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ABSTRACTS

ALGEBRAIC MODEL THEORY = CYLINDRIC SET ALGEBRAS

by H. Andréka

AMS classification code: 03C98, 03G30, 03G15.

The theory of cylindric set algebras (Cs-s) was elaborated in the book Henkin,L. Monk,J.D. Tarski,A. Andréka,H. Németi,I.: Cylindric Set Algebras, Lecture Notes in Mathematics 883, Springer-Verlag, 1982, with the aim to extend the algebraic logic approach of cylindric algebra theory to the more subtle issues of that part of logic which is called model theory. The cylindric algebraic counterparts of model theoretic notions, results and problems will be exhibited. E.g.: it will be proved that epimorphisms are surjective in the variety Gs of generalized Cs-s iff the solution of a certain open problem in definability theory is positive. (Definability theory is a branch of model theory.) It will be shown that the cylindric algebraic counterparts of model theoretic results are often stronger than the original ones e.g. the various ultraproduct theorems, the Keisler-Shelah theorem, results concerning elementary substructures, atomic models, definitional equivalence etc. Some of these have new model theoretic consequences first obtained by Cs-theory and not by "pure logic". Not only single models but also classes of models (e.g. axiomatizable classes) are represented by Gs-s. Logical connections between classes of models turn out to be nothing but special homomorphisms between Gs-s. (These are called base-homomorphisms.) Thus not only the Gs-s themselves but also their homomorphisms do have a rather characteristic model theoretic meaning. E.g. the reduct operator $\mathrm{Rd}_{\oplus} : BA \to Gr$ between the class BA of Boolean algebras and Gr of groups associating to each BA its reduct the only operation of which is the symmetric difference $\oplus$ is nothing but a certain base-homomorphism between two Gs-s. The category of base-homomorphisms is proved to be an iso-reflective subcategory of Gs if we assume the existence of an inaccessible cardinal. This category is isomorphic to the category of axiomatizable model classes and reduct-operators between them.

Some model theory for regularly closed fields

by Şerban A. Basarab

Given a class $\mathcal{G}$ of finite groups that is closed under homomorphic images, we associate to $\mathcal{G}$ the category $C_{\mathcal{G}}$ whose objects are the fields and whose morphisms are the field extensions F/K subject to: for every algebraic extension L of K and for every finite Galois extension M/L with $G(M/L) \in \mathcal{G}$, M and LF are linearly disjoint over L. Such field extensions are called $\mathcal{G}$-extensions.

Given a closed class $\mathcal{G}$ of finite groups [2] and an admissible [3] one $\mathcal{G}'$, we consider the full subcategory of $C_{\mathcal{G}}$, denoted by $C_{\mathcal{G},\mathcal{G}'}$, whose objects are the fields K for which the finite continuous homomorphic images of $G(K_s/K)$ are members of $\mathcal{G}'$. The category $C_{\mathcal{G},\mathcal{G}'}$ can be presented as the category of models of certain theory $\mathcal{G}\mathcal{G}'FL$ in a suitable extension of the customary language of rings with identity.

A field K is called $\mathcal{G},\mathcal{G}'$-regularly closed if K is an object of $C_{\mathcal{G},\mathcal{G}'}$ and is existentially complete in each regular extension F/K, where F is an object of $C_{\mathcal{G},\mathcal{G}'}$ too. The class of $\mathcal{G},\mathcal{G}'$-regularly closed fields is an elementary one and its existential theory is decidable if $\mathcal{G}$ and $\mathcal{G}'$ are recursive classes. Some other model theoretic properties of these fields are discussed. Extensions of some results from [1], [2], [3], [4], [5], [6] are obtained.

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AMS classification: 03C60, 12L05

A LATTICE OF DEGREES OF PROVABLE EXTENSIONS OF PEANO ARITHMETIC (ABSTRACT)

AMS subject classification 1980: 03F25 (relative consistency and interpretations)

It is well known that when looking for 'natural' binumerations of (extensions of) Peano arithmetic (P), we should restrict our attention to the primitive recursive (p.r.) ones (cf [Feferman]). But the work of [Feferman] and [Hájková] also seem to imply that among the p.r. binumerations of a theory there is no 'natural' choice. Therefore, when investigating the structure of extensions of P under some relation of relative consistency, it appears reasonable to choose an arbitrary but fixed p.r. binumeration of P, and look at the class of provable extensions of this binumeration. This course is followed in the present paper, where the above class is structured by a very strong notion of relative consistency.

Let $\pi(x)$ be any p.r. binumeration of P. A p.r. formula $\alpha(x)$ is a provable extension of $\pi(x)$ iff $P \vdash \pi(x) \rightarrow \alpha(x)$. Define a partial order on the provable extensions of $\pi(x)$ by

$$\alpha \leq \beta \text{ iff there is a p.r. term } t \text{ s.t.}$$

$$P \vdash \forall x(Prf_{\alpha}(\overline{0=1}, x) \rightarrow Prf_{\beta}(\overline{0=1}, tx)).$$

Let $\alpha \equiv \beta$ iff $\alpha \leq \beta$ and $\beta \leq \alpha$. Note that if $\alpha \leq \beta$ then $P \vdash con_{\beta} \rightarrow con_{\alpha}$ (we show that the converse is not true). Let the degrees be the equivalence classes under $\equiv$, partially ordered in the obvious way.

Let a, b be degrees. We show that every degree has an element of the form $\pi + \psi$ (i.e. $\pi(x) \vee x = \overline{\psi}$), where ψ is a Π^0_1-sentence, and that for $\pi + \phi \in a$ and $\pi + \psi \in b$, g.l.b. and l.u.b. may be defined by

$$a \cap b = d(\pi + \phi \vee \psi) \quad \text{and}$$

$$a \cup b = d(\pi + \theta), \quad \text{where } \theta \text{ is s.t.}$$

$$P \vdash \theta \leftrightarrow \forall y(Prf_{\pi}(\overline{\neg\phi}, y) \vee Prf_{\pi}(\overline{\neg\psi}, y) \rightarrow \exists z{<}y Prf_{\pi}(\overline{\neg\theta}, z)).$$

The structure thus obtained is shown to be a distributive, dense lattice with greatest and lowest elements. We also show a number of results concerning details of the lattice structure, e.g. the (non-)existence of relative complements, and investigate the relative positions of (binumerations) of certain extensions of P.

A comparison is made between this lattice and other similar structures (the lattice of types of interpretability, see [Lindström], and Hájková's lattice [Bin]).

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AMS 03C60

SUPERSTABLE GROUPS.

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Using Shelah's forking and Lascar's U-rank we initiate a classification of superstable groups and generalise to these groups Cherlin's [1] and Zil'ber's [4] results on ω-stable groups of finite Morley rank (in connection with algebraic groups over algebraically closed fields). We mention here some consequences easy to state.

Thanks to Poizat [3] one can associate to each superstable group G an ordinal U(G). We show for exemple that every superstable group G such that $U(G) \geqslant \omega^{\alpha}$, α any ordinal, has a definable abelian subgroup H such that $U(H) \geqslant \omega^{\alpha}$. This allows to give a short proof of the following difficult theorem of Cherlin: " Every superstable field is commutative".

