathrm{He}, 2\,{}^1\mathrm{H})\,{}^4\mathrm{He} \qquad \text{(PPI)}$$

$${}^3\mathrm{He}({}^4\mathrm{He}, \gamma)\,{}^7\mathrm{Be}(\mathrm{e}^-, \boxed{\nu_\mathrm{e}})\,{}^7\mathrm{Li}({}^1\mathrm{H}, {}^4\mathrm{He})\,{}^4\mathrm{He} \qquad \text{(PPII)}$$

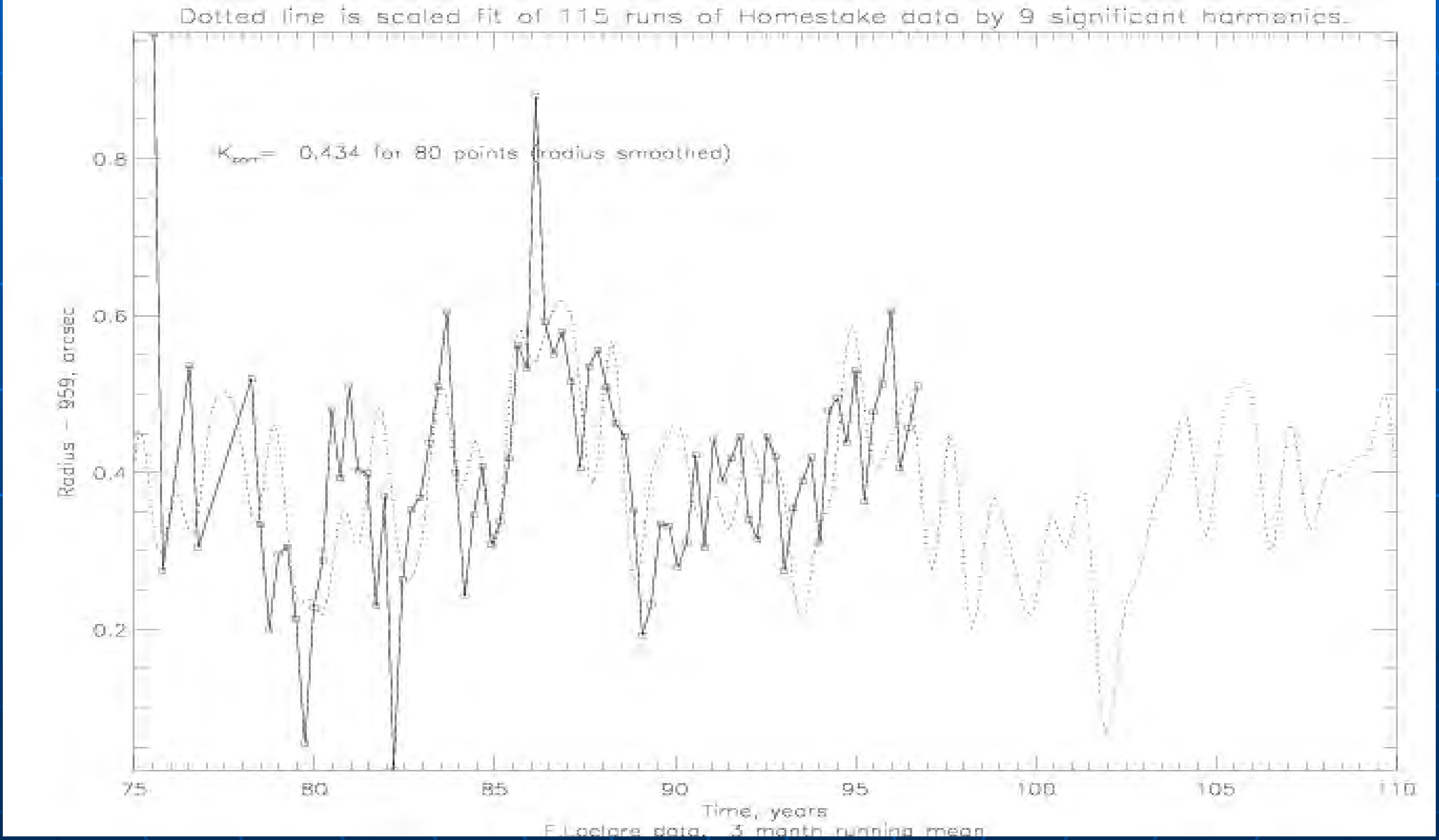
$$\Downarrow$$

$${}^7\mathrm{Be}({}^1\mathrm{H}, \gamma)\,{}^8\mathrm{B}(, \mathrm{e}^+\boxed{\nu_\mathrm{e}})\,{}^8\mathrm{Be}(, {}^4\mathrm{He})\,{}^4\mathrm{He} \qquad \text{(PPIII)}$$

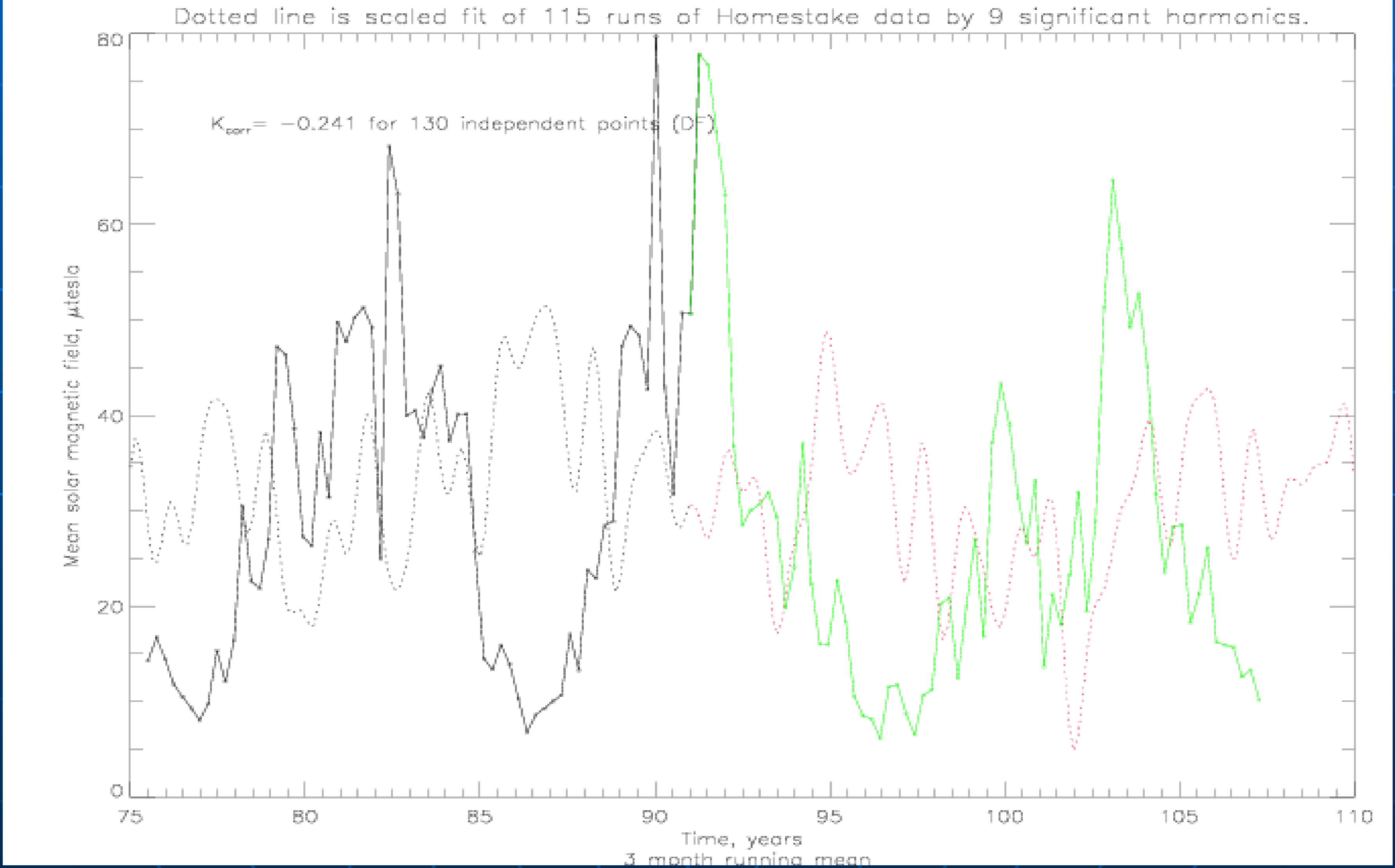
Neutrino detectors and Phenomenological model



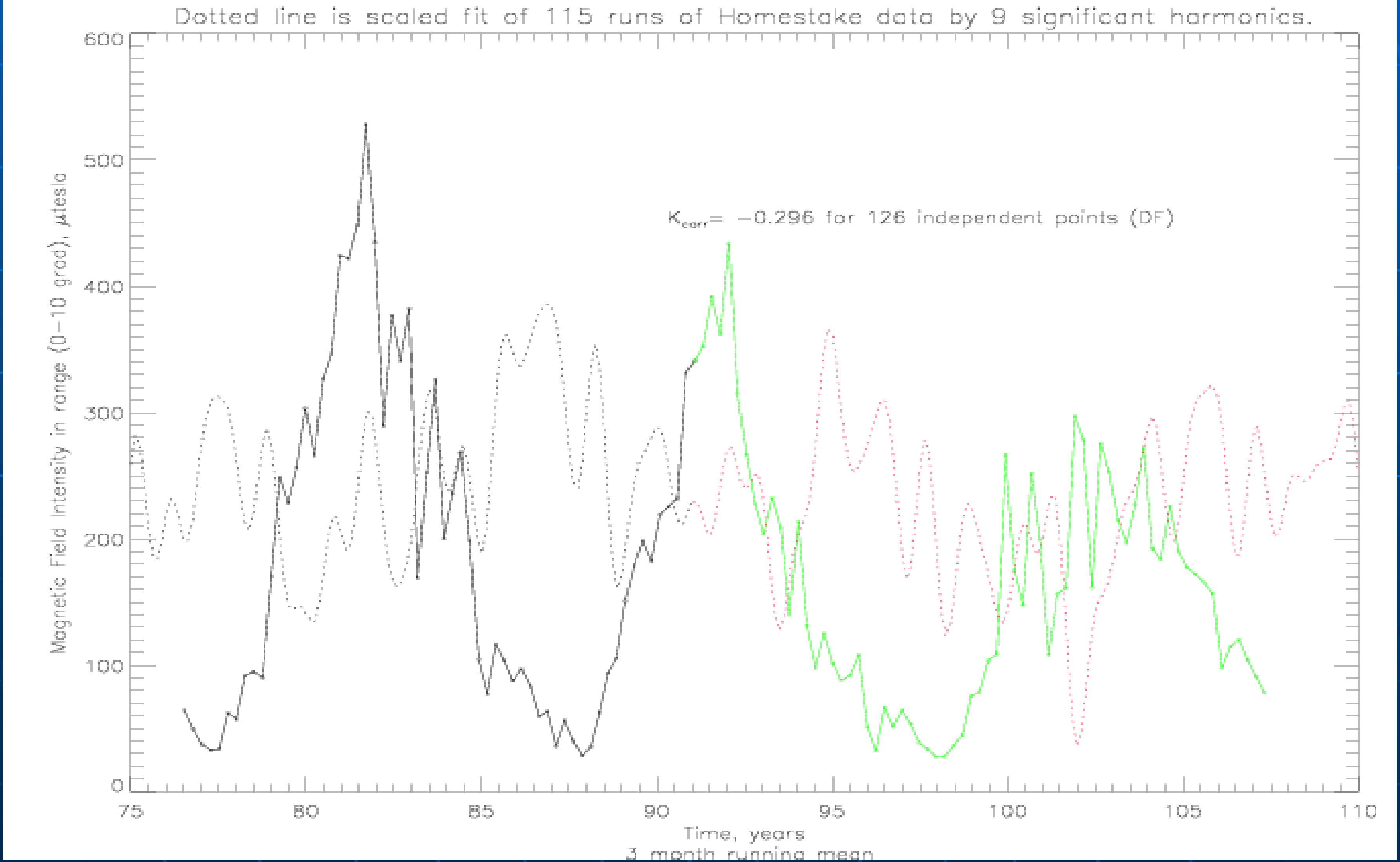
Radius and Phenomelogical Model of Neutrino counting rate

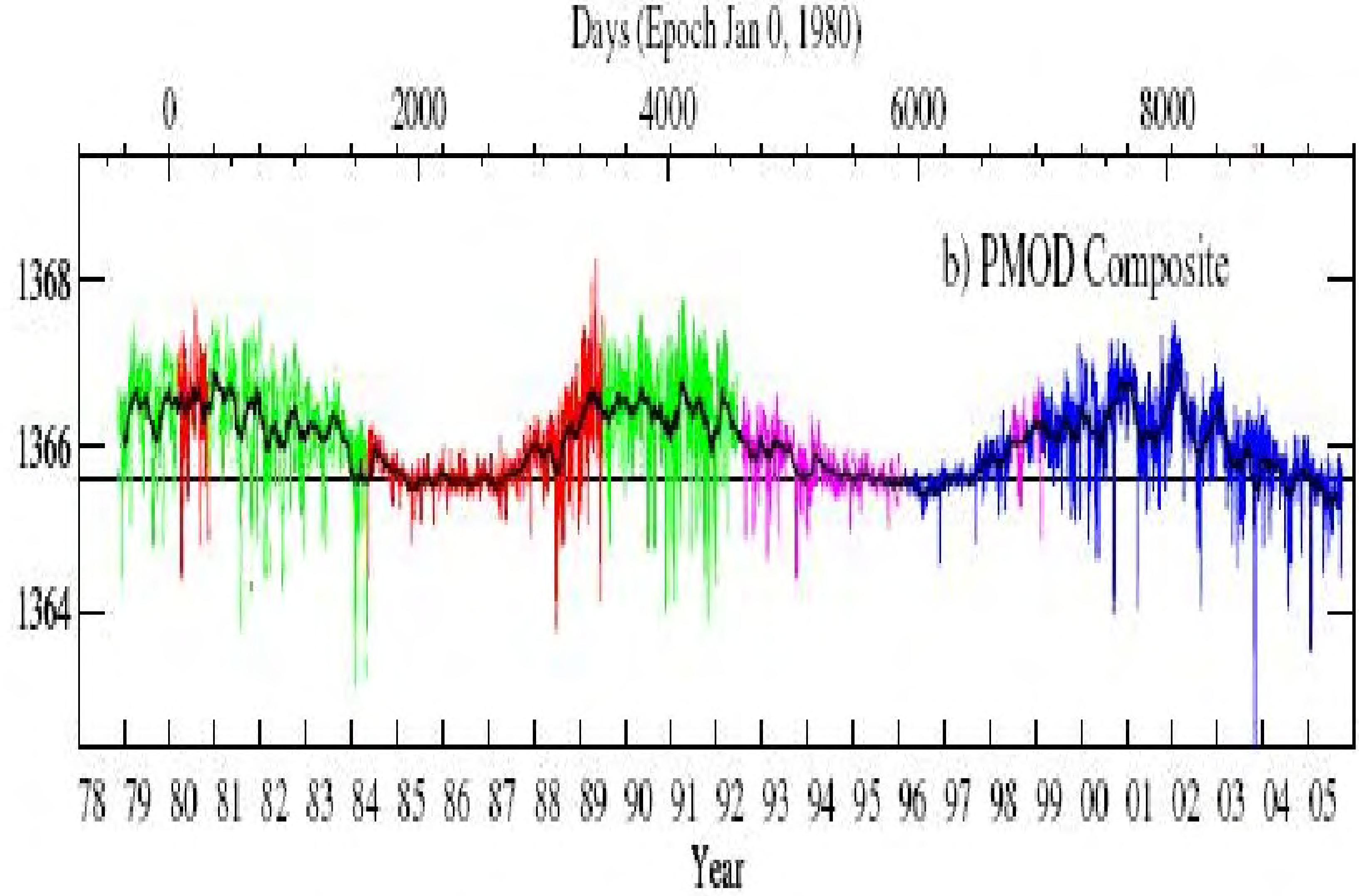


Mean solar magnetic field and Phenomenological model of neutrino counting rate variability



Magnetic field intensity in +/- 10 degrees equatorial zone and Phenomenological model of Neutrino flux variability in time



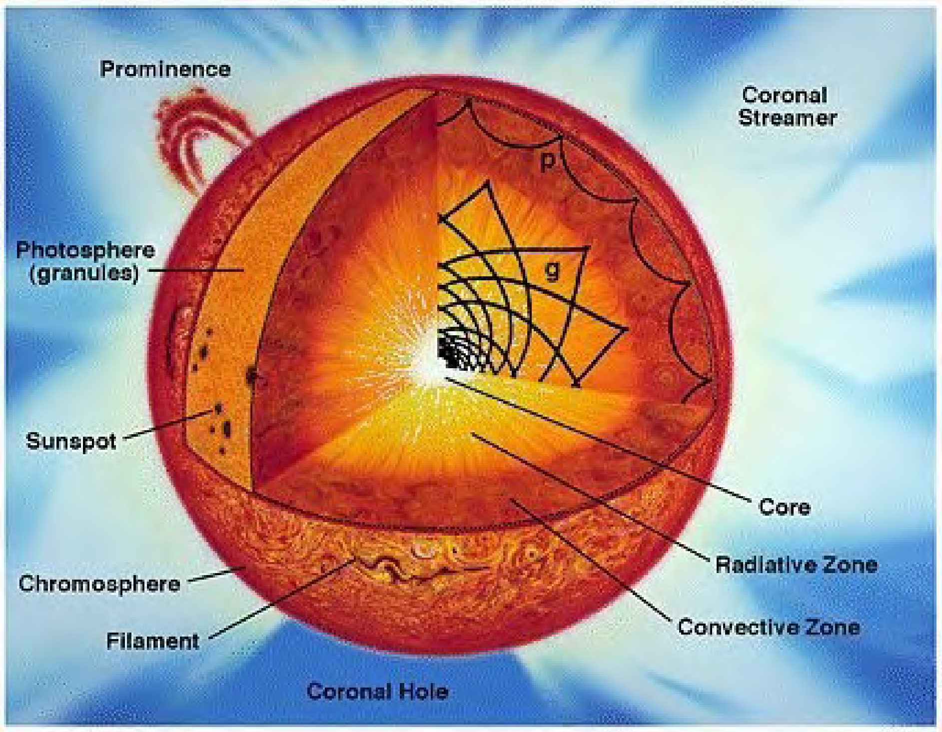


adapted and updated from: Quinn and Fröhlich, Nature, 401, p.841, 1999, with composite (vers d41_61_0510), ACRIM–IPIII (vers II:101001) and VIRGO 6_001_0510 data (Oct 07, 2005)

The Solar Interior

- Solar structure is inferred by modelling, based on bulk properties.
  - T_{surf}, $L_\odot$, $R_\odot$…
- Now supplemented by two direct probes:
  - Helioseismology
  - Solar neutrinos

GONG

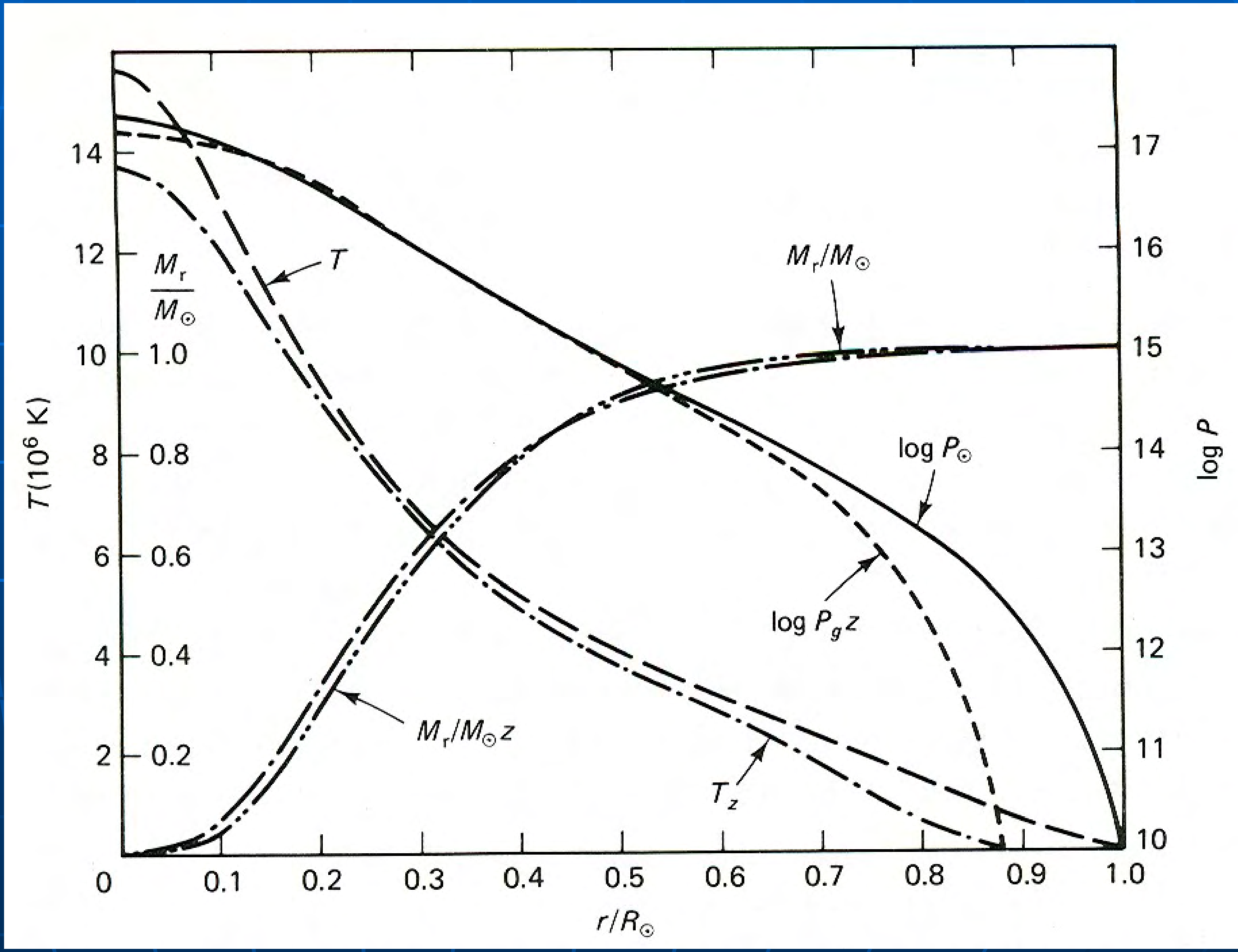


Standard solar model

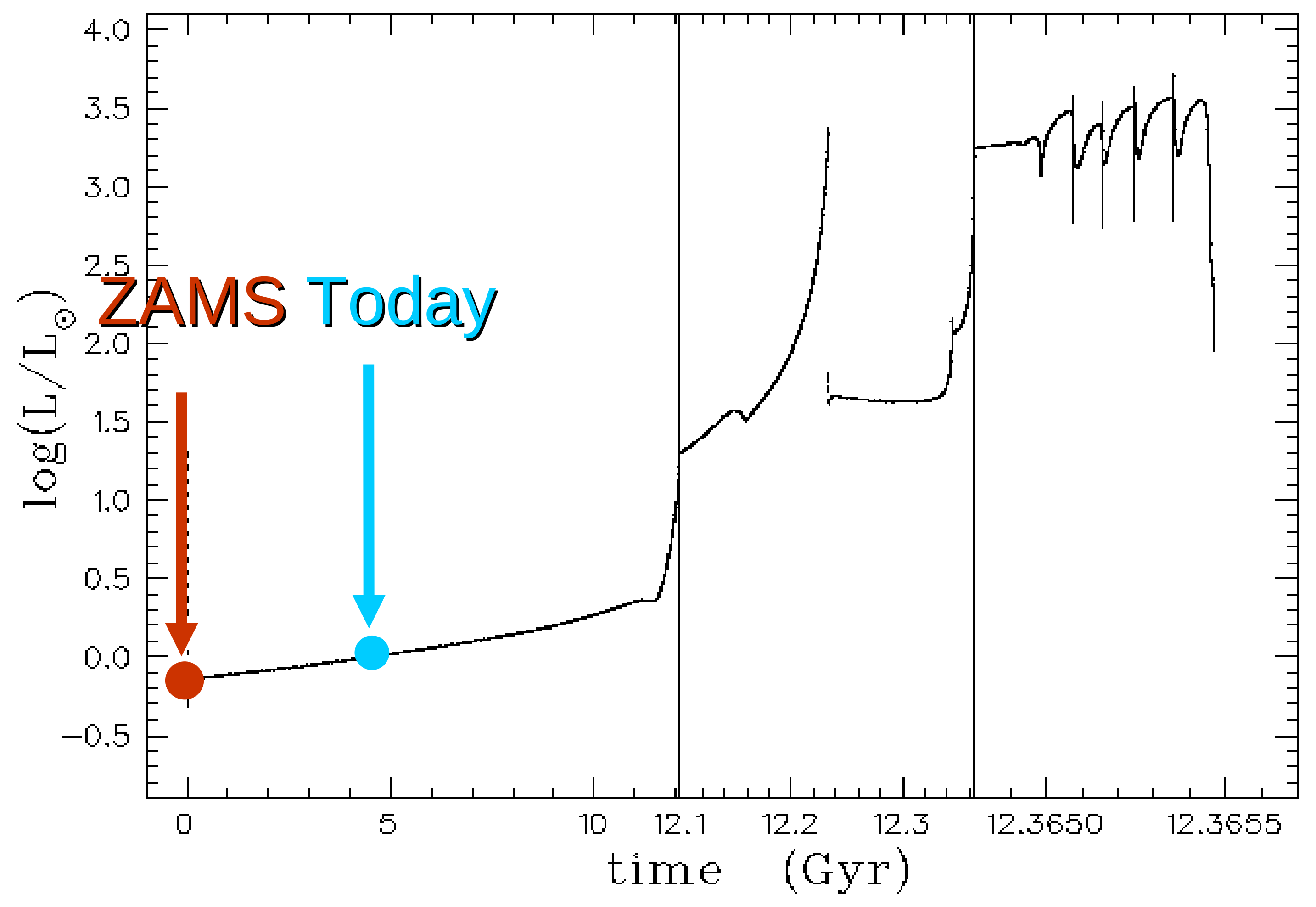
- Ingredients: Conservation laws and material dependent equations
  - Mass conservation
  - Hydrostatic equilibrium (= momentum conservation in a steady state)
  - Energy conservation
  - Energy transport
  - Equation of state
  - Expression for entropy
  - Nuclear reaction networks and reaction rates → energy production
  - Opacity
- Assumptions: standard abundances, no mixing in core or in radiative zone, hydrostatic equilibrium, i.e. model passes through a stage of equilibria (the only time dependence is introduced by the reduction of H and the build up of He in the core).

Internal structure of the Sun

- Internal models shown for ZAMS Sun (subscript z) and for present day Sun (radius reaching out to 1.0, subscript ⊙)



Evolution of Sun’s luminosity



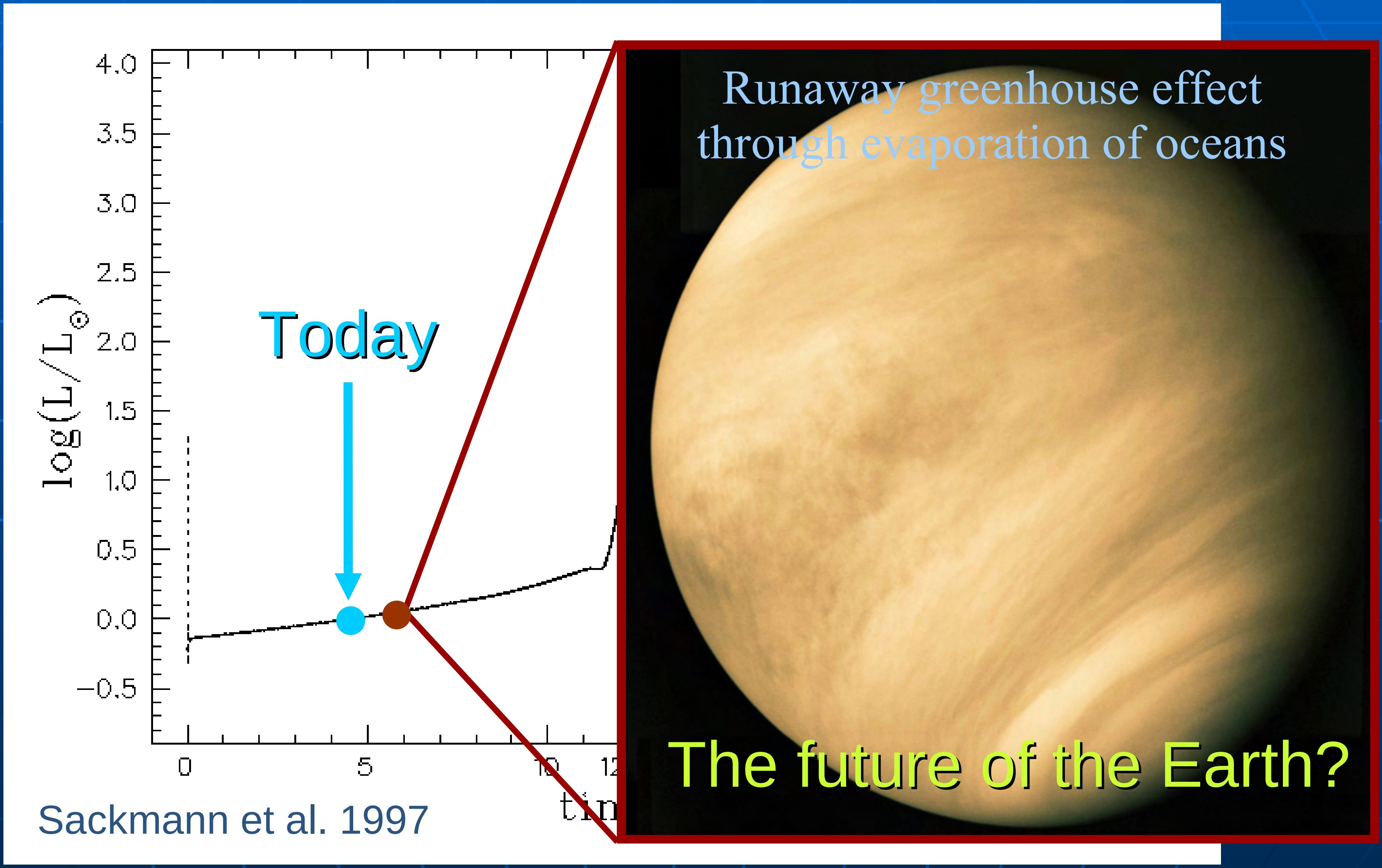
Faint young Sun paradox

- According to the standard solar model the Sun was approximately 30% less bright at birth than it is today
- Too faint to keep the Earth free of ice!
- Problem: Albedo of ice is so high that even with its current luminosity the Sun would not be able to melt all the ice away.
- Obviously the Earth is not covered with ice...
- So: Where is the mistake?

Possible resolution of the faint young Sun paradox

- The Earth's atmosphere was different 4 Gyr ago. More methane and other greenhouse gases. Higher insulation meant that even with lower solar input the Earth remained ice-free.
- As the Sun grew brighter life grew more abundant and changed the atmosphere of the Earth, reducing the greenhouse effect.
- Problem: what about Mars? Could it have had liquid water 4Gyr ago if Sun were so faint?
- Alternative: Sun was slightly more massive (1.04-1.07$M_\odot$ at birth and lost this mass (enhanced solar wind) in the course of time (Sackmann & Boothroyd 2003, ApJ). A more massive star on the ZAMS emits more light. Also agrees w. Mars data

Evolution of solar luminosity



Today

Runaway greenhouse effect through evaporation of oceans

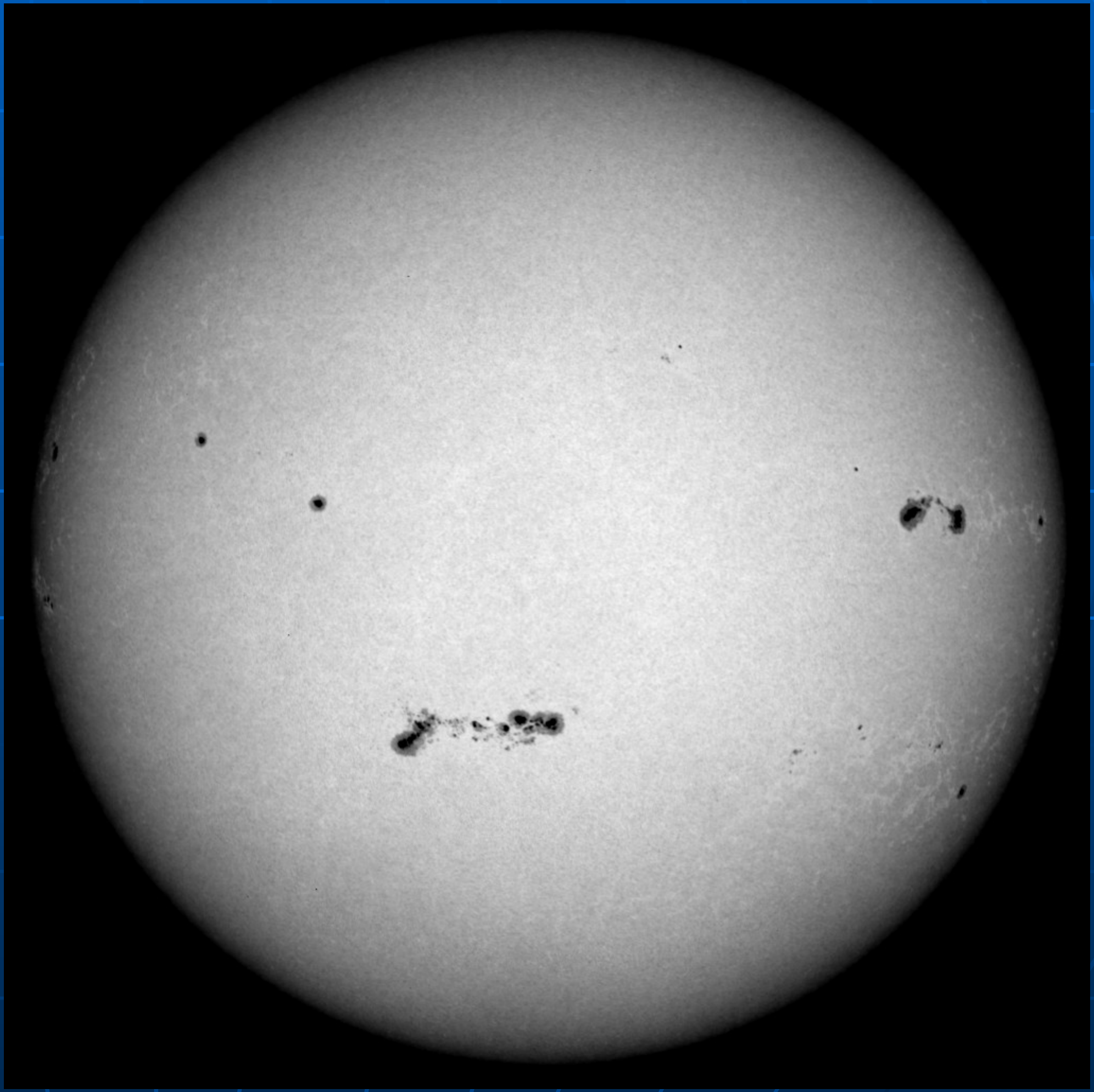
The future of the Earth?