Elsewhere we have as a corollary that every group which is elementarily equivalent to a superstable simple group is simple too.

Also we generalise to superstable groups of U-rank $\omega^{\alpha}2$ or $\omega^{\alpha}3$, α any ordinal, the structural results of Cherlin on ω-stable groups of Morley rank 2 or 3 and extended by Cherlin and Shelah [2] to superstable groups of ∞-rank 2 or 3 (hence of U-rank 2 or 3) and prove that"good" simple superstablegroups of U- rank $\omega^{\alpha}3$ are algebraic groups.

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APPLICATIONS TO LOGIC OF ERŠOV NUMERATION THEORY

by Claudio Bernardi and Franco Montagna

Eršov numeration theory has been deeply studied both from a categorical point of view and from a recursive - theoretical one. Visser first applied it to logic, pointing out interesting examples and consequences. Our purpose is to study connections between positive numerations, regarded as positive equivalence relations on ω, and formulas of *PA*.

A useful lemma is the following: *any precomplete positive equivalence relation is complete with respect to 1 - reducibility.* The converse is not true: a counter-example is the equivalence relation $\sim$ which associates two numbers x,y iff x,y are Gödel numbers of provably equivalent sentences. Actually, we have:

THEOREM. - *A positive equivalence relation* $\mathfrak{R}$ *is recursively isomorphic to* $\sim$ *iff 1) it admits a total recursive diagonal function* Δ *(i.e.* $\Delta x \not\mathfrak{R} x$*) and 2) it is u.f.p. (i.e. every partial recursive function having a finite range can be made total in a uniform way modulo* $\mathfrak{R}$ *).*

(Note that a precomplete relation always satisfies 2, but never satisfies 1).

Now, for every formula $F(v)$ we define the equivalence relation $\sim_F$ as follows: $x \sim_F y$ iff $\vdash F(x) \longleftrightarrow F(y)$.

THEOREM. - *Every positive equivalence relation coincides with* $\sim_F$ *for a suitable formula* $F(v)$.

As regards formulas $G(v)$ which preserve provable equivalence, we have:

THEOREM.- *An equivalence relation is isomorphic to* $\sim_G$ *for a suitable* $G(v)$ *iff it is 1) total, or 2) isomorphic to* $\sim$ *, or 3) precomplete.*

An example of formula of the kind 3 is $Theor(v)$. So, for every recursive function f, there exists a sentence α such that $\vdash Theor(\ulcorner\alpha\urcorner) \longleftrightarrow Theor(f\ulcorner\alpha\urcorner)$.

Another logical consequence of numeration theory, which generalizes a classical result of Putman - Smullyan, is the following:

THEOREM. - *Let* $(\alpha_i)_{i\in\omega}$ *and* $(W_i)_{i\in\omega}$ *be r.e. sequences of* Σ_n*-formulas and r.e. sets respectively, such that if* $\vdash \alpha_i \longleftrightarrow \alpha_j$ *then* $W_i = W_j$ *and if* $\nvdash \alpha_i \longleftrightarrow \alpha_j$ *then* $W_i \cap W_j = \emptyset$. *Then there exists a* Σ_{n+1}*- formula* $H(v)$ *such that* $\vdash H(x) \longleftrightarrow \alpha_i$ *iff* $x \in W_i$.

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INDEPENDENT INSTANCES FOR SOME UNDECIDABLE PROBLEMS

By

Cristian CALUDE and Gheorghe PAUN

Let T be a formalized theory having the following four properties : a) T is recursively axiomatizable, b) T is consistent, c) all the theorems deductible in T are true at the level of metalanguage, d) T is rich enough to contain the recursive arithmetic. With respect to any such theory we prove that the Emptiness Problem, the Finiteness Problem, the Totality Problem, the Halting Problem, and the Post Correspondence Problem have independent instances (which can be algorithmically built). As consequences, a diophantine equation can be effectively found for which the solution existence is an independent question and for each basic undecidable problem in Formal Language Theory an independent instance can be effectively constructed.

Abstract for "On the complexity of winning strategies for clopen games"

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By title

In [1] Blass considered the effective version of the Gale-Stewart theorem that every open game is determined. We present a refinement of Blass' results. A clopen set $S \subseteq \omega^\omega$ for the Baire topology is given by sets $A, B \subseteq \omega^{<\omega}$ such that

$$S = \{ f \in \omega^\omega : \exists s \in A \quad s = f \upharpoonright lh(s) \}$$

$$\bar{S} = \omega^\omega - S = \{ f \in \omega^\omega : \exists \ s \in B \quad s = f \upharpoonright lh(s) \}.$$

The pair A,B can be taken minimal in the sense that

$$\forall s, t \in \omega^{<\omega} [s \in A \cup B \ \& \ s = t \upharpoonright lh(s) \ \& \ s \neq t \longrightarrow \quad t \notin \ A \cup B].$$

Let $T = \{ t \in \omega^{<\omega} : \exists s \in A \cup B \quad t = s \upharpoonright lh(t) \}$.

For A,B minimal the set T is a tree without any infinite branch, so as usual one can define the height of T. For $S \subseteq \omega^\omega$ clopen the height of S denoted ht(S) is the smallest ht(T) where T corresponds to A,B determining S.

Theorem 1. Let $S \subseteq \omega^\omega$ be clopen with recursive code. Let $n < \omega$ and λ denote a recursive limit ordinal with fundamental sequence $\lambda_n \uparrow \lambda$.

Then

$ht(S) \leq 2 \longrightarrow$ one of the players has a recursive w.s.

$ht(S) \leq n+3 \longrightarrow$ either player 1 has a w.s. σ recursive in 0^n or player 2 has a w.s. recursive in 0^{n+1}

$ht(S) \leq \lambda \longrightarrow$ either player 1 has a w.s. σ recursive in 0^{λ_n} some $n < \omega$ or player 2 has a w.s. σ recursive in 0^λ

$ht(S) \leq \lambda+1 \longrightarrow$ either player 1 or player 2 has a w.s. recursive in 0^λ

$ht(S) \leq \lambda+n+2 \longrightarrow$ either player 1 has a w.s. σ recursive in $0^{\lambda+n}$ or player 2 has a w.s. recursive in $0^{\lambda+n+1}$.

With the same notations from Theorem 3. we prove

Theorem 2. There are clopen $S \subseteq \omega^\omega$ with recursive code satisfying

$ht(S) = n+3$	and	player 2 wins and 0^{n+1} is uniformly recursive in every winning strategy
$ht(S) = \lambda$	and	player 2 wins and 0^λ is uniformly recursive in every winning strategy
$ht(S) = \lambda+1$	and	player 1 wins and 0^λ is uniformly recursive in every winning strategy
$ht(S) = \lambda+n+2$	and	player 2 wins and $0^{\lambda+n+1}$ is uniformly recursive in every winning strategy.

Noting that if A,B are minimal then A B is well-ordered lexicographically, define the order type of $S \subseteq \omega^{\omega}$, for S a clopen set, to be the smallest order type of $A \cup B$, where A,B determine S.