Sackmann et al. 1997

Solar radiation and spectrum

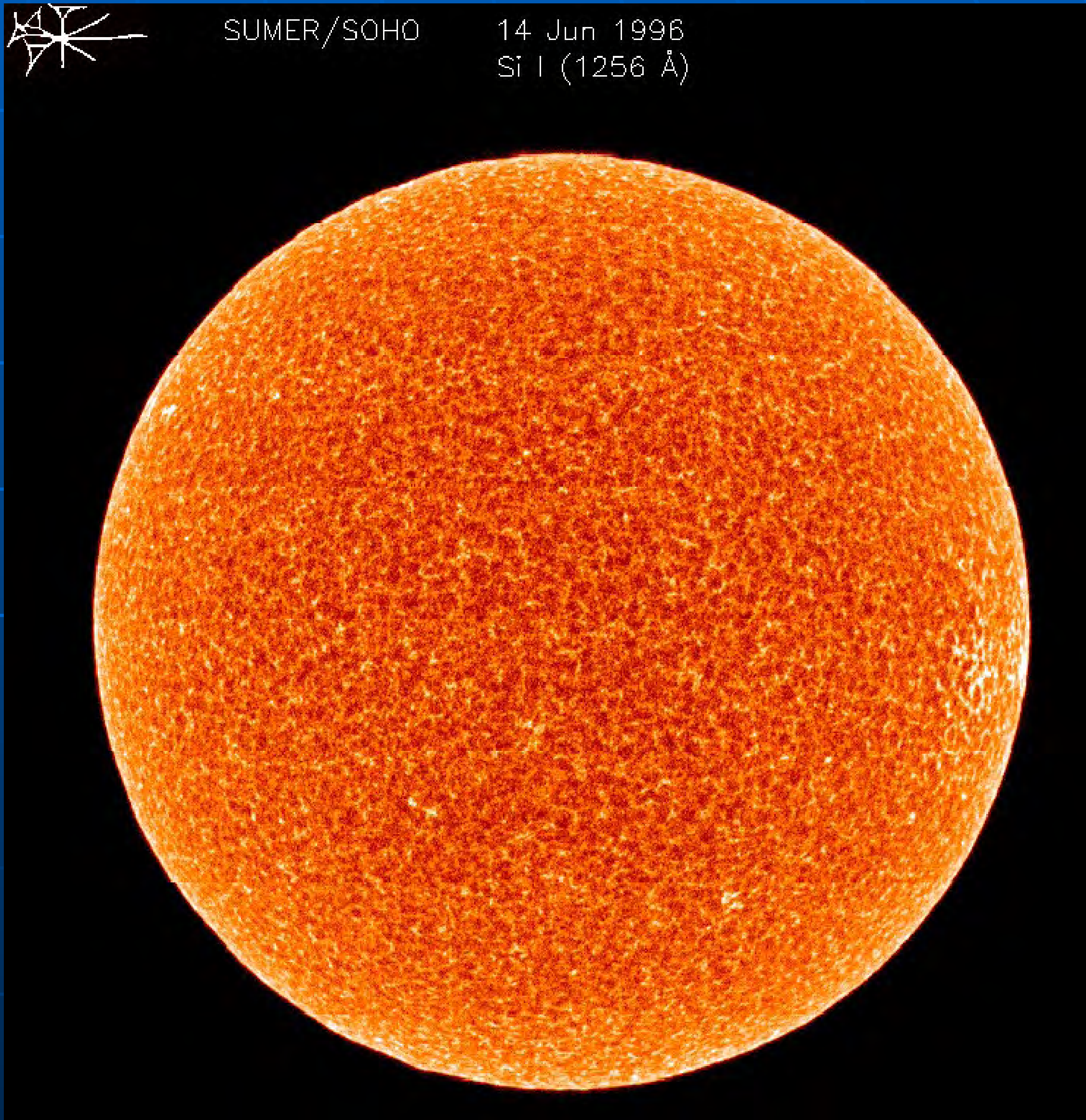
The Sun in white light: Limb darkening

- In the visible, the Sun's limb is darker than the centre of the solar disk (Limb darkening)
- Since intensity ~ Planck function, $B_\nu(T)$, T is lower near limb.
- Due to grazing incidence we see higher near limb: T decreases outward



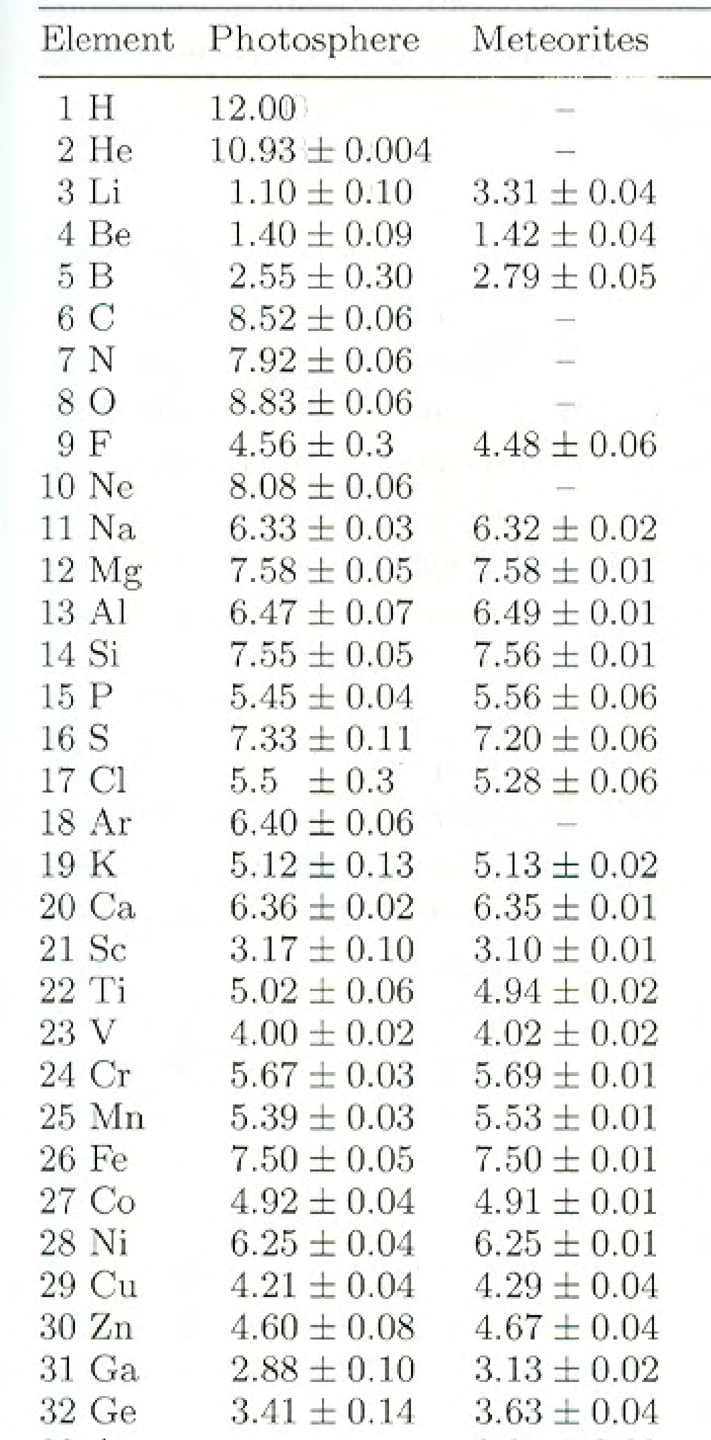
The Sun in the EUV: Limb brightening

- Limb brightening in optically thin lines does not imply that the Sun’s temperature increases outwards (although by chance it does in these layers....)



Elemental abundances

- **Photospheric values**
- Logarithmic (to base 10) abundances of the 32 lightest elements on a scale on which H has an abundance of 12
- Heavier elements all have low abundances
- Note that in general the solar photospheric abundances are very similar to those of meteorites, with exception of Li, with is depleted by a factor of 100.



Element	Photosphere	Meteorites
1 H	12.00	–
2 He	10.93 ± 0.004	–
3 Li	1.10 ± 0.10	3.31 ± 0.04
4 Be	1.40 ± 0.09	1.42 ± 0.04
5 B	2.55 ± 0.30	2.79 ± 0.05
6 C	8.52 ± 0.06	–
7 N	7.92 ± 0.06	–
8 O	8.83 ± 0.06	–
9 F	4.56 ± 0.3	4.48 ± 0.06
10 Ne	8.08 ± 0.06	–
11 Na	6.33 ± 0.03	6.32 ± 0.02
12 Mg	7.58 ± 0.05	7.58 ± 0.01
13 Al	6.47 ± 0.07	6.49 ± 0.01
14 Si	7.55 ± 0.05	7.56 ± 0.01
15 P	5.45 ± 0.04	5.56 ± 0.06
16 S	7.33 ± 0.11	7.20 ± 0.06
17 Cl	5.5 ± 0.3	5.28 ± 0.06
18 Ar	6.40 ± 0.06	–
19 K	5.12 ± 0.13	5.13 ± 0.02
20 Ca	6.36 ± 0.02	6.35 ± 0.01
21 Sc	3.17 ± 0.10	3.10 ± 0.01
22 Ti	5.02 ± 0.06	4.94 ± 0.02
23 V	4.00 ± 0.02	4.02 ± 0.02
24 Cr	5.67 ± 0.03	5.69 ± 0.01
25 Mn	5.39 ± 0.03	5.53 ± 0.01
26 Fe	7.50 ± 0.05	7.50 ± 0.01
27 Co	4.92 ± 0.04	4.91 ± 0.01
28 Ni	6.25 ± 0.04	6.25 ± 0.01
29 Cu	4.21 ± 0.04	4.29 ± 0.04
30 Zn	4.60 ± 0.08	4.67 ± 0.04
31 Ga	2.88 ± 0.10	3.13 ± 0.02
32 Ge	3.41 ± 0.14	3.63 ± 0.04

Solar convection

The convection zone

- Through the outermost 30% of solar interior, energy is transported by convection instead of by radiation
- In this layer the gas is convectively unstable. The unstable region ends just below the solar surface. I.e. the visible signs of convection are actually due to overshooting.
- Due to this, the time scale changes from the time scale for a random walk of the photons through the radiative zone (due to high density, the mean free path in the core is well below a millimeter) to the convective transport time:
- $t_{\text{radiative}} \sim 10^6$ years $>> t_{\text{convective}} \sim$ months

Increasing size of convective cells with depth

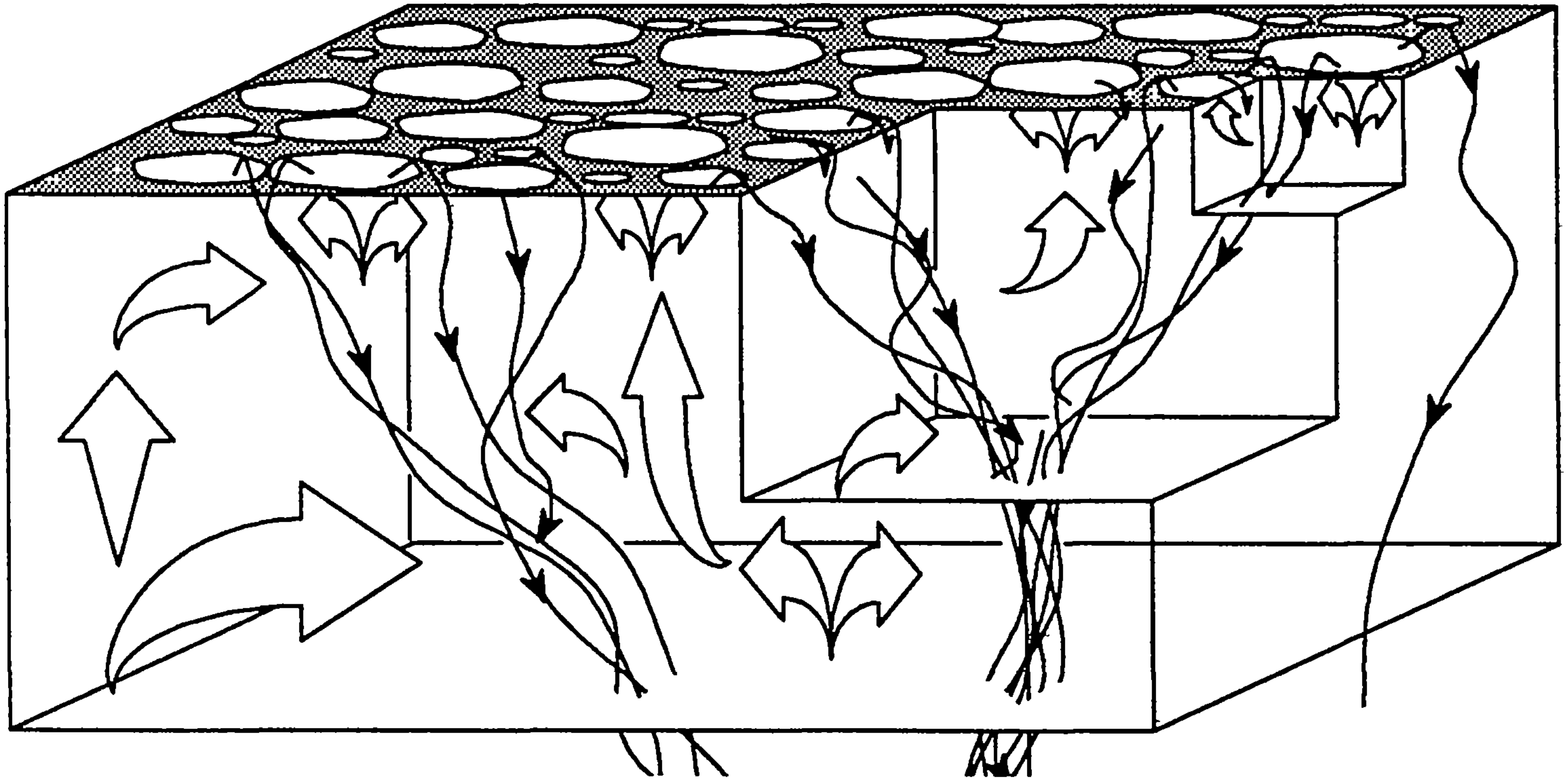
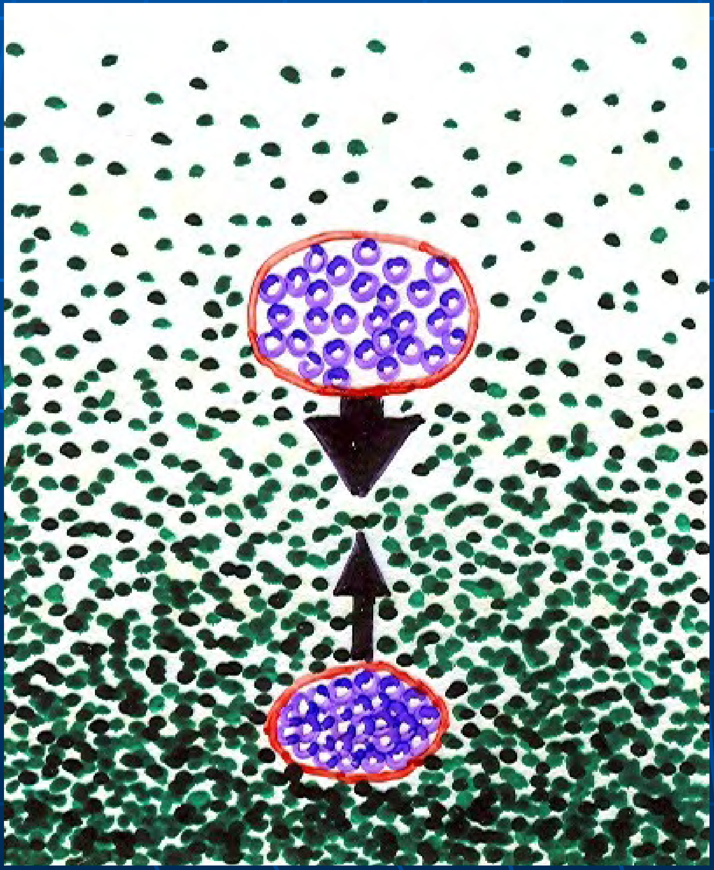


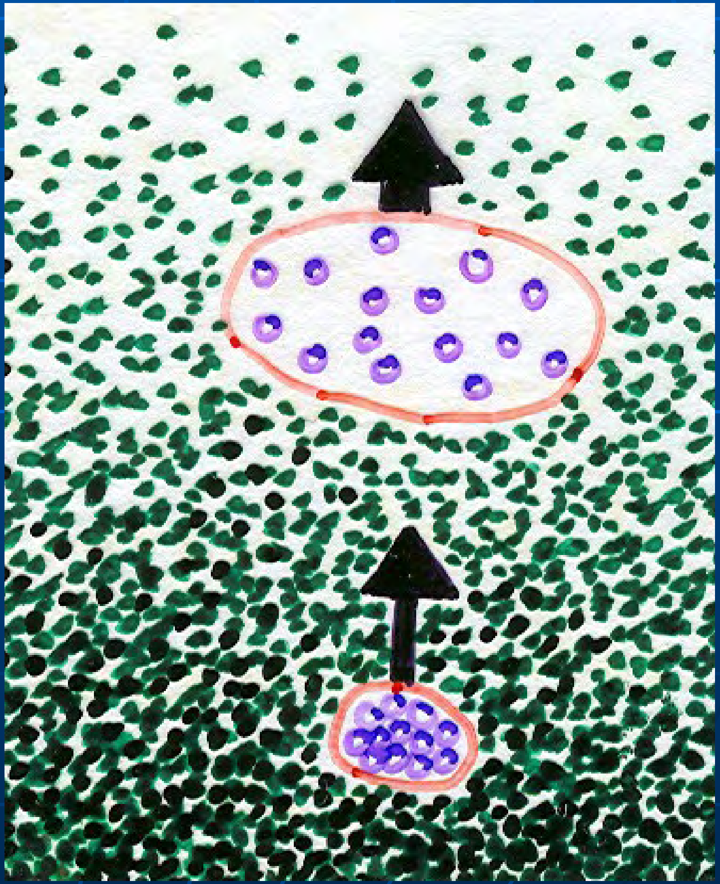
Figure 7 Flow lines showing the merging of the downdrafts on successively larger scales (schematic). The boxes cut out illustrate how the same process occurs on (in this illustration) three different scales.

Illustration of convectively stable and unstable situations

Convectively stable

Convectively unstable

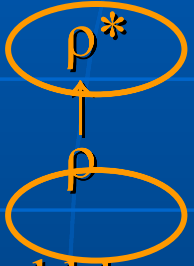




Onset of convection

Schwarzschild's instability criterion

Consider a rising bubble of gas:



ρ^*	ρ	$z-\Delta z$
ρ	$\rho_0 = \rho$	z
bubble	surroundings	depth

Condition for convective instability: $\rho^* < \rho_0$

For small Δz, bubble will not have time to exhange heat with surroundings: adiabatic behaviour. Convectively unstable if:

$$[d\rho/dz - (d\rho/dz)_{adiab}]\ \Delta z < 0$$

$d\rho/dz$: true stellar density gradient,

$(d\rho/dz)_{adiab}$: adiabatic gradient

Why an outer convection zone?

- Why does radiative grad exceed adiabatic gradient?
- Mainly: radiative gradient becomes very large due to ionization of H and He below the solar surface.
- Expression for radiative gradient (for Eddington approximation):

$$\nabla_{rad} = (3F_r / 16\sigma g)\,(\kappa_{gr}\, P_g / T^4)$$

  - F_r = radiative flux (≈ constant)
  - σ = Stefan-Boltzmann constant
  - g = gravitational acceleration (≈ constant)
  - κ_{gr} = absorption coefficient per gram. As H and He become ionized with depth, κ_{gr} increases rapidly, leading to large radiative gradient.

Ionisation of H and He

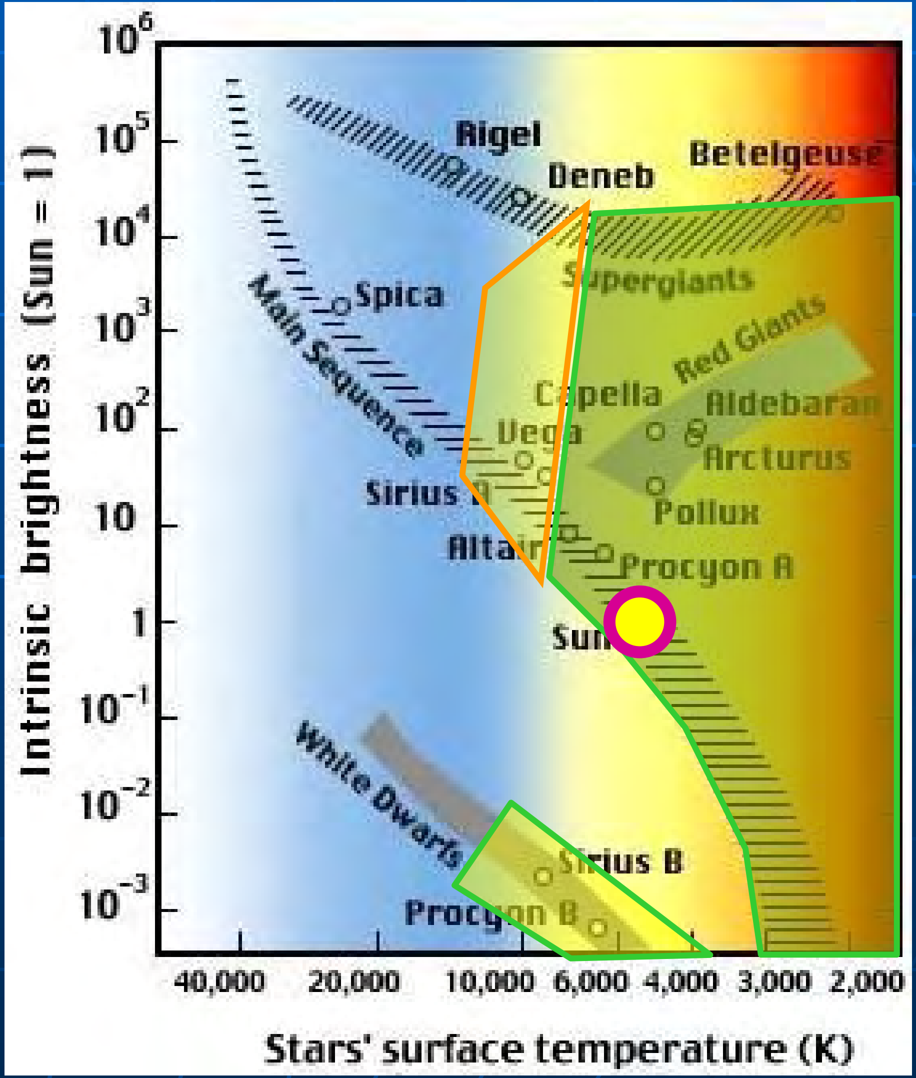
- Ionisation balance is described by Saha's equation: degree of ionisation depends on T and n_e
- H ionisation happens just below solar surface
- $He \rightarrow He^+ + e^-$ happens 7000 km below surface
- $He^+ \rightarrow He^{++} + e^-$ happens 30'000 km below surface
- Since H is most abundant, it provides most electrons (largest opacity) and drives convection most strongly
- At still greater depth, other elements also provide a minor contribution.