<u>Theorem</u> 3. Let λ be any recursive limit ordinal.
Then (1) for every $S \subseteq \omega^{\omega}$ clopen with recursive code and order type $\leq \omega^{\lambda}$ either player 1 has a w.s. recursive in 0^{λ_n} for some $n < \omega$ (where λ_n is a fundamental sequence for λ) or player 2 has a w.s. recursive in 0^{λ}
and (2) there is $S \subseteq \omega^{\omega}$ clopen with recursive code and order type ω^{λ} where 2 wins the game G_S and 0^{λ} is uniformly recursive in every w.s. of player 2. ◼

<u>Remark</u>. The essential point of these results is that, as $\Delta^1_1 = \bigcup_{\alpha \text{ rec}} \Sigma^0_{\alpha}$, winning strategies for clopen games with recursive code are simply generalized Skolem functions for infinitary $L_{\omega_1^{ck},\omega}$ formulae. The most obvious evaluation of the complexity of these Skolem functions (basis result) is in general the best possible (anti-basis result). The height of the clopen set corresponds to a generalized way of counting the number of quantifier alternations.

1 A. Blass "Complexity of Winning Strategies" Discrete Mathematics
Volume 3 Number 4 1972

On the existence of finitely determinate models for some theories in Stationary Logic. By Joël Combase.

AMS Subject classification: 03 C 80.

Given a first order Language L, let L^{aa} be the corresponding language for stationary logic.

If A is a set whose cardinal number K is regular and $>\omega$, define $P<(A)$ to be the set of subsets of A of cardinally $<K$ and let $D<(A)$ be the closed unbounded filter on $P<(A)$. A natural model for a theory T in L^{aa} is a first order structure $OL=\langle A,\ldots\rangle$ for L where it is under stood that the satisfaction relation obeys the rule

$$OL \models aas\ \phi(s) \text{ iff } \{a\in P<(A):\ OL \models \phi(a)\}\in D<(A)$$

An aa, - theory is a theory in L^{aa} whose every axiom is of the form $aa\ \vec{s}\ \Theta$, where Θ is a 1st order formula.

Theorem 1. Let T be an aa, - theory. If for every n, T has a natural model whose cardinality is n-ineffable, then T has a finitely determinate (natural) model.

Theorem 2. Let T be a finitely determinate theory. If for every n, T has a natural model whose cardinal is n-ineffable, then T has a natural model in every regular uncountable cardinality.

A NEW FOUNDATION FOR THE THEORY OF RELATIONS

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ABSTRACT

A polygroupoid is a partial multivalued algebra

$$\mathfrak{M} = \langle M, \cdot, I, ^{-1} \rangle$$

where $\cdot$ is a partial multivalued binary operation on M, $I \subseteq M$, and $^{-1}$ is an operation on M that satisfies the following axioms:

(i) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all $x,y,z \in M$,

(ii) $I \cdot x = x = x \cdot I$ for all $x \in M$,

(iii) the formulas $x \in y \cdot z$, $y \in x \cdot z^{-1}$, and $z \in y^{-1} \cdot x$ are equivalent for all $x,y,z \in M$.

Denote the complex algebra of $\mathfrak{M}$ (see[1,Definition 3.8]) by $C[\mathfrak{M}]$.

Section 5 of [1] presents a relationship between certain relation algebras and (generalized) Brandt groupoids. The results below extend these connections.

THEOREM 1. The complex algebra $C[\mathfrak{M}]$ of a polygroupoid $\mathfrak{M}$ is a complete atomic relation algebra with $0 \neq 1$. Conversely, if

$$\mathfrak{A} = \langle A, +, \cdot, 0, 1, ;, 1', ^{\smile} \rangle$$

is a complete atomic RA with $0 \neq 1$, M the set of all atoms of $\mathfrak{A}$, and $I = \{x \in M : x \leq 1'\}$, then $\mathfrak{M} = \langle M, ;, I, ^{\smile} \rangle$ is a polygroupoid (called the atomic structure of $\mathfrak{A}$) and $\mathfrak{A} \cong C[\mathfrak{M}]$.

COROLLARY 2. Every relation algebra is embeddable in the complex algebra of a polygroupoid.

A polygroupoid $\mathfrak{M}$ is connected if for all $x,y \in I$ there exist $z \in M$ such that $x \cdot z = z$ and $z \cdot y = z$.

THEOREM 3. If $\mathfrak{M}$ is a connected polygroupoid, $C[\mathfrak{M}]$ is a simple RA. Conversely, if $\mathfrak{A}$ is a simple RA, its atomic structure $\mathfrak{M}$ is connected.

COROLLARY 4. Every simple RA is embeddable in the complex algebra of a connected polygroupoid.

These results show that multivalued systems can be used to provide an alternative to the usual approach to the study of relations. Examples will be given.

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COMPLETENESS OF TYPE ASSIGNMENT IN CONTINUOUS LAMBDA MODELS

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The completeness of Curry's rules for assigning type schemes to terms of the pure lambda-calculus has been proved in [1] , [2] using models of synctactic nature. A first result of this paper is a completeness proof for the model P_ω (as asked in [3]). Moreover an extension of Curry's system in which types can be assigned to the fixpoint combinator Y is introduced, together with a notion of type semantics for which it is proved sound and complete (answering a question of [4]). Also in this case completeness is proved in the model P_ω . All results hold for the different notions of type semantics proposed in [1] , [3] .

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BETA STRONG REDUCTION IN COMBINATORY LOGIC: PRELIMINARY REPORT

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Combinatory logic differs from λ-calculus in that in the former the operation of abstraction (the λ-operation) is defined and not primitive. The rule

(ξ) $X = Y \rightarrow \lambda x.X = \lambda x.Y$

is part of the replacement property in λ-calculus, but it must be postulated separately in combinatory logic. It is well known that it is possible to define equality relations corresponding to λβ- and λβη-conversion by adding a finite number of axioms to weak (ordinary) combinatory equality. But for reduction the situation is not so simple. There is strong reduction, which corresponds to λβη-reduction, but to define it by starting with weak reduction, one must either postulate (ξ) or else use an infinite number of axioms; furthermore, it is necessary to define abstraction in such a way that the property

(η) $\lambda x.Ux = U$, where x does not occur (free) in U,

is incorporated in the definition as a syntactic identity between terms. For this reason, there is not yet any agreement on a combinatory analogue for λβ-reduction.

This talk presents two candidates for a combinatory analogue for λβ-reduction and discusses the criteria such a relation should satisfy.

AMS Classification 1980 : 03B45 (Modal Logic)
03F99 (Proof Theory)

Kosta Došen : Sequent-Systems for Modal Logic

Abstract

The purpose of this work is to present Gentzen-style formulations of S5 and S4 based on sequents of higher levels. Sequents of level 1 are like ordinary sequents, sequents of level 2 have collections of sequents of level 1 on the left and right of the turnstile, etc. Rules for modal constants involve sequents of level 2, whereas rules for customary logical constants of first-order logic with identity involve only sequents of level 1. A restriction on Thinning on the right of level 2, which when applied to Thinning on the right of level 1 produces intuitionistic out of classical logic (without changing anything else), produces S4 out of S5 (without changing anything else).