Convection on other stars

- F, G, K & M stars posses outer convection zones and show observable effects of convection (also WDs)
- Observations are difficult since surfaces cannot be resolved.

➔Use line bisectors: independent of spatial resolution

- A,F stars show inverse bisectors: granulation has different geometry.



Oscillations and helioseismology

5-minute oscillations

- The entire Sun vibrates from a complex pattern of acoustic waves, with a period of around 5 minutes
- The oscillations are best seen as Doppler shifts of spectral lines, but also as intensity variations.
- Identified as acoustic waves, called p-modes
- Spatio-temporal properties of oscillations best revealed by 3-D Fourier transforms.

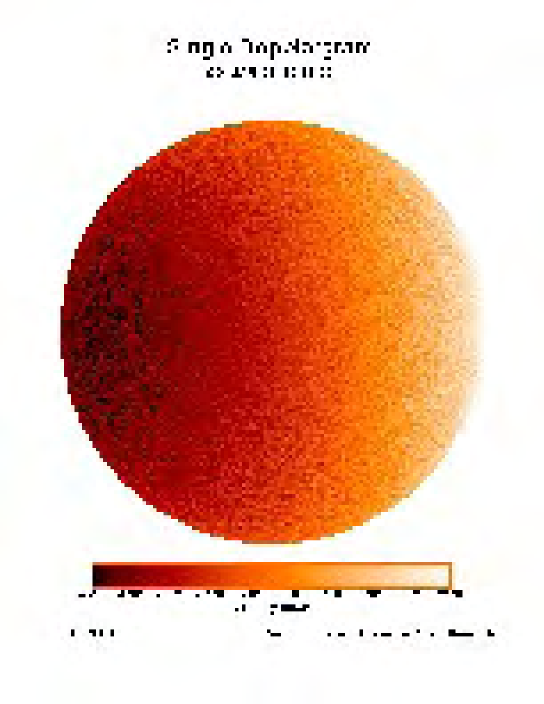
Hear the Sun sing!

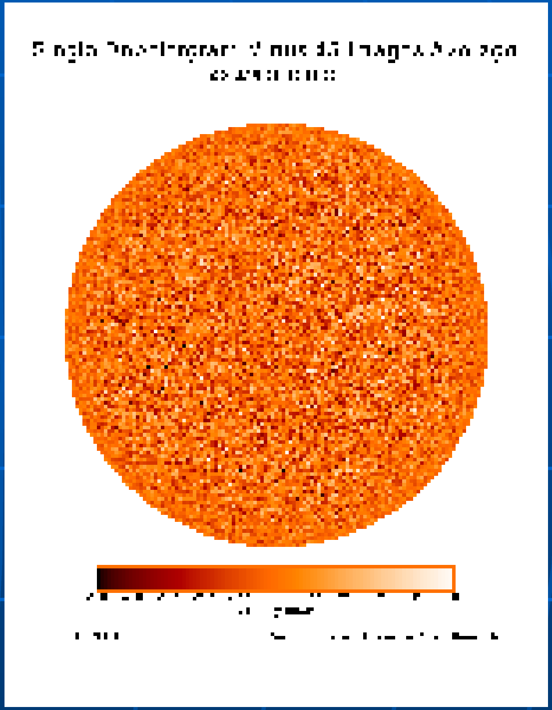
Sound waves speeded up 42,000 times

Doppler shift

Helioseismic Waves Observed

GONG

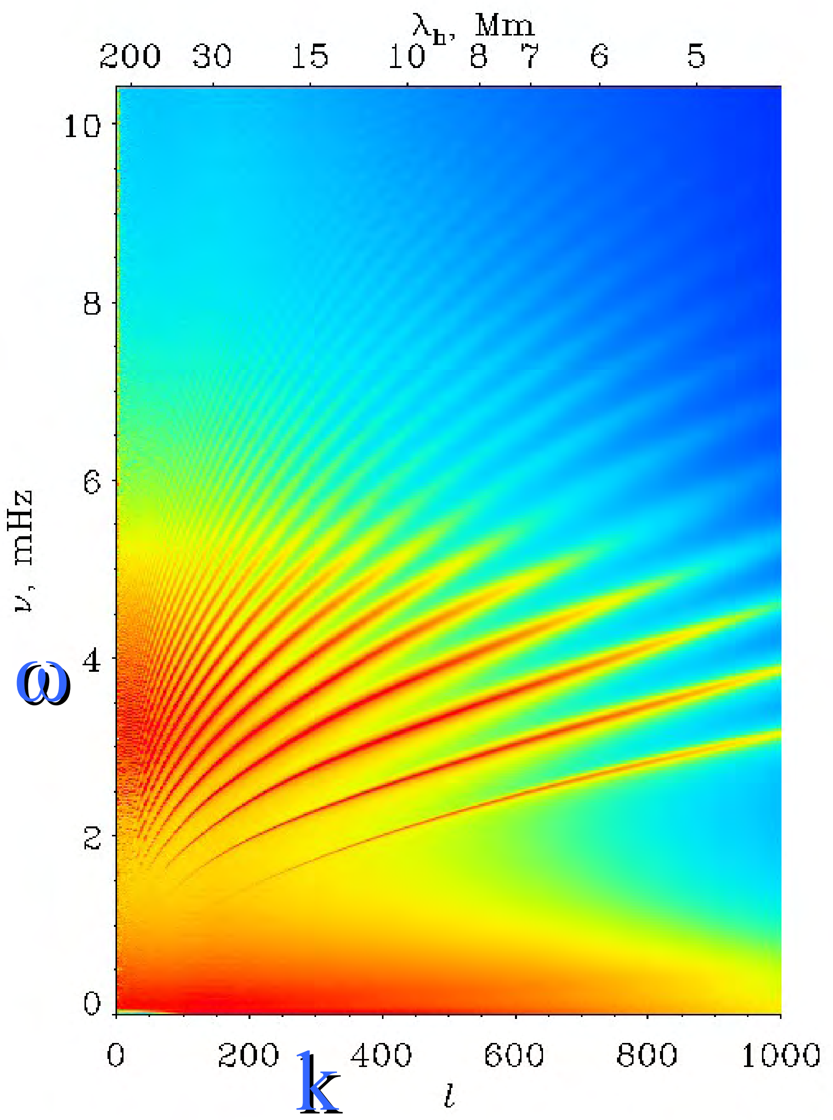




- Wave patterns are observable by measuring Doppler shifts as a function of position on the solar surface.
  - Thousands of normal modes have been detected in this way.

Solar Eigenmodes

- The p-modes show a distinctive dispersion relation (k-ω diagram: $k \sim \omega^2$)
- Important: there is power only in certain ridges, i.e. for a given k^2 (= $k_x^2 + k_y^2$), only certain frequencies contain power.
- This discrete spectrum suggests the oscillations are trapped, i.e. eigenmodes of the Sun.



Global oscillations

- The Sun's acoustic waves bounce from one side of the Sun to the other, causing the Sun's surface to oscillate up and down. They are reflected at the solar surface.
- Modes differ in the depth to which they penetrate: they turn around because sound speed ($C_S \sim T^{1/2}$) increases with depth (refraction)
- p-modes are influenced by conditions inside the Sun. E.g. they carry info on sound speed
- By observing these oscillations on the surface we can learn about the structure of the solar interior

Description of solar eigenmodes

- Eigen-oscillations of a sphere are described by spherical harmonics
- Each oscillation mode is identified by a set of three parameters:
  - n = number or radial nodes
  - l = number of nodes on the solar surface
  - m = number of nodes passing through the poles (next slide)

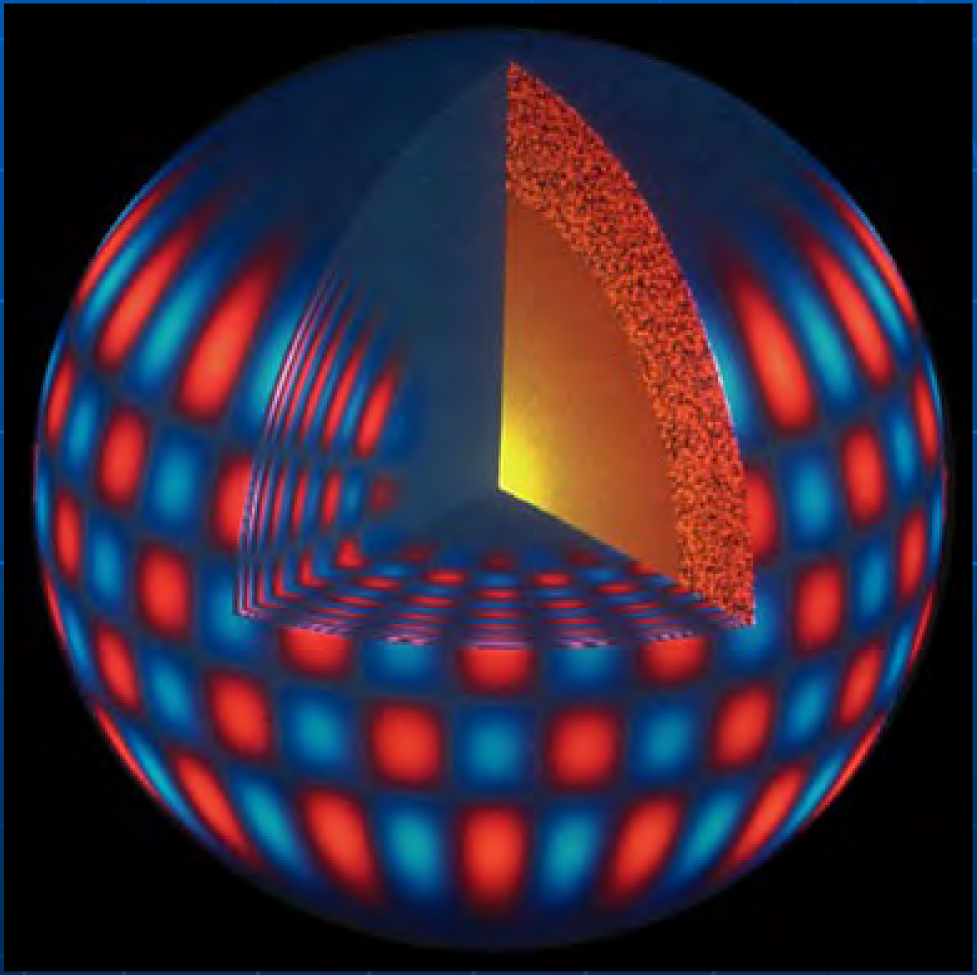
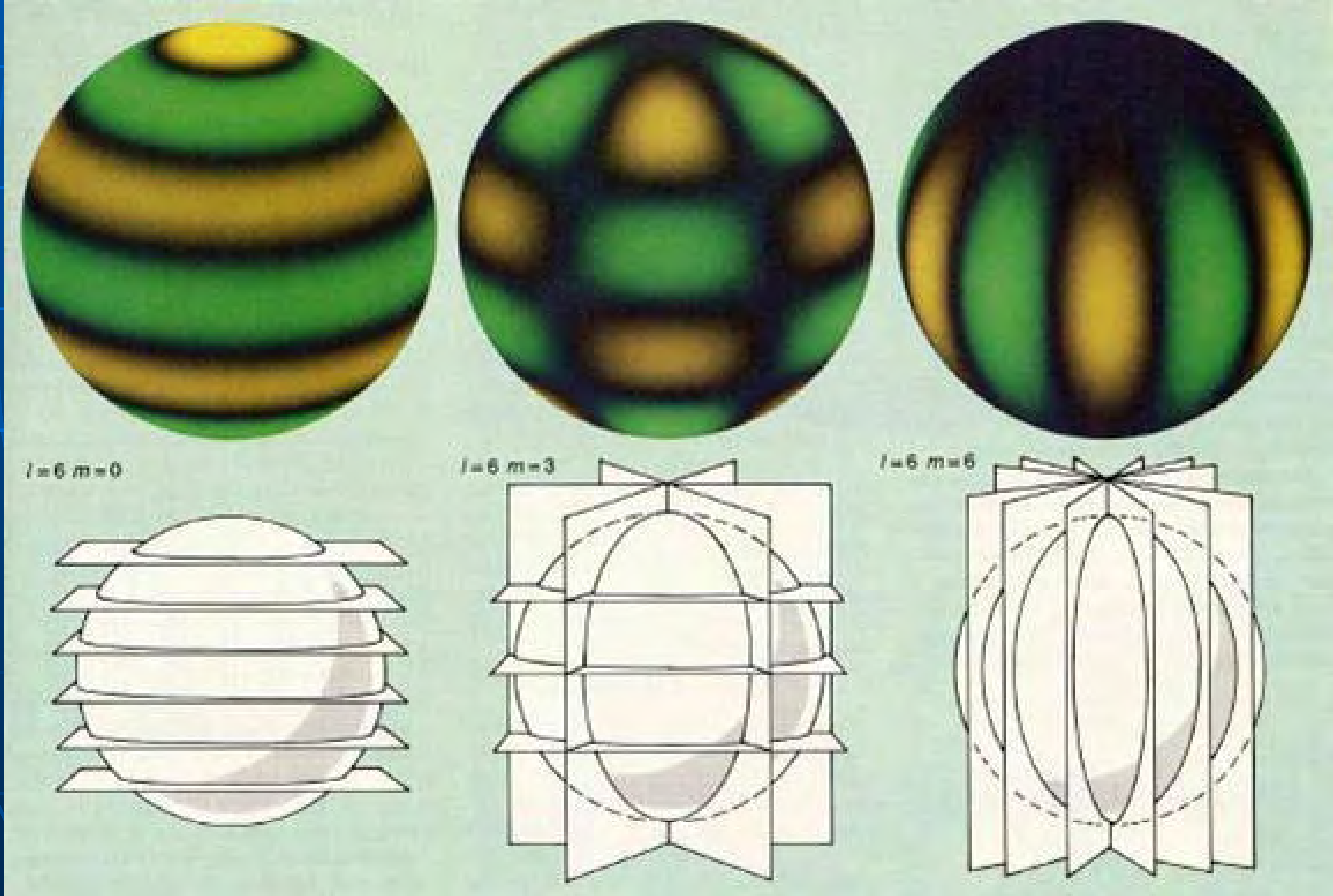


Illustration of spherical harmonics

- l = total number of nodes (in images: $l = 6$) = degree
- m = number of nodes connecting the “poles”



Small perturbations around an equilibrium

No motion: $\mathbf{v}_0 = 0$

Hydrostatic equilibrium: $\nabla p_0 = \rho_0 \mathbf{g}_0$

Gravity: $\mathbf{g}_0 = -\frac{G m_0}{r^2} \mathbf{a}_r$

Energy: $\rho_0 \, \epsilon_0 = \mathrm{div}\, \mathbf{F}_0 = \frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r}(r^2 F_0) = \frac{1}{4\pi r^2} \frac{\mathrm{d}L_0}{\mathrm{d}r}$

Perturbations: $p(\mathbf{r}, t) = p_0(r) + p'(\mathbf{r}, t)$, etc.

Velocity: $\mathbf{v}' = \frac{\partial \boldsymbol{\delta}\mathbf{r}}{\partial t}$

Eulerian (p') and Lagrangian (δp) perturbations:

$$\delta p = p' + \boldsymbol{\delta}\mathbf{r} \cdot \nabla p_0 ,$$

Basic linearized equations

Continuity equation

$$\rho' + \mathrm{div}(\rho_0 \boldsymbol{\delta}\mathbf{r}) = 0 \; .$$

Momentum equation

$$\rho_0 \frac{\partial^2 \boldsymbol{\delta}\mathbf{r}}{\partial t^2} = \rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p' + \rho_0 \mathbf{g}' + \rho' \mathbf{g}_0 \; ,$$

Poisson's equation

$$\nabla^2 \Phi' = 4\pi G \rho' \; , \qquad \mathbf{g}' = -\nabla \Phi'$$

Adiabaticity

$$\delta p = \frac{\gamma_{1,0} p_0}{\rho_0} \delta \rho = c_0^2 \delta \rho \; ,$$

Boundary conditions

At centre

$$\xi_r \simeq l\xi_{\rm h} \,, \qquad \text{for } r \to 0 \,.$$

At surface

$$\Phi' = A\,r^{-l-1} \,, \qquad \frac{\mathrm{d}\Phi'}{\mathrm{d}r} + \frac{l+1}{r}\Phi' = 0 \qquad \text{at } r = R \,.$$

$$\delta p = p' + \xi_r \frac{\mathrm{d}p}{\mathrm{d}r} = 0 \qquad \text{at } r = R \,.$$

Equations and boundary conditions determine frequencies ω_{nl}

$$\frac{\delta h_{\rm rms}}{\delta r_{\rm rms}} = \frac{\sqrt{l(l+1)}}{\sigma^2} \qquad \text{at } r = R \,, \qquad \sigma^2 = \frac{R^3}{GM}\omega^2$$

Spherical harmonics

- Let $v(\theta,\varphi,t)$ be the velocity, e.g. as measured at the solar surface over time t. Then:

$$v(\theta,\varphi,t)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}a_{lm}(t)Y_l^m(\theta,\varphi)$$

- The temporal dependence lies in a_{lm}, the spatial dependence in the spherical harmonic Y_l^m.

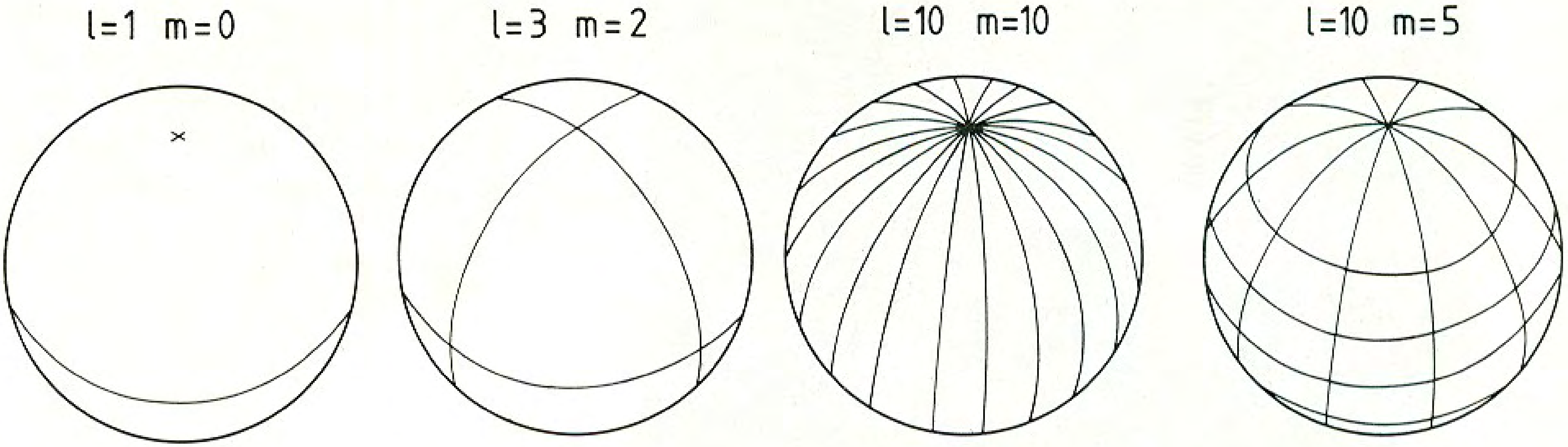
$$Y_l^m(\theta,\varphi)=P_l^{|m|}(\theta)\exp(im\varphi)$$

$$P_l^{|m|}(\theta)=\text{associated Legendre Polynomial}$$

- Due to the normalization of the spherical harmonic, the Fourier power is given by $F(a)F(a)^*$
- Here $F(a)$ is the Fourier transform of the amplitude a_{lm}

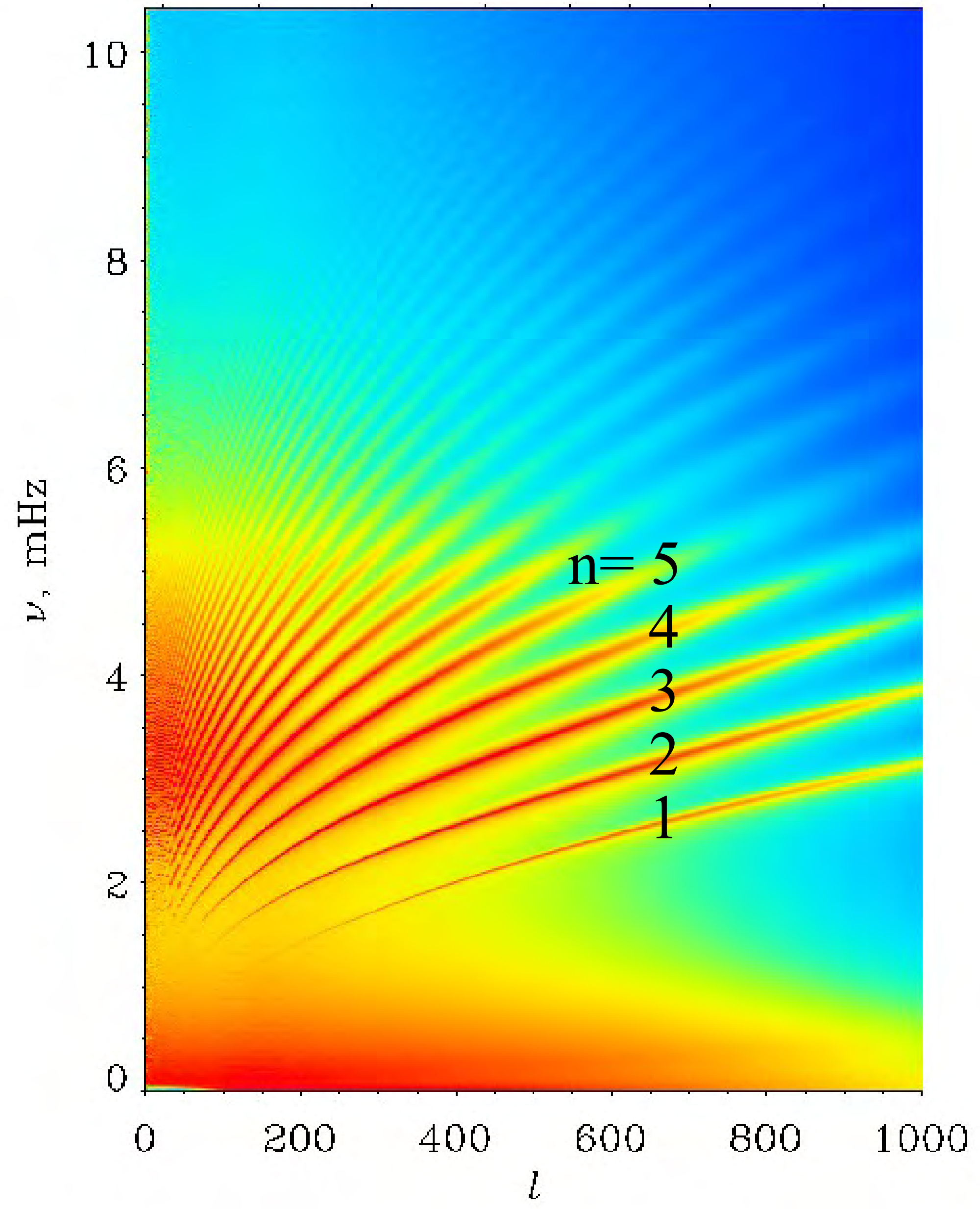
More examples and a problem with identifying spherical harmonics

- General problem: Since we see only half of the Sun, the decomposition of the sum of all oscillations into spherical harmonics isn’t unique.
- This results in an uncertainty in the deduced *l* and *m*



Interpretation of *k-ω* or *ν-l* diagram

- At a fixed *l*, different frequencies show significant power. Each of these power ridges belongs to a different order *n* (*n* = number of radial nodes), with *n* increasing from bottom to top.
- Typical are small values of *n*, but intermediate to large degree *l*.



A few observational remarks

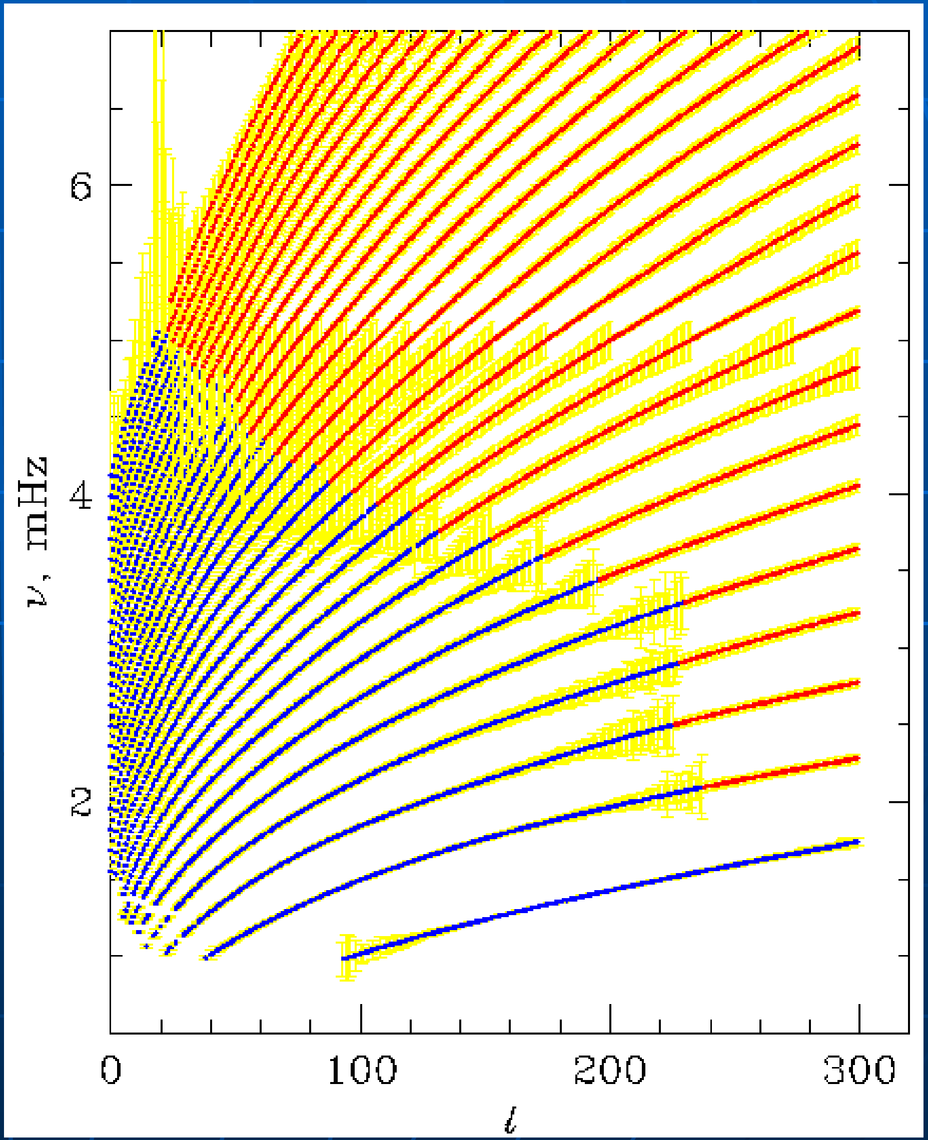
- 10^7 modes are present on the surface of the Sun at any given time (and interfering with each other).
- Typical amplitude of a single mode: < 20 cm/s
- Total velocity of all 10^7 modes: a few 100 m/s
- Accuracy of current instruments: better than 1 cm/s
- Frequency resolution ~ length of time series (Heisenberg's uncertainty principle) ~ lowest detectable frequency

→ Longer time series are better

- Gaps in time series produce side lobes (i.e. spurious peaks in the power spectrum)
- Highest detectable frequency ~ cadence of obs.

Accuracy of frequency measurements

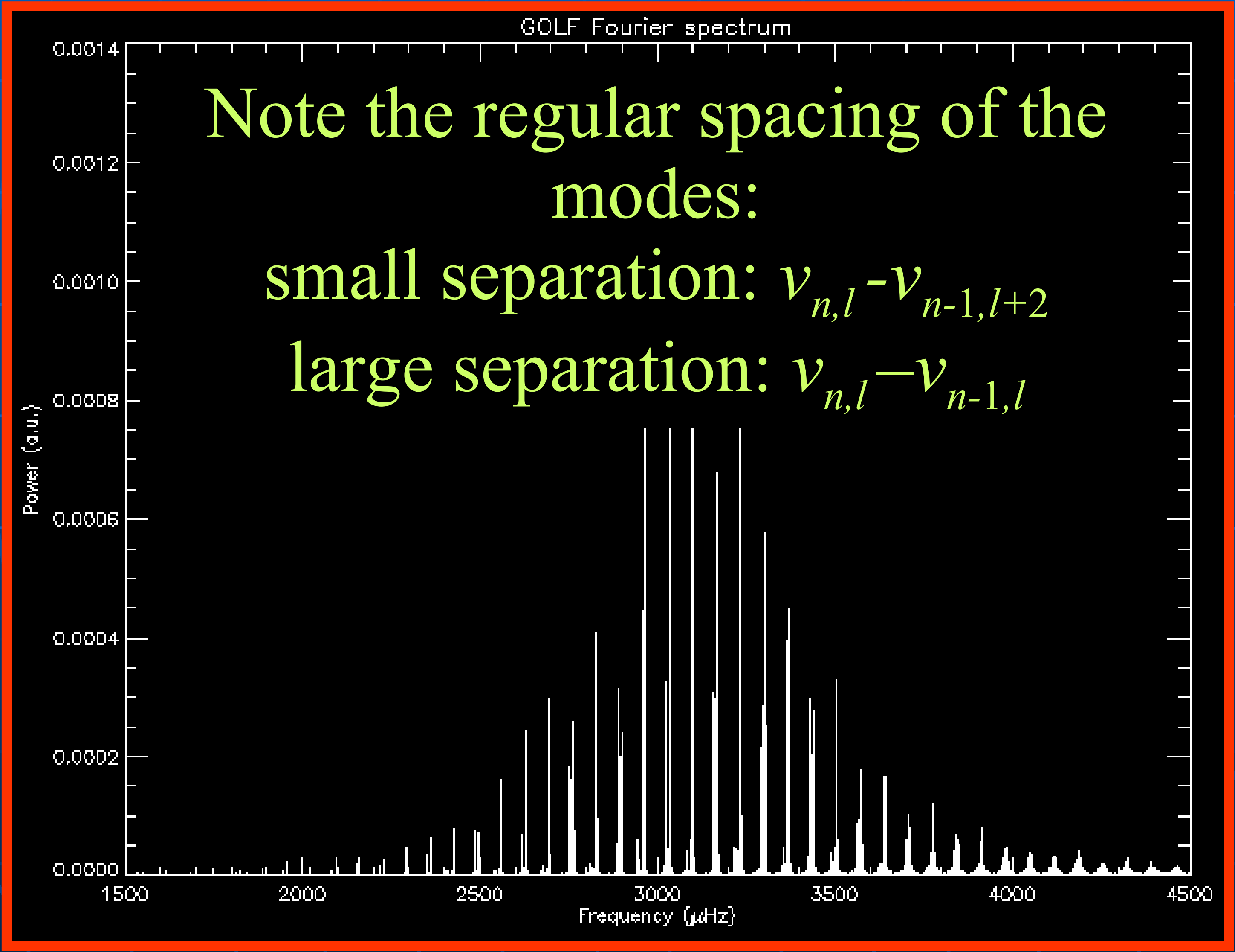
- Plotted are identified frequencies and error bars (yellow; 1000σ for blue freq., 100σ for red freq. below 5 mHz and 1σ for higher freq.)
- Best achievable freq. resolution: a few parts in 10^5; limit set by mode lifetime ~100 d



Frequency vs. amplitude

- Frequencies are the important parameter, more so than the amplitudes of the modes or of the power peaks.
- The amplitudes depend on the excitation, while the frequencies do not. They carry the main information on the structure of the solar interior.
- p-modes are excited by turbulence, which excites all frequencies. However, only at Eigenfrequencies of the Sun can eigenmodes develop.
- Frequencies (being more constant) are also measured with greater accuracy.

Best current low-*l* power spectrum



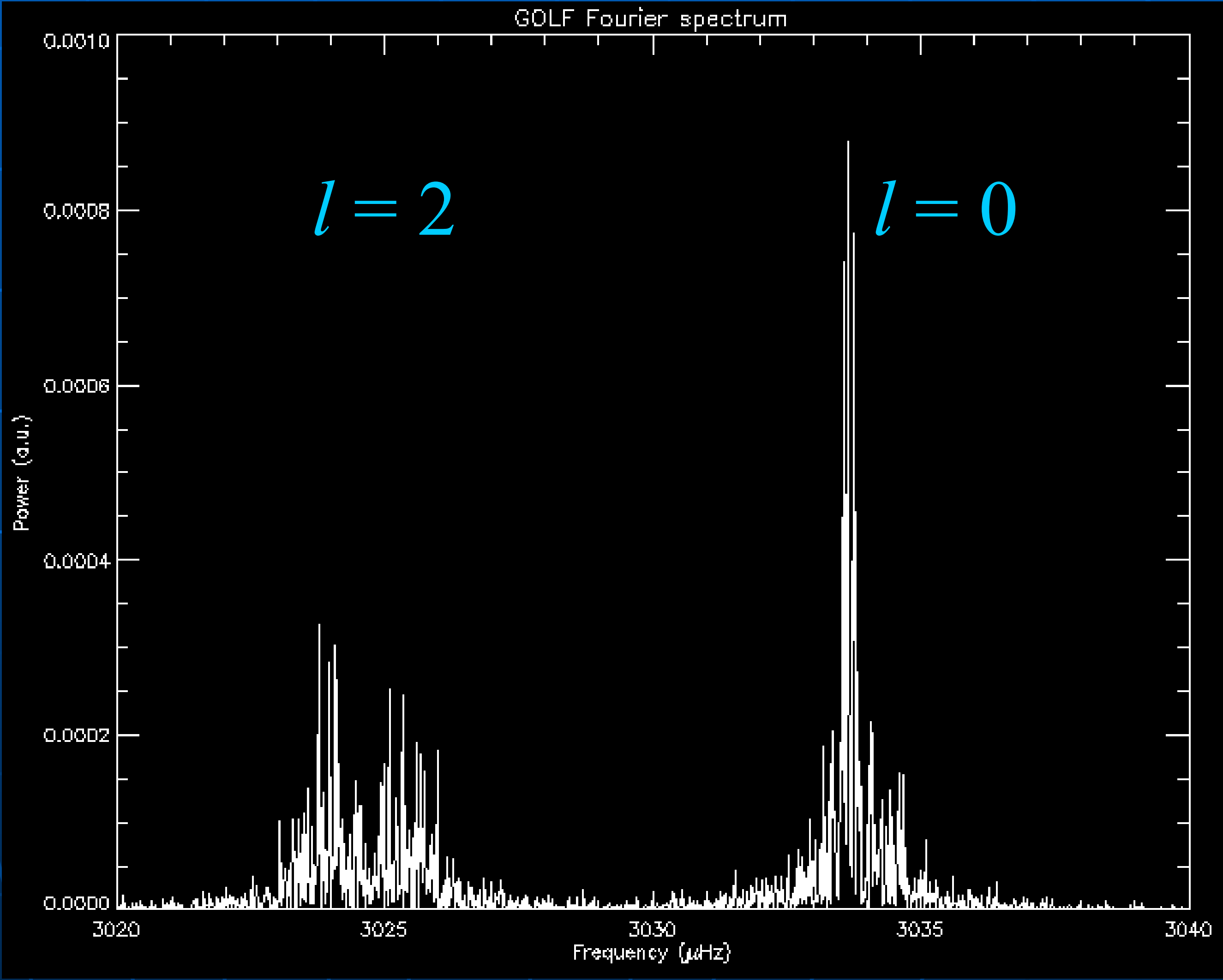
Note the regular spacing of the modes:

small separation: $\nu_{n,l} - \nu_{n-1,l+2}$

large separation: $\nu_{n,l} - \nu_{n-1,l}$

Mode structure of low *l* spectrum

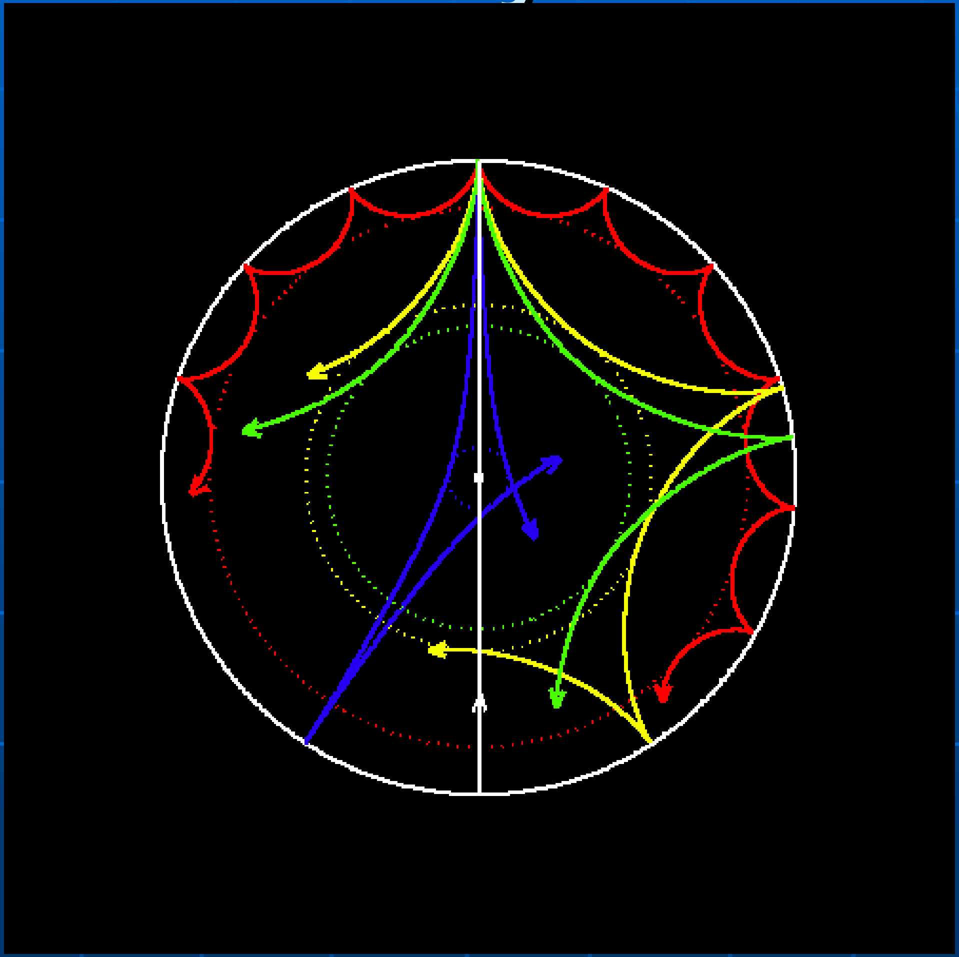
- GOLF/SOHO observations showing a blowup of the power spectrum with an $l = 0$ and an $l = 2$ mode.
- The noise is due to random re-excitation of the oscillation mode by turbulence