This characterization of modal constants with sequents of level 2 is unique in the following sense. If constants which differ only graphically are given a formally identical characterization, they can be shown inter-replaceable (not only uniformly) with the original constants salva provability. Customary characterizations of modal constants with sequents of level 1, as well as characterizations in Hilbert-style axiomatizations, are not unique in this sense. This parallels the case with implication, which is not uniquely characterized in Hilbert-style axiomatizations, but can be uniquely characterized with sequents of level 1.

These results bear upon theories of philosophical logic which attempt to characterize logical constants syntactically. They also provide an illustration of how alternative logics differ only in their structural rules, whereas their rules for logical constants are identical.

ADJ SEMANTICS AS MODELS

Let G be a formal grammar, not necessarily context-free, and Pars*(G) the strict monoidal category associated to the parses (duals of derivations) of G. Call* G^o *the grammar obtained from G by 'forgetting' the terminal letters in the productions defining G. One defines easily a trivial Horn theory* $T(G^o)$ *associated to* G^o*, hence (following Bénabou-Coste or Reyes-Dionne) a category* $\underline{T}(G^o)$*.*

Theorem: Let $\underline{E}$ be a Grothendieck topos. One can define a Grothendieck topology J_G on Pars(G) so that to each $\underline{T}(G^o)$-algebra in $\underline{E}$ is associated a continuous functor from the site (Pars(G) , J_G) to the (site underlying the) topos $\underline{E}$.

Cor. 1: Any $\underline{E}$-valued ADJ-semantics is a model (ie a continuous functor) from the site (Pars(G) , J_G) to the topos $\underline{E}$.

Cor. 2: The ordinary ADJ-semantics is a model from the site (Pars(G) , J_G) to the topos Set.

Designate by ScOrd *the category of Scott-continuous ordered sets.*

Cor. 3: The Scott-continuous ADJ-semantics is a model from the site (Pars(G),J_G) to the site (ScOrd , J_c), where J_c is the canonical topology on ScOrd.

Hence ADJ-semantics are models in the sense of Reyes, ie continuous functors between sites.

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INTUITIONIST IMPLICATION IN SOME MODAL LOGICS OF TYPE S4

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We consider a wide class of modal logics of type S4 axiomatized in Gödel's style (i.e. with L primitive and axioms $Lp \to p$, $L(p \to q) \to (Lp \to Lq)$, $Lp \to LLp$, and Rule of Necessity $p \vdash Lp$) and we work in the corresponding algebraic models, namely topological algebras (Boolean, pseudo-Boolean, Hilbert, and other implicative algebras). If $\langle A, I, \cdot \rangle$ is such a topological algebra (where I is an interior operator) then we have the closure system $\mathfrak{D}$ of all the deductive systems of A ($D \subset A$ such that $1 \in D$, if $a \in D$ and $a \cdot b \in D$ then $b \in D$, and if $a \in D$ then $Ia \in D$) and so we have the associated consequence operator $\mathbf{D}$. We see that the main properties of this operator are well described by the intuitionist implication:

$$a \Rightarrow b = I(Ia \cdot Ib)$$

which was defined by A. Monteiro . The two main results are the following:

Deduction Theorem: $b \in \mathbf{D}(X,a)$ iff $a \Rightarrow b \in \mathbf{D}(X)$ $\forall a,b \in A$, $\forall X \subset A$.

Theorem: $\forall D \subset A$, D is a deductive system of A iff $1 \in D$ and if $a \in D$ and $a \Rightarrow b \in D$ then $b \in D$.

This operation allows us to give "implicative characterizations" of several concepts as completely irreducible and maximal deductive systems, radical, and semisimplicity. We also see that the abstract logic $L = \langle A, D \rangle$ associated with D is of type the same as the non-modal logic actually used.

Finally we can show the

Theorem: If $\langle A, \cdot \rangle$ is an algebra of the class quoted above and $\Rightarrow$ is a binary operation on A such that, for all $a,b,c \in A$:

(1) $a \Rightarrow a = 1$;

(2) $a \Rightarrow (b \Rightarrow c) = (a \Rightarrow b) \Rightarrow (a \Rightarrow c)$;

(3) $a \Rightarrow (b \cdot c) \leqslant a \Rightarrow (b \Rightarrow c)$; and

(4) $(a \Rightarrow b) \Rightarrow c \leqslant (a \Rightarrow b) \cdot c$,

then if we define $Ia = 1 \Rightarrow a$ $\forall a \in A$, $\langle A, I, \cdot \rangle$ is a topological algebra and $\Rightarrow$ is exactly the intuitionist implication associated with I .

From this we conclude that we can give a formalization of modal logics of type S4 (icluding S4 itself) by using none of the classical modal operators (L,M) but the intuitionist implication $\Rightarrow$.

AMS/AMOS 1980 # 03G , 03B45, 03B20, 06A ...

CAN CARDINAL ORDERING BE UNIVERSAL ?

by

Marco Forti* and Furio Honsell+

It is well known that if the Axiom of Choice is not assumed cardinal ordering is quite arbitrary. Jech and others proved that every partial ordering can be represented, in a model of ZF, by the injective ordering of cardinals.

The question arises whether the following axiom is consistent relatively to ZF:

$$CU \;:\; \forall x \forall r \,(\, r \text{ p. orders } x \rightarrow \exists f : x \rightarrow u \text{ bijective s. t. } \forall z,y \in x \;(\, z\, r\, y \longleftrightarrow |f(z)| \leq |f(y)| \,)\,)$$

i. e. any partial ordering r is isomorphic to the cardinal ordering on some set u.

In Gödel-Bernays-Von Neumann set theory without the Axiom of Foundation and with the axiom "there is a proper class of autosingletons" (which is equiconsistent with ZF_0), we define a generalized symmetric model $\mathfrak{N}$ taking the hereditarily symmetric sets for a suitable filter of permutations on the class of autosingletons. Then the following holds:

THEOREM : *For any partially ordered set x in $\mathfrak{N}$ such that fix x belongs to the generalized normal filter, there is an injective function g on P(x) such that*

$$\mathfrak{N} \models \forall p,q \subseteq x \left(p \subsetneq q \longleftrightarrow |g(p)| \leq |g(q)|\right)$$

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<u>Corollary</u> *The following axioms are consistent with ABC*

CU lin : *Any linear ordering is isomorphic to the cardinal ordering on some set.*

CU reg : *Any partial ordering on a well founded set is isomorphic to the cardinal ordering on some set.*

Note that CU in its full strenght fails in the model. Its consistency with ABC or ABCD is still an open problem, as is the consistency of the previous axioms with ZF (including regularity), for the transfer methods of Jech and Pincus cannot be directly generalized to fit this situation.