Types of oscillations

- Solar eigenmodes can be of 2 types:
  - p-modes, where the restoring force is the pressure, i.e. normal sound waves
  - g-modes, where the restoring force is gravity (also called buoyancy modes)
- So far only p-modes have been detected on the Sun with certainty.
- They are excited by the turbulence associated with the convection, mainly the granulation near the solar surface (since there the convection is most vigorous).
- Being p-modes, they travel with the sound speed C_S. They dwell longest where C_S is lowest. Since $C_S \sim T^{1/2}$, this is at the solar surface.

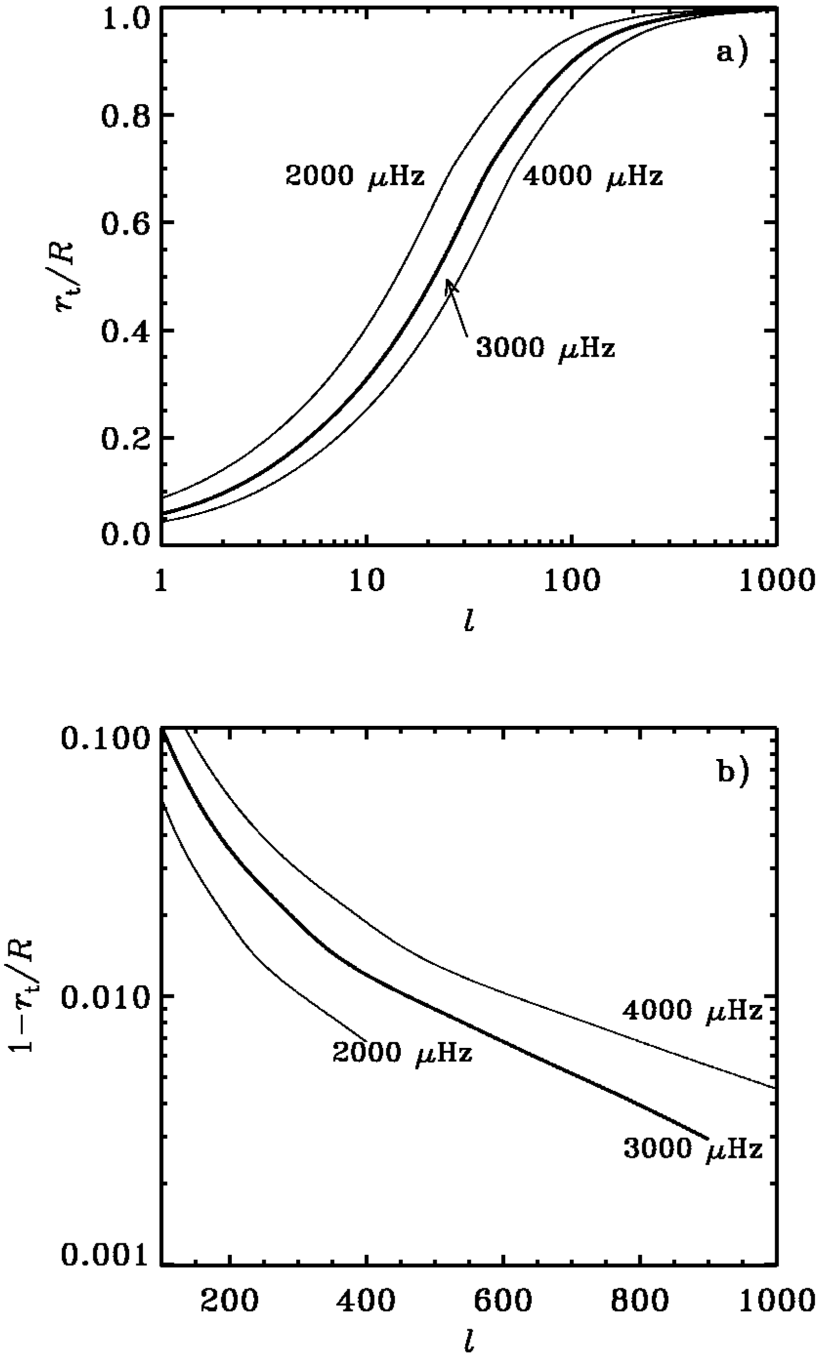
Rays



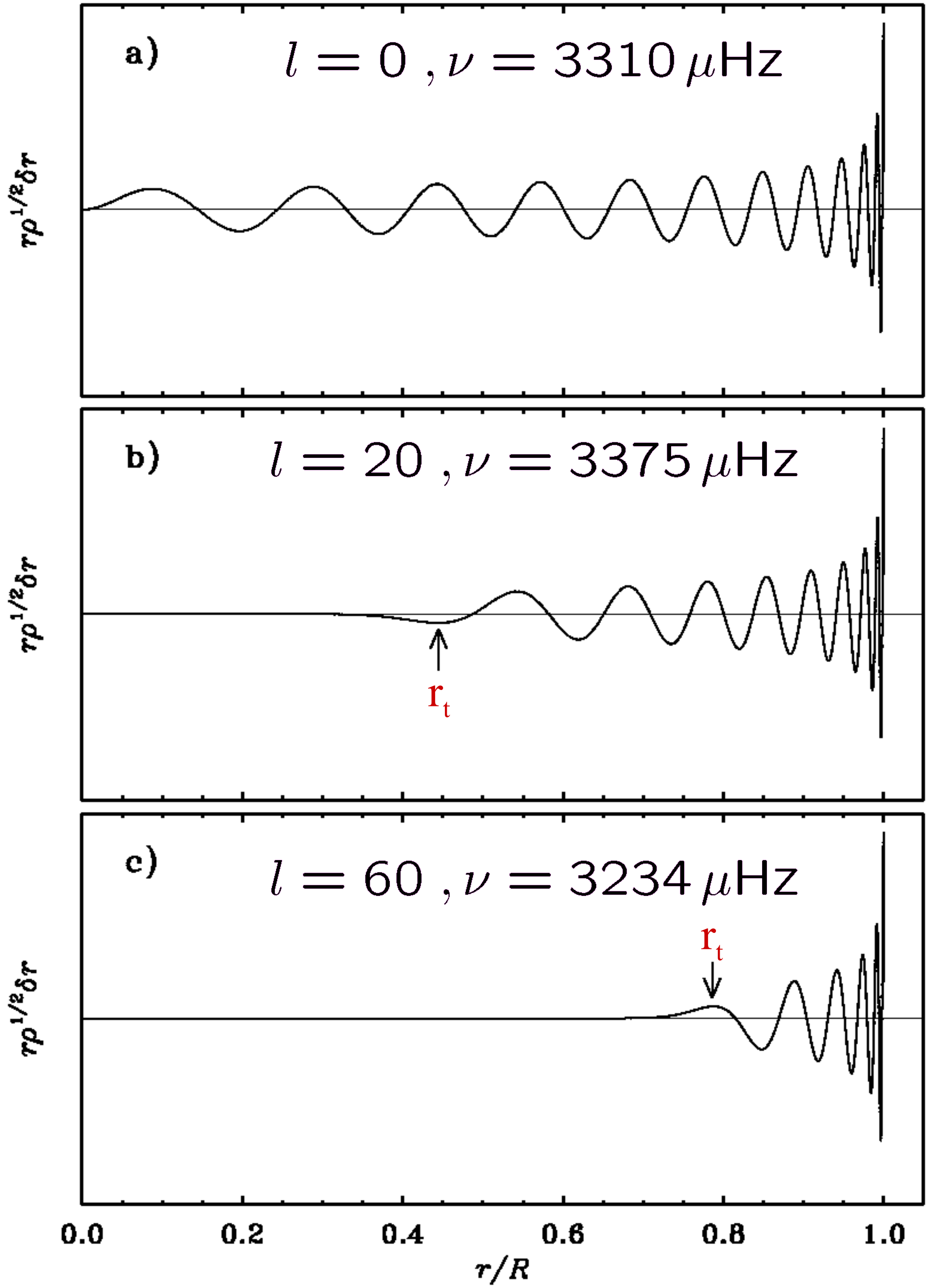
$$k_r = \left[\frac{\omega^2}{c^2} - \frac{l(l+1)}{r^2}\right]^{1/2}$$

Turning point: $\frac{c(r_{\rm t})}{r_{\rm t}} = \frac{\omega}{\sqrt{l(l+1)}}$

Location of turning point



Effect on eigen-functions

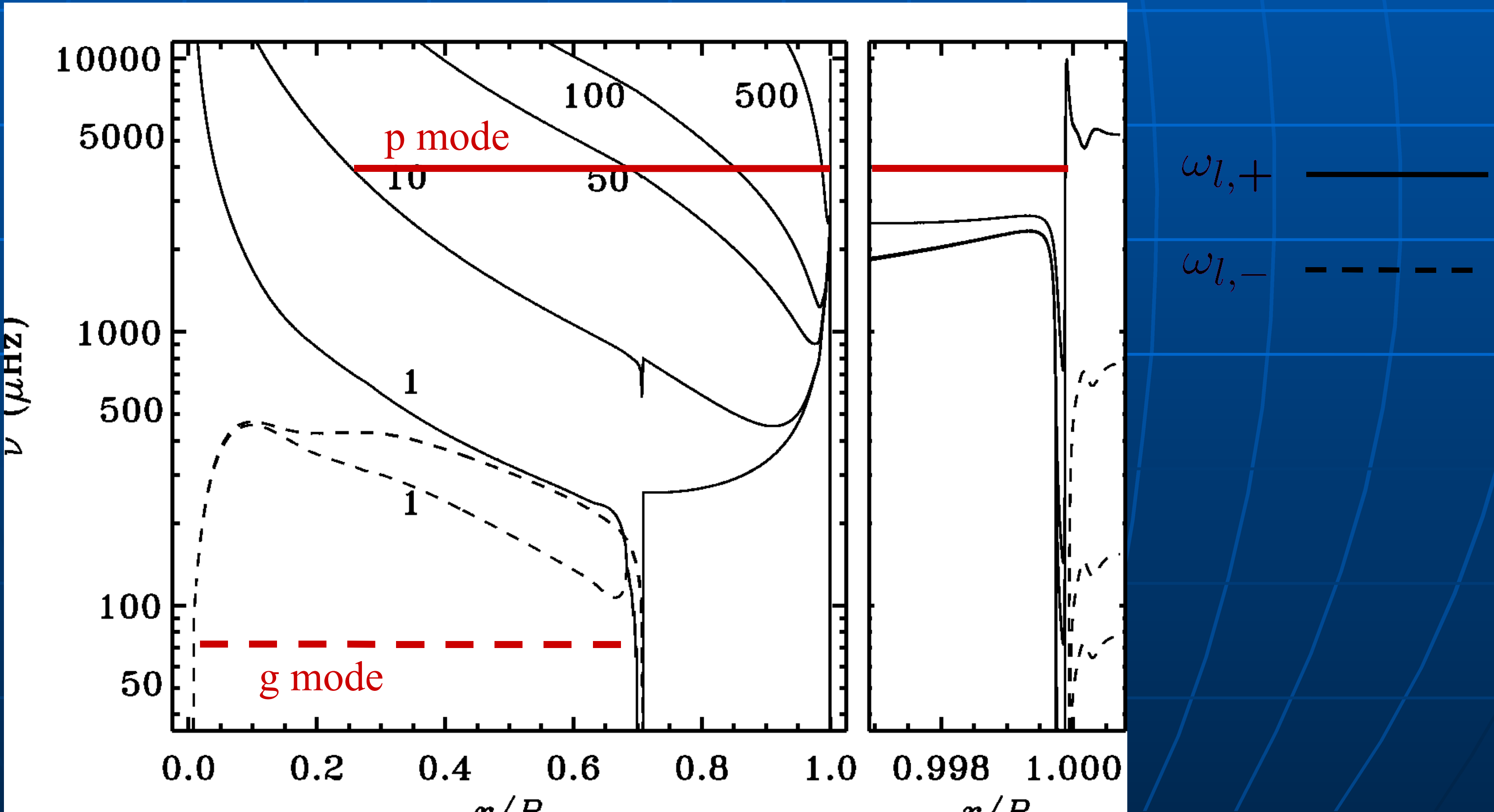


p-modes vs. g-modes

- p-modes propagate throughout the solar interior, but are evanescent (later slide) in the solar atmosphere
- g-modes propagate in the radiative interior and in the atmosphere, but are evanescent in the convection zone (their amplitude drops exponentially there, so that very small amplitudes are expected at the surface). Convection means buoyancy instability; oscillations require stability.
- g-modes are expected to be most sensitive to the very core of the Sun, while p-modes are most sensitive to the surface
- Current upper limit on solar interior g-modes lies below 1 cm/s.

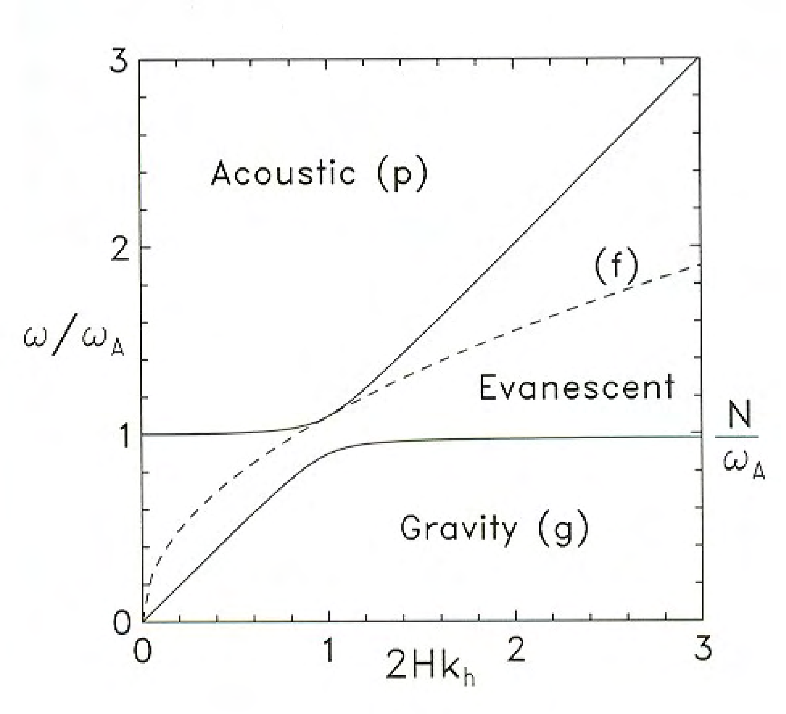
A more rigorous asymptotic analysis (III)

$$\frac{\mathrm{d}^2X}{\mathrm{d}r^2} = -\frac{1}{c^2}\left[S_l^2\left(\frac{N^2}{\omega^2}-1\right)+\omega^2-\omega_{\mathrm{c}}^2\right]X \equiv -\frac{\omega^2}{c^2}\left(1-\frac{\omega_{l,+}^2}{\omega^2}\right)\left(1-\frac{\omega_{l,-}^2}{\omega^2}\right)X$$



Regimes of oscillation

- In regimes of acoustic and gravity waves $k_r^2 > 0$, while in regime of evanescent waves $k_r^2 < 0$ (exponential damping). The solid lines show $k_r^2 = 0$.
- Evanescent waves occur when the period is so long that the whole (exponentially stratified) medium has time to adapt to the perturbation, achieving a new equilibrium. Therefore the wave does not propagate, but rather the medium as a whole oscillates.