ALGEBRAIC LOGIC FOR THE QUANTIFIER "THERE EXISTS UNCOUNTABLE MANY"

George Georgescu

Let $(A,I,S,\exists,E)$ be a locally finite polyadic algebra of infinite degree. For any $p \in A$ we shall denote by J_p the minimal support of p. Consider a family of unary operations of A: $\{Q(i):A \to A \mid i \in I\}$ such that for any $i \in I$ and $p,q \in A$ we have the following properties:

(Q_1) $Q(i)\, 1 = 1$;

(Q_2) $Q(i)(E(i,j) \vee E(i,k)) = 0$ for $i \neq j,k$;

(Q_3) $\forall(i)(p \to q) \leq (Q(i)\, p \to Q(i)\, q)$;

(Q_4) $Q(i)\, p = Q(j)\, S(j/i)\, p$, where $j \notin J_p$;

(Q_5) $Q(j) \exists(i)\, p \leq \exists(i)\, Q(j)\, p \vee Q(i) \exists(j)\, p$, for any $j \in I$;

(Q_6) If J is a support of p, then $J-\{i\}$ is a support of $Q(i)\, p$;

(Q_7) For any $\sigma \in I^I$ such that $\sigma|_{\sigma^{-1}(\{i\})}$ is injective, we have $Q(i)\, S(\sigma) = S(\sigma)\, Q(j)$, where $\sigma(j) = i$.

A Q-algebra is a polyadic algebra $(A,I,S,\exists,E)$ with a family of unary operations $\{Q(i): i \in I\}$ such that the axioms (Q_1) – (Q_7) are verified.

The Q-algebras are the adequate algebraic structures for the logic L(Q) with the quantifier "there exists uncountable many". Any polyadic algebra of the form $F(X^I,O)$ has a canonical structure of Q-algebra if for any $i \in I$ we define $Q_o(i)$ by: $Q_o(i)\, p(x) = 1$ iff $\{u \in X \mid p((u/i)_* x) = 1\}$ is uncountable.

Representation Theorem. Let $\langle A, Q(i): i \in I\rangle$ be a countable Q-algebra of countable degreee and Γ a proper filter of Boolean algebra $B(A) = \{ p \in A \mid J_p = \emptyset \}$.

Then there exist $X \neq \emptyset$ and a morphism of Q-algebras:

$$\Phi : \langle A, Q(i): i \in I\rangle \to \langle F(X^I,O), Q_o(i): i \in I\rangle$$

such that card $(X) = \omega_1$ and $\Phi(p) = 1$ for any $p \in A$.

Another result is an omitting types theorem formulated in Q-algebras.

EXTRACTING LISP PROGRAMS FROM CONSTRUCTIVE PROOFS: A FORMAL THEORY OF CONSTRUCTIVE MATHEMATICS BASED ON LISP

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We present a formal theory of constructive mathematics based on the programming language Lisp, and introduce its formalized q-realizability interpretation. Since the realizers of our realizability interpretation are finite sequences of Lisp-programs, we can extract Lisp-programs from $\forall\exists$ -theorems of the theory. Our formal theory LM (Lisp Mathematics) is a variant of Feferman's formal theories of classes and functions introduced to formalize Bishop's constructive mathematics. The main differences between Feferman's formal theories and LM are as follows:

1. Feferman's theories have no intended interpretations of domains. On the other hand, the domain of LM is intended as the set of the data types of Lisp.

2. Algorithms (functions) in Feferman's theories are discussed by combinators. In LM, Lisp-programs are used instead of combinators.

The main result is Church's rule for LM:

Theorem. If D is a class of LM and $\forall x \in D\, \exists y A(x,y)$ is a theorem of LM, then we can effectively find a Lisp-program f from its proof such that $LM \vdash \forall x \in D\, \exists y (f[x] \simeq y \ \&\ A(x,y))$.

We have extracted a program of the Wang-algorithm of propositional logic by hand.

AMS '80, 03 F XX 50, 68 G

A partial predicate calculus and forcing.

Albert HOOGEWIJS

In order to get a better insight in the model theory for the partial predicate calculus we introduced in [1],we consider a forcing relation which may be defined in the following way (see also [2]).

Definition. Finite sets of atomic, Δ-atomic and negated atomic sentences of a countable language L_C, which are consistent with a consistent theory T of L_C are called conditions for T.

Definition. The relation $p \Vdash \alpha$ for a condition p and a formula α is defined inductively on the complexity of α.

0. If α is atomic, then $p \Vdash \alpha$ iff $\Delta\alpha \in p$, and
   $p \Vdash \alpha$ iff $[\alpha \in p$ and $Aq \geq p\ Er \geq q\ r \Vdash \Delta\alpha]$.
1. $p \Vdash \Delta\Delta\beta$
2. $p \Vdash \Delta\neg\beta$ iff $p \Vdash \Delta\beta$.
   $p \Vdash \neg\beta$ iff $Aq \geq p$ [not $q \Vdash \beta$ and $Er \geq q\ r \Vdash \Delta\beta]$
3. $p \Vdash \Delta\bigvee\Phi$ iff $E\beta\in\Phi[p \Vdash \beta$ and $p \Vdash \Delta\beta]$ or $A\beta\in\Phi[p \Vdash \Delta\beta]$
   $p \Vdash \bigvee\Phi$ iff $E\beta\in\Phi[p \Vdash \beta]$ and $Aq \geq p\ Er \geq q[r \Vdash \Delta\bigvee\Phi]$
4. $p \Vdash \Delta\exists x\beta(x)$ iff $Ec\in C[p \Vdash \beta(c)$ and $p \Vdash \Delta\beta(c)]$ or $Ac\in C[p \Vdash \Delta\beta(c)]$
   $p \Vdash \exists x\beta(x)$ iff $Ec\in C[p \Vdash \beta(c)]$ and $Aq \geq p\ Er \geq q\ [r \Vdash \Delta\exists x\beta(x)]$

then the generic model theorem holds and we show the following form of the

Omitting types theorem

Let M be an $\forall\bigvee\exists$ class and let $\varphi_n = \forall x_1 \ \forall x_{m_n} \psi_n(\vec{x})$ be a countable sequence of $\forall\bigvee\exists$ sentences. Suppose that for each n, each finite piece p of M, and each m_n-tuple $\vec{c} \in C^{m_n}$, $p \cup \{\psi_n(\vec{c})\}$ is satisfiable in M. Then M contains a countable model in which each φ_n holds.

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EASY TERMS: INCLUSION PROBLEMS AMONG CLASSES OF λ-TERMS

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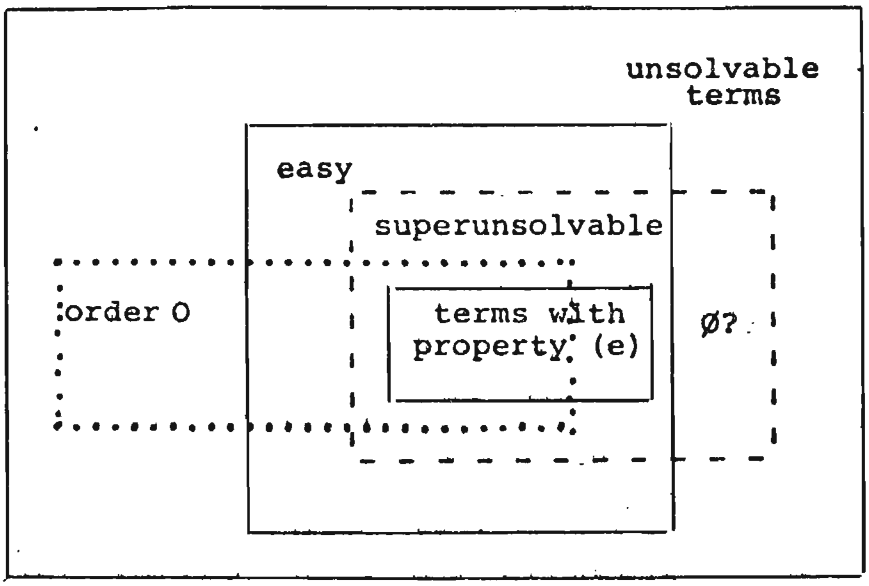
(AMS Classification: 03B40)

Abstract - Extensions of $\lambda\eta$ (by the terminology in [1]) can be obtained by adding a set of equations T = M, with $T,M \in \Lambda^0$, as axioms.

By defining $T \in \Lambda^0$ to be easy iff $\lambda\eta + T{=}M \nvdash$ with M arbitrary in Λ^0, in [3] some set of terms, namely the set of head recurrent terms of order 0, have been shown to be easy (i.e. to be such that every element of the set is easy) by using, essentially, a sufficient condition for ease, there referenced as (e).

In the paper it is shown that (e) is not a necessary condition for ease, and the obtained results concern classes of terms for which the following inclusion scheme holds, where superunsolvable terms are defined in the paper.



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Some modifications of Scott's theorem on injective spaces

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This paper is related to Scott result from [3], [2]. The notion of the retract was introduced by Borsuk [1].

Let α, δ be regular cardinal numbers and let ∞ denote the class of all cardinal numbers. A $\langle\alpha, \delta\rangle$-closure space is an ordered pair $X = \langle X, \mathcal{F}\rangle$ such that: (1) $\emptyset, X \in \mathcal{F}$, (2) if $\emptyset \neq \mathcal{R} \subseteq \mathcal{F}$, then $\bigcap \mathcal{R} \in \mathcal{F}$ (3) if $\overline{\overline{\mathcal{R}}} < \alpha$, $\mathcal{R} \subseteq \mathcal{F}$ then $\bigcup \mathcal{R} \in \mathcal{F}$, (4) if $\mathcal{R} \subseteq \mathcal{F}$ is δ-directed, then $\bigcup \mathcal{R} \in \mathcal{F}$

In the sequel $L = \langle L, \leq\rangle$ be a complete lattice. We call $\nabla \subseteq L$ is $\langle\alpha, \delta\rangle$-filter in L provided that the following two conditions are satisfied: (i) ∇ is upper set, (ii) for every $Z \subseteq \nabla$, $\overline{\overline{Z}} < \delta$ if Z is downward α-directed, then $\inf_L Z \in \nabla$. Let us denote by $\nabla_{\alpha,\delta}(L)$ the $\langle\alpha, \delta\rangle$-closure space of all $\langle\alpha, \delta\rangle$-filters in L and by $S_{\alpha,\delta}(L)$ the smallest $\langle\alpha, \delta\rangle$-closure space such that for every principial filter F in L, F is closed in $S_{\alpha,\delta}(L)$. Let π be a function defined as follows: $\pi(\alpha, \delta) = 1$ iff for every m $\nabla_{\alpha,\delta}(\langle \mathcal{P}(m), \subseteq\rangle) = S_{\alpha,\delta}(\langle\mathcal{P}(m), \subseteq\rangle)$. Observe that if $\alpha \geq \delta$ then $1 = \pi(\alpha, \delta) = \pi(\alpha, \infty) = \pi(0, \delta)$.

L is said to be $\langle\alpha, \delta\rangle$-lattice if for every family $\{a_{t,s}\}_{t\in T, s\in S}$ of elements of L

$$\inf_{t\in T} \sup_{s\in S} a_{t,s} = \sup_{\varphi\in S^T} \inf_{t\in T} a_{t,\varphi(t)},$$

provided that

(i) $\overline{\overline{T}} < \delta$,

(ii) for every s the family $\{a_{t,s}\}_{t \in T}$ is downward α-directed,

(iii) for every $T' \subseteq T$, $\overline{\overline{T'}} < \alpha$ there exists $t' \in T$ such that for every $t \in T'$ and $s \in S$ we can find $s' \in S$ such that $a_{t',s'} \leq a_{t,s}$.

Theorem 1

If $\pi(\alpha, \delta) = 1$, then a closure space X is an absolute retract in the category of $\langle \alpha, \delta \rangle$-closure spaces iff contraction of X is closure space of $\langle \alpha, \delta \rangle$-filters in a $\langle \alpha, \delta \rangle$-lattice.

It is easy to observe that by the above theorem we obtain:

Corollary 1 (Scott [3])

A topological space X is an absolute retract in the category of topological spaces iff there is a continuous lattice L such that contraction of X is Scott's topology on L^{o}.

Corollary 2

A closure space X is an absolute retract in the category of all closure spaces iff contraction of X is a closure space of all principial filters in a completely distributive complete lattice.

Corollary 3

A closure space X is absolute retract in the category of all closure space which satisfies the compactness theorem iff contraction of X is a closure space of all filters in a complete Heyting lattice.

Let $X = \langle X, \mathcal{T} \rangle$ be a closure space and let $d,a,b \in X$. We shall say that d _is disjunction of_ a _and_ b _in_ X provided that $C(\{d\}) = C(\{a\}) \cap C(\{b\})$. A set $F \in \mathcal{T}$ _is prime in_ X provided for every $d \in F$ if d is disjunction of a and b, then $a \in F$ or $b \in F$.

A closure space $X = \langle X, \mathcal{T} \rangle$ is _regular_ provided that X satisfies the compactness theorem and there is a family $\mathcal{B} \subseteq \mathcal{T}$ in which every element of $\mathcal{B}$ is prime in X and X is the smallest closure space $X' = \langle X, \mathcal{T}' \rangle$ such that $\mathcal{B} \subseteq \mathcal{T}'$ and X' satisfies the compactness theorem.

Theorem 2

A closure space X is absolute retract in the category of all regular closure spaces (with continuous functions preserving all finite disjunctions as morphism) iff contraction of X is a closure space of all filters in a complete Boolean algebra.

On account of the theorem 2 we obtain

Corollary 4 (Sikorski [4])

A Boolean algebra A is an absolute retract in the category of Boolean algebras iff A is a complete Boolean algebra.

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ABSTRACT

A NEW PROOF OF THE THEOREM
OF DAVIS PUTNAM AND ROBINSON

by James P. Jones* and Ju. V. Matijasevič

One of the historically important first steps in the eventual solution of Hilbert's tenth problem was the 1961 Theorem of Martin Davis, Hilary Putnam and Julia Robinson which states that every recursively enumerable set, A can be represented in so called exponential diophantine form

$$x \in A \iff \exists x_1, \ldots, x_n \; [R(x, x_1, \ldots, x_n) = S(x, x_1, \ldots, x_n)].$$

Here $x_1, x_2, \ldots x_n$ range over natural numbers $0, 1, 2, \ldots$ and R and S are functions built up from $x, x_1, \ldots x_n$ by the operations of addition, $A+B$, multiplication, AB and exponentiation, A^B.