Cutoff frequency for acoustic waves in a stratified medium:

$$\omega_C = C_S/2H$$

Global helioseismology

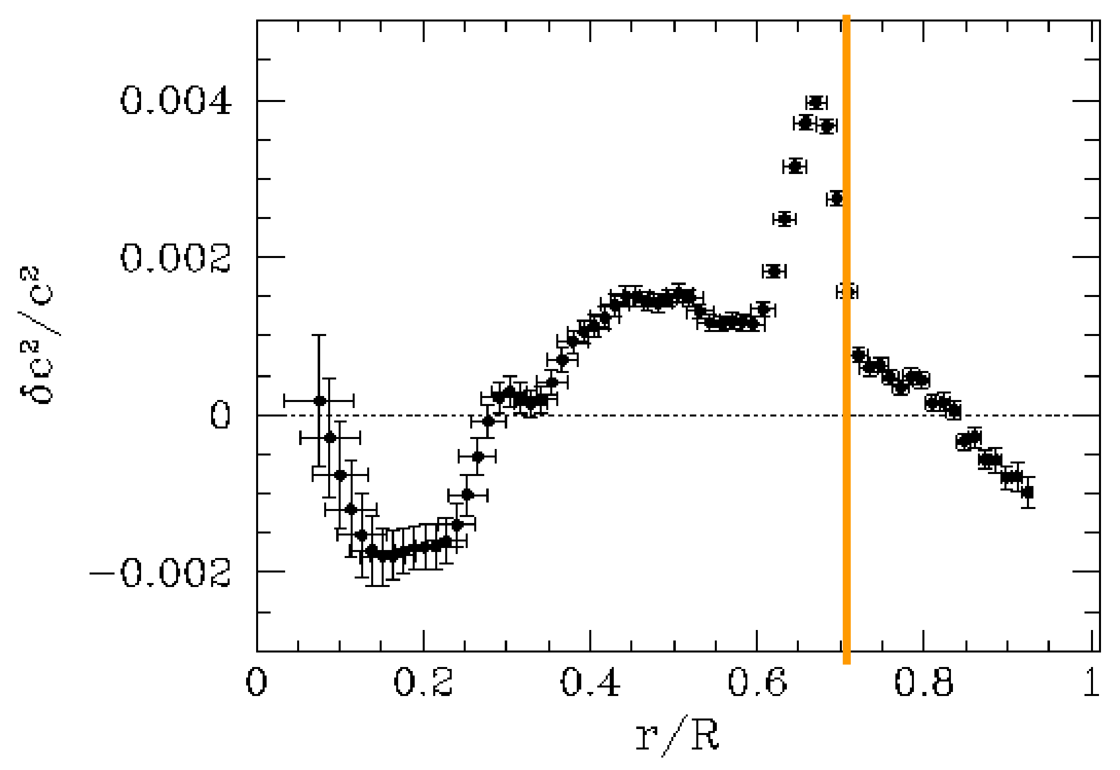
- Use frequencies of many modes.
- Basically two techniques for deducing information on the Sun’s internal structure
  - Forward modelling: make a model of the Sun’s internal structure (e.g. standard model discussed earlier), compute the frequencies of the eigenoscillations of the model and compare with observations
  - Inverse technique: Deduce the sound speed and rotation by inverting the oscillations (i.e. without any comparison with models)
- Note that forward modelling is required in order to first identify the modes. Only after that can inversions be carried out.

Deducing internal structure from solar oscillations

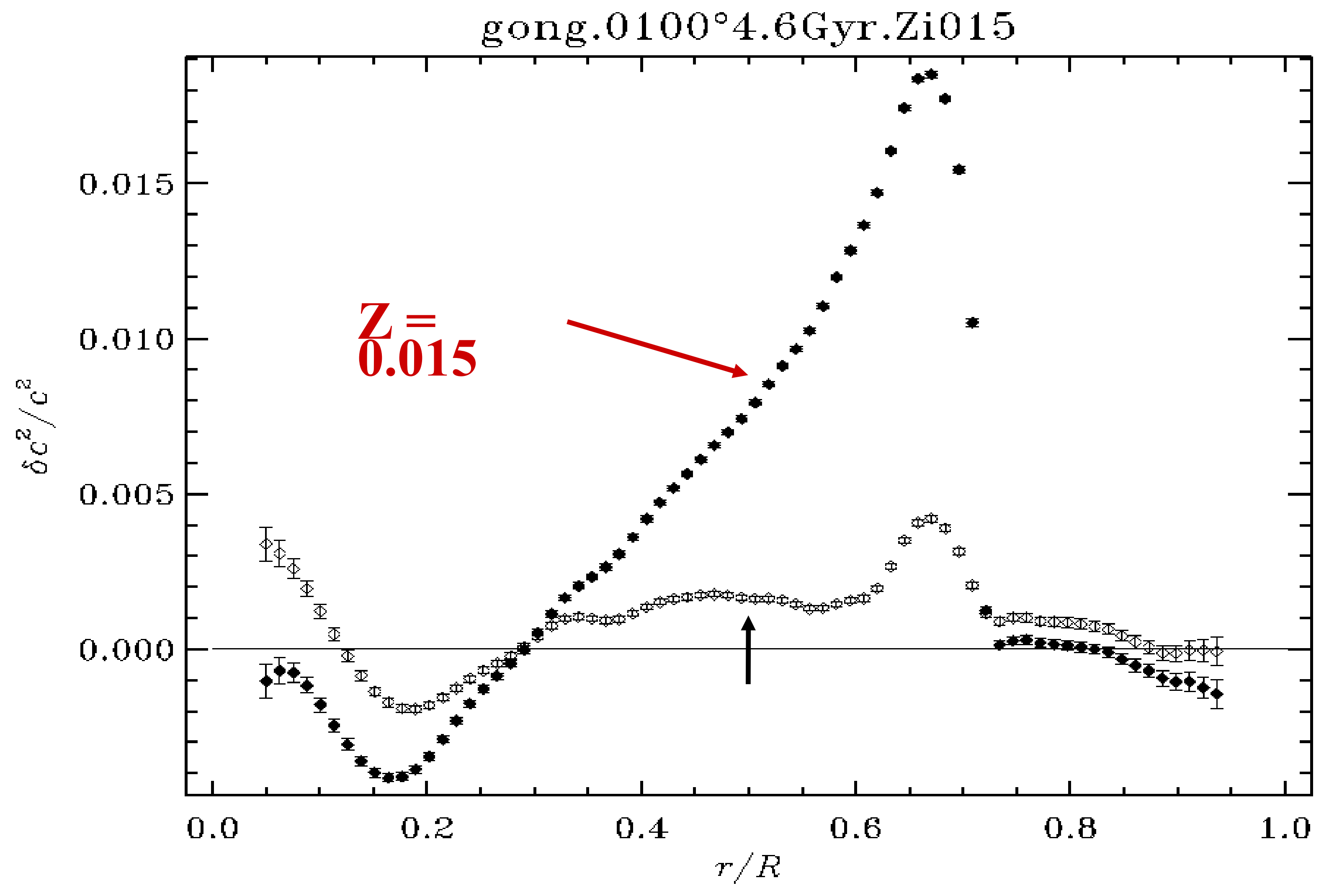
- Global helioseismology: Gives mainly the radial dependence of solar properties, although latitudinal dependence can also be deduced (ask R. Mecheri).
  - Radial structure of sound speed
  - Structure of differential rotation
- Local helioseismology: Allows in principle 3-D imaging of solar interior. E.g. time-distance helioseismology does not measure frequencies, but rather the time that a wave requires to travel a certain distance (relatively new)

Testing the standard solar model: results of forward modelling

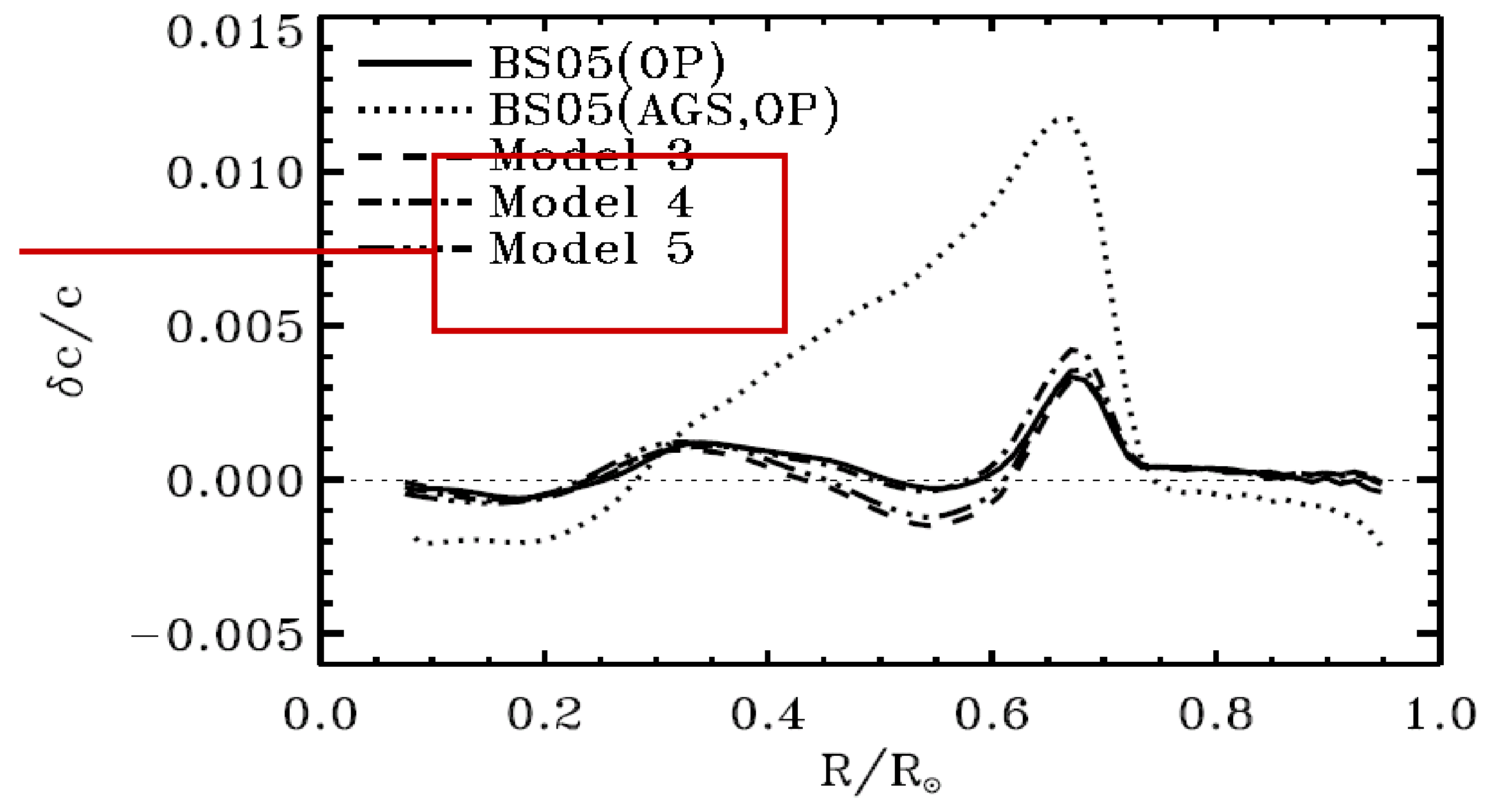
- Relative difference between C_S^2 obtained from inversions and from standard solar model plotted vs. radial distance from Sun centre.
- Typical difference:
  ➔ good!
- Typical error bars inversion:
  poor!
- Problem areas:
  - solar core
  - bottom of CZ
  - solar surface



Revision of solar surface abundances



The neon story



Ne x 2.5

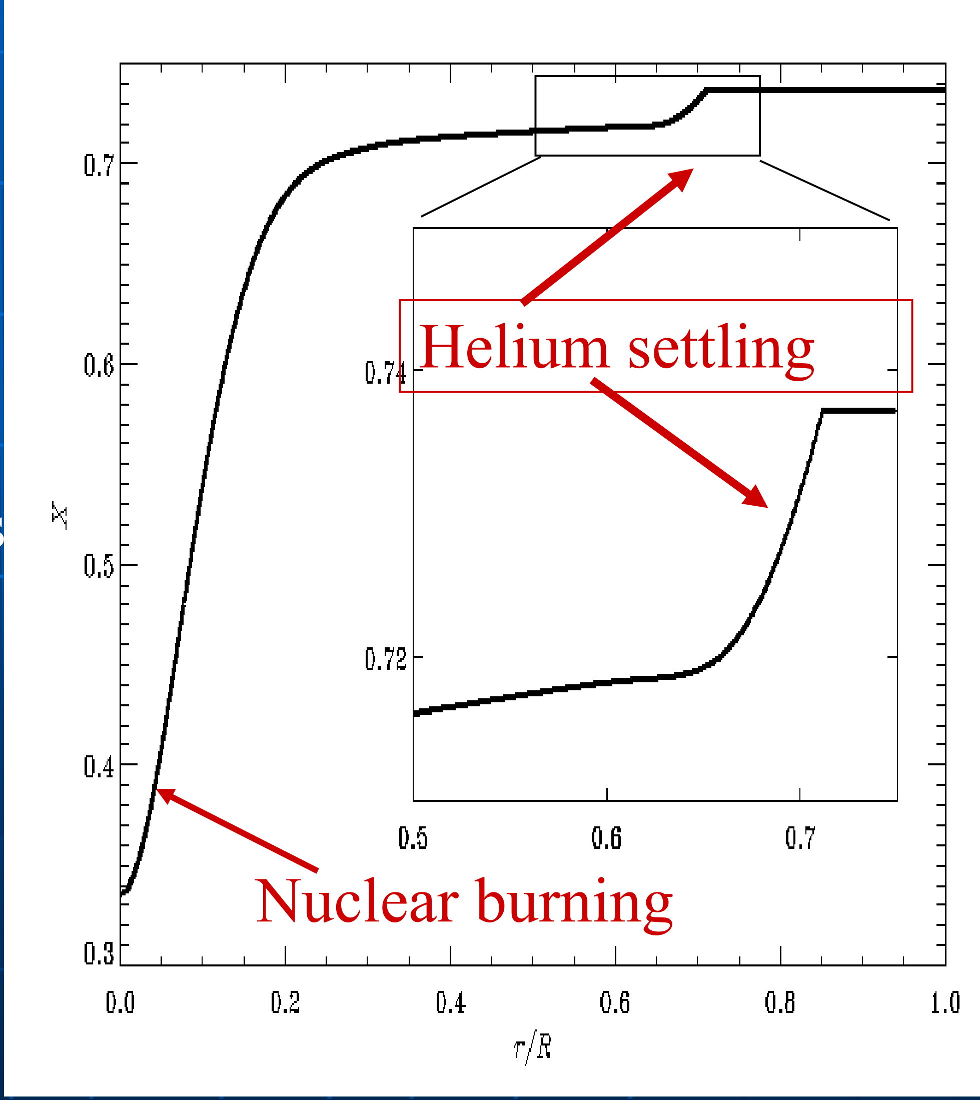
Bahcall et al. (2005; ApJ, in the press [astro-ph/0502563])

Drake & Testa (2005; Nature, in the press [astro-ph/0506182 v1]): X-ray observations of nearby stars indicate such a neon increase

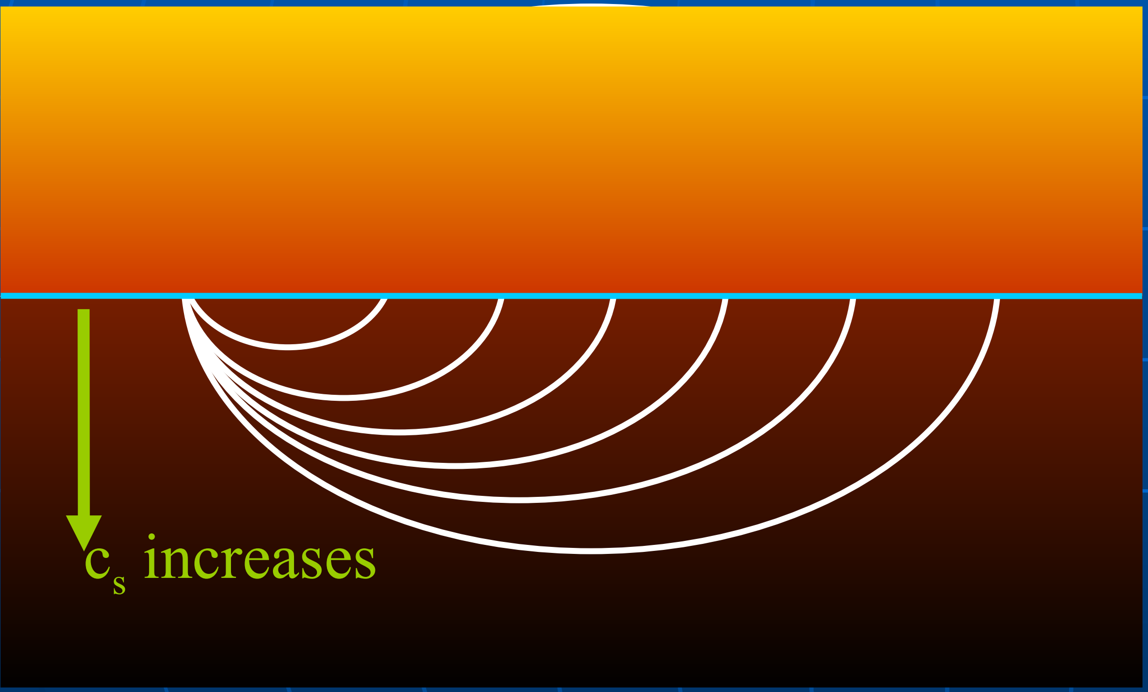
Changes in composition

The evolution of stars is controlled by the changes in their interior composition:

- Nuclear reactions
- Convective mixing
- Molecular diffusion and settling
- Circulation and other mixing
- processes outside convection zones



Local excitation of wave by a flare

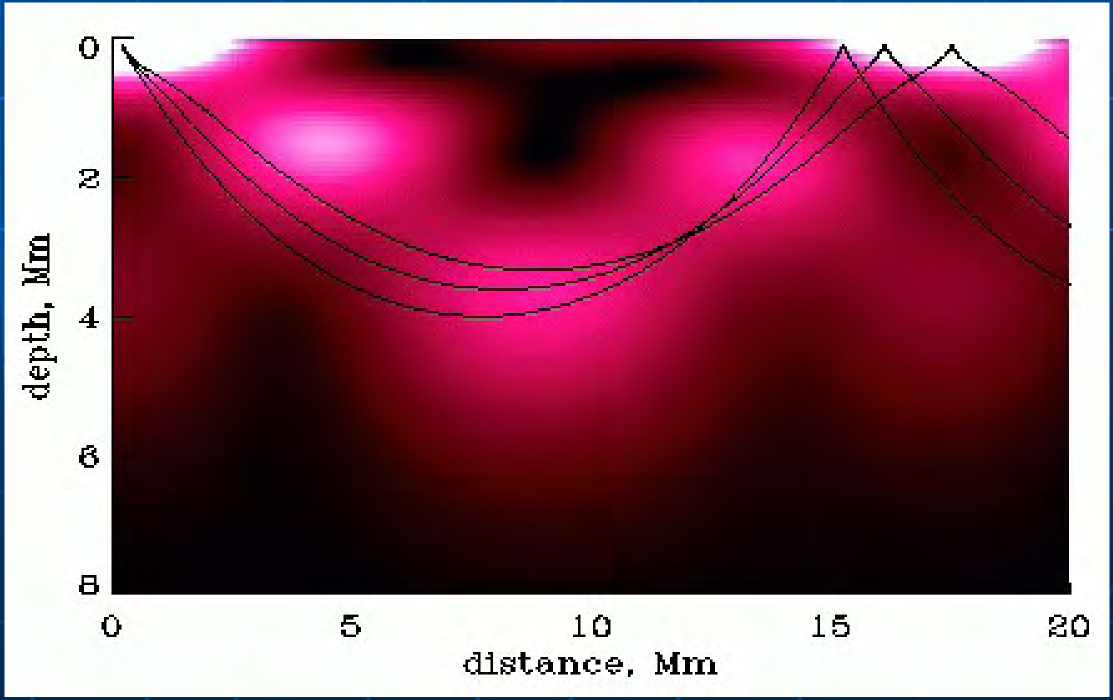


- Clear example of wave being triggered.
- The wave is not travelling at the surface, but rather reaching the surface further out at later times. Note how it travels ever faster. Why?