Recently a new and very much simpler proof of this theorem was found by James P. Jones and Ju. V. Matijasevič. This new proof is based on register machines. It is a very simple and direct translation of the work of register machines into exponential diophantine equations.

The new proof uses only very elementary number theory, specifically we need only the partial ordering relation, $\preceq$, defined by

$$k \preceq n \iff \binom{n}{k} \equiv 1 \pmod 2.$$

The relation $k \preceq n$ has the effect of a bounded universal quantifier. For $k \preceq n$ holds if, and only if, each binary digit of k is less than or equal to the corresponding binary digit of n.

AMS 1980 subject classification numbers: 03D10, 03D25, 10N05

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D_∞ As a model for ω-order intuitionistic Logic (λ-Logic).

ABSTRACT

The main aim of this paper is to formulate natural foundations for type-free illative λ-calculus. We extend Scott's D_∞ model for the pure λ-calculus to include a lattice algebraic model for propositional logic by proving the following

Theorem. Let D_0 be a finite relatively pseudo-complemented lattice. Then D_∞ is a complete relatively pseudo-complemented lattice with the operation of relative pseudo-complementation given by

$$(x \Rightarrow y)_n = \bigcap_{k=0}^{\infty} \{(x_n \Rightarrow y_n), \Psi_{n+1,n}(x_{n+1} \Rightarrow y_{n+1}), \ldots, \Psi_{n+k,n}(x_{n+k} \Rightarrow y_{n+k}), \ldots\}$$

This shows that the D_∞ model is amenable to an axiomatization of intuitionsitic propositional calculus. We have found that it is not possible to strengthen this result to Boolean logic due to the following result:

Theorem. The maximal Boolean subalgebra of D_∞ is contained in D_0^∞.

Using the model provided by the above two theorems we are able to resolve the so-called classical paradoxes of illative combinatory logic (e.g. Curry's) by restricting the notion of application and abstraction to (lattice) continuous terms. All ordinary λ-calculus terms are continuous. Based on this interpretation of implication we build an axiom system for illative logic. As in all model-based axiomatizations we are guaranteed consistency through the existence of a model. Because of the completeness of the Scott lattice, this theory may be extended to ω-order predicate calculus.

1980 Mathematics subject classification: 03B40 (Combinatory Logic and λ-calculus).

EXTENDING PARTIAL COMBINATORY ALGEBRAS

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ABSTRACT

In [2] it is asked (question 11, posed by H.Barendregt, G.Mitschke and D.Scott) whether every partial combinatory algebra (p.c.a.) A can be completed to a total c.a. A^*. Here a p.c.a. $A = (A,k,s,\cdot)$ is a structure with a partially defined binary operation $\cdot$ called application and equipped with two distinguished elements k,s such that for all $x,y,z \in A$:

(i) kx, sx, sxy are defined

(ii) $kxy \simeq x$, $sxyz \simeq xz(yz)$.

(Definition: if t_1, t_2 are two applicative expressions then $t_1 \simeq t_2$ iff t_1, t_2 are both undefined or both defined and equal.)

The question is now: *can every p.c.a. A be completed to a c.a. A^* by adding some elements and completing the application operation?*

(Note that A^* has to have 'the same s and k'.)

We will answer the question negatively by constructing a p.c.a. which cannot be completed.

Secondly, a rather natural condition will be established which guarantees the existence of a completion. This condition is for instance satisfied by Kleene's recursion theoretic p.c.a. where application is defined by app(m,n) = {m}n.

Finally it will be shown that a p.c.a. which satisfies the 'combinatory axioms' (A_β, see p.157 of [1]) is already total.

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A First Step for Analyzing the Semantics of Parallelism in Computations Using a Categorical Approach

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ABSTRACT

The aim of our work is twofold: (i) on one hand we show how one can provide a categorical semantics for languages which allow parallelism; (ii) on the other hand we compare the different notions of parallelism and communications in two programming languages by analyzing at abstract level their categorical models. We stress the fact that we would like to present, more than the technical results themselves, the method which has been used. Other approaches have been used in the past for giving semantics to programming languages: the operational one, the axiomatic one, the denotational one, the translational one, etc. They all have their advantages, but they seem to be not particularly useful for analyzing and comparing the abstract notions of parallelism.

A couple of years ago, Category theory has been used for defining the semantics of a specification language [1]. We will use the same categorical approach for the semantics of two programming languages, in which parallel operations are specified by distinguished operators: they are CCS [3] and CSP [2]. By a "CCS theory of parallelism" we mean a set of CCS terms well formed from some basic operators (terms are usually written as a sum of guards), together with a given set of equations. Those equations identify terms which should be considered equivalent, i.e. denoting the "same" parallel behaviour. Terms denote nondeterministic computing agents interacting with each other.

In the theories we consider the fundamental notion of parallelism is by "handshaking". We can associate to one of these theories a category $\mathcal{E}$, whose objects are the terms and morphisms are recursively defined on the structure of the terms. A class of monomorphisms in $\mathcal{E}$, namely the inbeddings, will be also considered. One sees that the summation corresponds to categorical coproduct in the class of monomorphisms, NIL [3] is the initial object and composition is expressible as a pushout in $\mathcal{E}$. Relabelling operation is nothing more than a functor $S: \mathcal{E} \longrightarrow \mathcal{E}$ that satisfies given conditions. Usual equations are obtained requiring commutativity of some diagrams.

A similar categorical structure can be associated to CSP [2]. In this case, every agent is supposed to be the set of its failures in the interaction with the environment. Notice that by the intersection operation [2] we model the communication between agents regarded as "handshaking". Between the symbols of the alphabet there is a distinguished one denoting the successful termination of a process. If A is a set of all possible traces, an agent, i.e. an object of $\mathcal{E}'$, will be a function

$$P: \quad A^{*} \longrightarrow P(P_{fin}(A))$$

It is quite easy to find a suitable class of morphisms and a class of monomorphisms among them, such that STOP is the initial object and RUN the terminal one in $\mathcal{E}'$, CHAOS the terminal object and nondeterministic composition the coproduct in the class of monomorphisms. Parallel composition by intersection corresponds to the product in $\mathcal{E}'$.

What we have done is to consider an abstract categorical structure and CCS and CSP as its models; comparisons between them have then been made at this abstract level. An outcome of this comparison is, for example, the following: summation in CCS and nondeterministic composition in CSP have turned to be the same categorical construction.

As general remark we can notice that the presence in CSP of a distinguished symbol denoting successful termination of a process, requires, for its complete formalisation, a more sophisticated categorical framework than that we need for CCS.