Local helioseismology

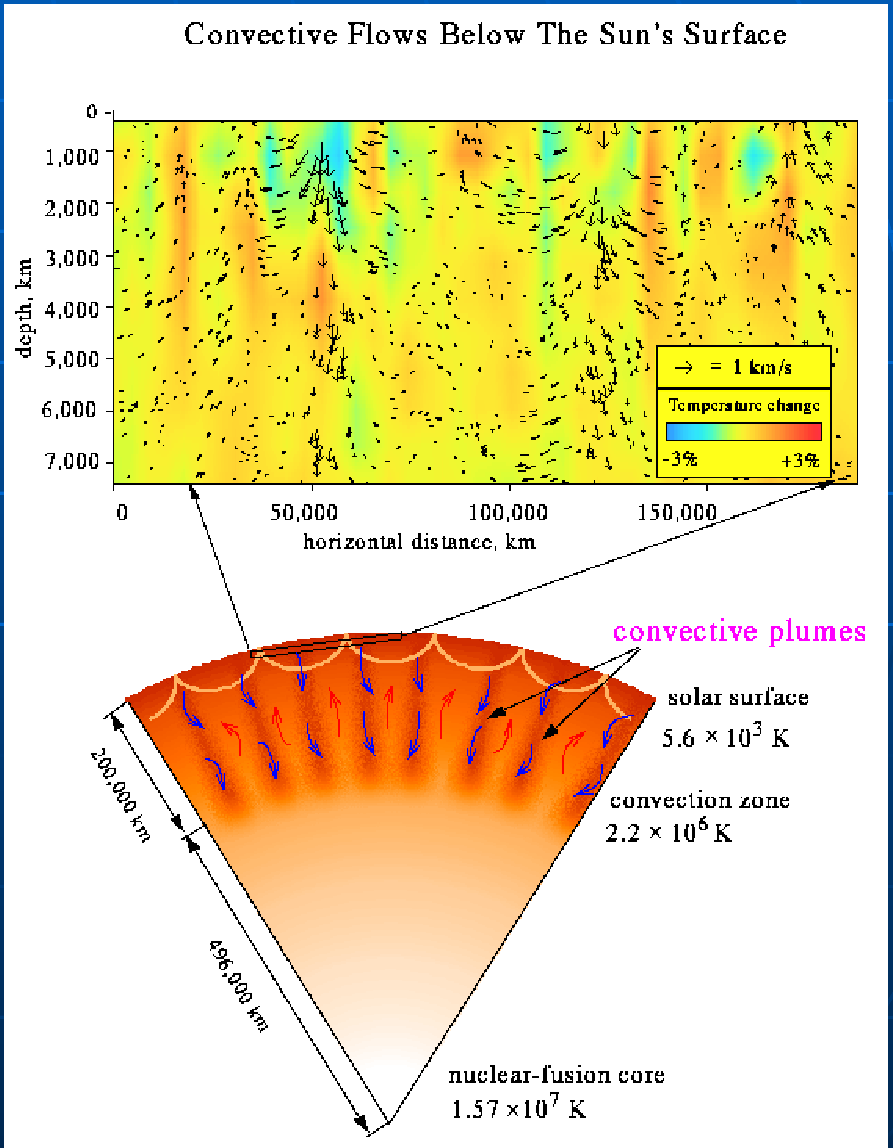
- Does not build upon measuring frequencies of eigenmodes, but rather measures travel times of waves through the solar interior, between two “bounces” at the solar surface (for particular technique of time-distance helioseismology).

- The travel time between source and first bounce depends on the structure of C_S below the surface. By considering waves following different paths inhomogeneous distributions of C_S can be determined.



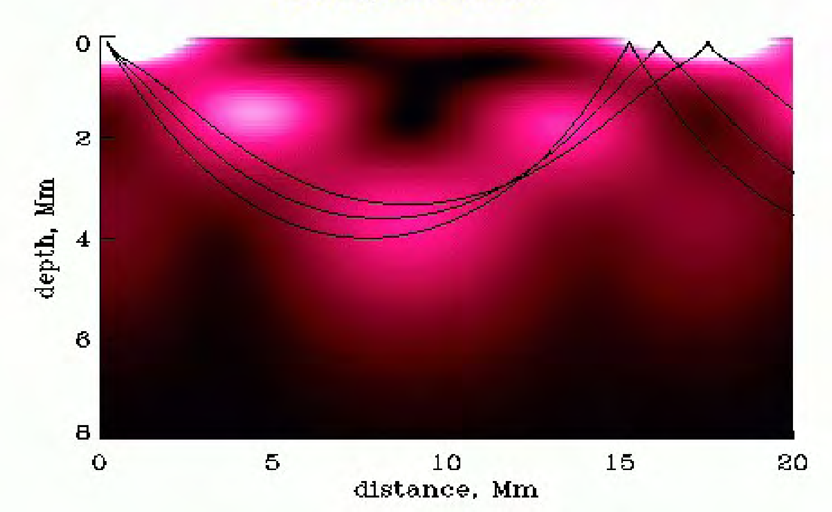
Local helioseismology II

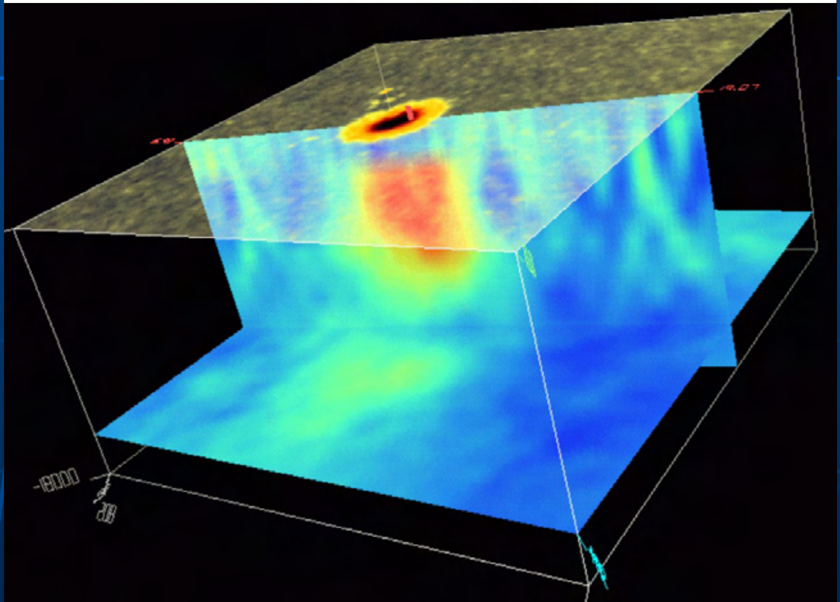
- Temperature and velocity structures can be distinguished, since a flow directed with the wave will affect it differently than a flow directed the other way (increase/decrease the sound speed).
- By considering waves passing in both directions it is possible to distinguish between T and velocity.
- At right: 1st images of convection zone of a star!



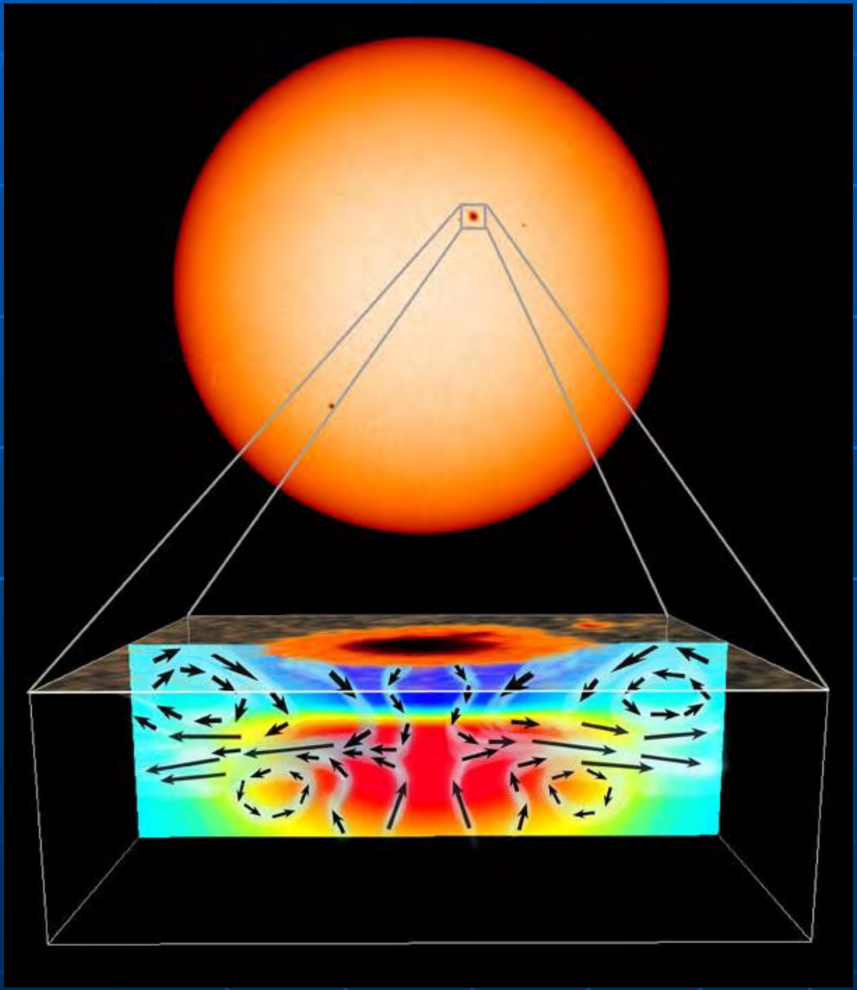
Time-Distance Helioseismology of a sunspot

- Subsurface structure of sunspots
- Sunspots are good targets, due to the large temperature contrast.
- Major problem: unknown influence of the magnetic field on the waves.





Time-Distance Helioseismology of a sunspot II



Kosovichev et al. 2000

Zhao et al. 2004

Helioseismology instruments

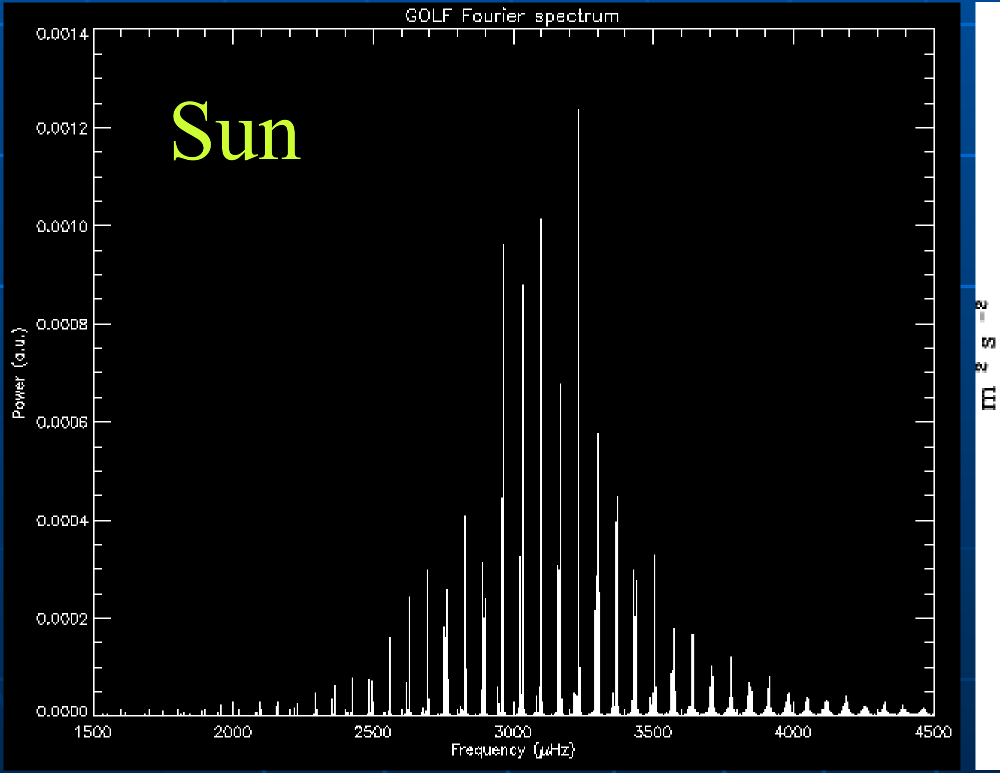
- Needed:
  - uninterrupted, long time series of observations
  - Either high velocity sensitivity, or high intensity sensitivity (and extremely good stability)
  - Low noise
  - Spatial resolution better than 1” (for local helioseismology)
- Instruments are either:
  - Ground based global networks (GONG+, BiSON)
  - Space based instruments in special full-Sun orbits (advantage of lower noise relative to ground-based networks; MDI, GOLF, VIRGO on SOHO, HMI on SDO)
  - Usually filter instruments with high spectral or intensity fidelity

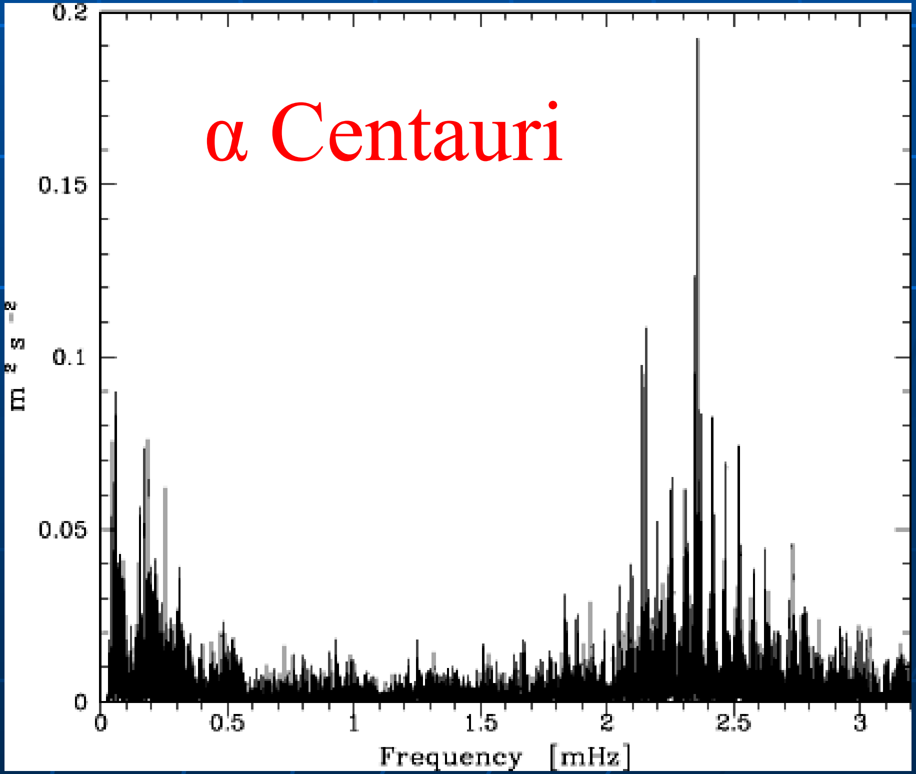
Instruments and projects

- Ground (networks of 3 or more telescopes aimed at reducing the length and number of data gaps)
  - GONG+
  - BISON
  - TON
- Space (uninterrupted viewing, coupled with lack of noise introduced by the atmosphere)
  - SOHO MDI, GOLF and VIRGO (running)
  - SDO HMI (being built)
  - Solar Orbiter VIM (planned)

Asteroseismology

First reliable detection of oscillations on the near solar analogue, α Centauri, and other Sun-like stars. Note the shift in the p-mode frequency range to lower values for α Centauri, which is older than the Sun (note also factor 10^3 difference in ν scale)

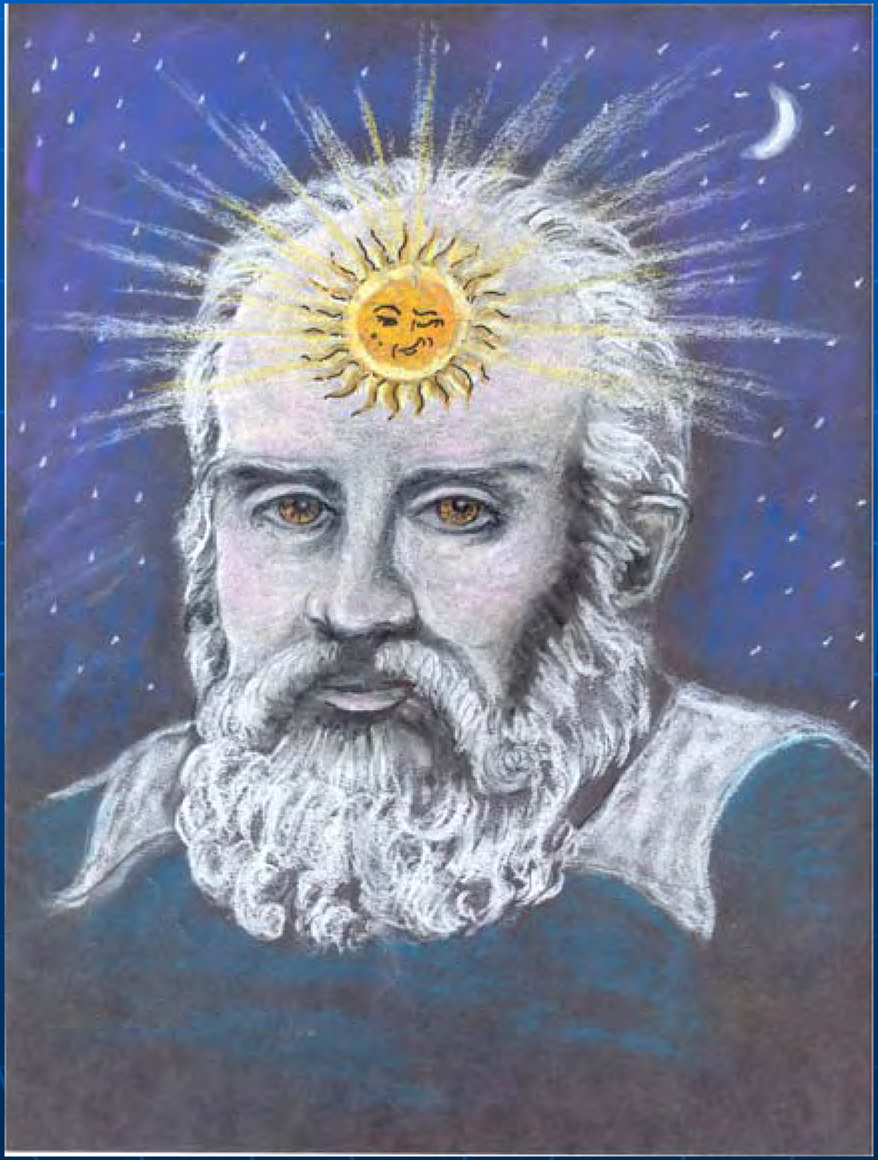




Projects

- Major asteroseismic Space missions:
  - COROT
  - Kepler
- Ground based:
  - ESO 3.6m (HARPS)
  - ESO VLT (UVES)
  - Networks of smaller Telescopes

Some more questions and theoretical answers ... some later ...



Some more questions and experimental answers in the future …

Some more images ...

Do you like it ?