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Inadequate and Weakly Incompatible Modal Systems

an abstract

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Call a modal system S inadequate if $\vdash_S \phi \supset \Box\phi$. Call two modal systems S_1 and S_2 weakly incompatible if the system that results when they are combined is inadequate.

In Section I, I survey the known results on weakly incompatible systems. In Section II, I prove some new results on the weak incompatibility of various modal systems. For example, systems including both Sobociński's system K1 and the Brouwerische axiom are shown to be inadequate; thus systems containing K1 and systems containing B are weakly incompatible.

UNITIES, SEMANTICS AND REALIZATIONS.

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In the intent of describing semantics for propositional calculi, we give the following

Definition 1. Let $\mathbb{P} = (P, \leq)$ be a bounded poset and A a non-empty set. A triple $\mathbb{M} = (\mathbb{P}, \models, \dashv)$ is a pre-model of A, if the following conditions hold, for all $a \in A$, $p,q \in P$:
(1) $\underline{0} \models a$; (2) $\underline{1} \dashv a$; (3) if $p \leq q$ and $q \models a$, then $p \models a$; (4) if $p \leq q$ and $p \dashv a$, then $q \dashv a$; (5) if $p \models a$ and $q \dashv a$, then $p \leq q$.

We compare this concept and that of unity of a relation (see [1]) , which we present in a slightly modified version, more suitable for a Kripke-style semantics. Subsequently, we define realizable elements of a pre-model.
The definition of a model $\mathbb{M} = (\mathbb{P}, \models, \dashv)$ of $\mathbb{F}(A)$, the propositional language on a set A of propositional letters, is a natural extension of Definition 1, e.g. take $p \dashv \alpha \vee \beta$ iff $p \dashv \alpha$ and $p \dashv \beta$.

Definition 2. A formula α of $\mathbb{F}(A)$ is hereditarily realizable in a model $\mathbb{M}$ of $\mathbb{F}(A)$, if every subformula of α is realizable in $\mathbb{M}$.

A first result is similar to Theorem 5 in [2] :

Theorem 1. There exists a poset $\mathbb{P}$ such that for any fragment of $\mathbb{F}(A)$ (i.e. a subset of $\mathbb{F}(A)$, closed under subformulae), there is a model $\mathbb{M}(F)$ of the form $(\mathbb{P}, \models(F), \dashv(F))$ in which every element of F is hereditarily realizable.

Theorem 6 in [2] can be reformulated, with suitable restriction on the model $\mathbb{M}$ of $\mathbb{F}(A)$ and on the fragment F; the core of its statement is as follows:

Theorem 2. There exists a model $\mathbb{M}[F]$ which is an extension of $\mathbb{M}$, such that every element of F is hereditarily realizable in $\mathbb{M}[F]$.

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Model-theoretic properties of Parovičenko space

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E.K.van Douwen and J.van Mill introduced the notion of Parovičenko space, [1], and proved the part (←) of the following theorem, began by I.I.Parovičenko:

<u>Theorem</u> CH is equivalent to the statement that every Parovičenko space is homeomorphic to growth$\omega = \beta\omega - \omega$.

The proof presented in [1] is based on a construction of two Parovičenko spaces X,Y so that X has a point of the character $\aleph_1$, but every point in Y has the character $c=2^{\aleph_0}$. We give a purely model-theoretic construction of the second space which is more involved. It is based on the following facts which might be of an independent interest. If M is a Boolean notion then M^* denotes its topological dual, and BA stands for Boolean algebra.

1° [2], a space X is a Parovičenko space iff there is an atomless ω_1-saturated BA B, $|B|=c$, so that $X=B^*$.

2° If D is the filter of cofinite subsets of an infinite set I, then the reduced product of BA's B_i, $i \in I$, modulo D has the following dual

$$\left(\prod_D B_i\right)^* = \text{growth} \sum_i B_i^*$$

3° (B.Jonsson, P.Ollin) If D is the filter of cofinite subsets of ω then D is ω_1-saturative.

4° (Ž.Perović) Let A be a free BA with $k \geq \aleph_0$ free generators. If D is a proper filter over ω, then every ultrafilter p of A^ω/D has the character $\geq k$.

<u>Corollary</u> Let B be a free BA with c free generators, and D the filter of cofinite subsets of ω. Then $(B^\omega/D)^*$ is a Parovičenko space in which every point has the character $\geq c$.

<u>Remarks</u> 1° $(B^\omega/D)^* = \text{growth}(\omega \times 2^c)$, as B^* is the Cantor space 2^c. This space is the space T in [1].

2° The part (→) of the above theorem immediately follows from the uniqueness of saturated models of a complete theory (in this case of atomless BA's) of the same cardinality and 1°.

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A NOTE ON $(\lambda,\mu)^*$-COMPACTNESS

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ABSTRACT

The concept of $(\lambda,\mu)^*$-compactness of an arbitrary logic defined by Makowsky and Shelah is shown to be equivalent to (λ,μ)-set compactness. Using this notion elementary proofs are given for the following results of Makowsky and Shelah:

(i) If a logic is $(cf(\lambda),cf(\lambda))^*$-compact, then it is $(\lambda,\lambda)^*$-compact;

(ii) A logic is $(\lambda,\omega)^*$-compact if and only if it is $(\kappa,\kappa)^*$-compact for all regular κ with $\omega \leq \kappa \leq \lambda$.

A simple topological characterization of $(\lambda,\mu)^*$-compactness is given and result (ii) above is shown to have been essentially formulated and proved by Alexandroff and Urysohn in 1929.

AMS Subject Classification: 03C95

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RIGHT AND LEFT INVERTIBILITY IN λ-β-CALCULUS

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It is well known [3] that the set Λ of λ-$\beta(\eta)$ terms builds a semigroup, having as identity the combinator $I \equiv \lambda x.x$ and as composition the operation $\circ$ so defined : $X \circ Y = BXY$, where $B \equiv \lambda xyz.x(yz)$. The problem of characterizing the normal forms having inverse has been studied both in λ-β-calculus and in λ-β-η-calculus and it has been proved that λ-terms without normal forms cannot have inverse in λ-β-η-calculus [1], [2], [4].
In the present paper we use the notion of direct approximation and the partial order relation $\sqsubseteq$ introduced in [5]. Firstly we notice that every left (right) inverse of a λ-term X is a left (right) inverse for every λ-term Y such that $X \sqsubseteq Y$. In order to characterize the set of terms having left and/or right inverse the concept of Böhm tree is used and two "operations" on the set of Böhm trees: terminal extension and initial extension, are introduced. We prove that a λ-term X has left (right) inverse if and only if there exists at least a λ-term Y such that $Y \sqsubseteq X$ and Y can be obtained from I applying a sequence of terminal (initial) extensions. Moreover we characterize the λ-terms left (right) invertible having one and only one left (right) inverse and we prove that in the other cases there exists an infinite number of left (right) inverses. Finally we discuss the above results about invertibility on the graph model P_ω and we show that the two functions which map an element of P_ω into the set of all its right or left inverses, respectively, are not monotonic, i.e. we can have that for some X, Y of P_ω : $X \subsetneq Y$ (where $\subseteq$ is the usual order relation of P_ω) and there exists a left (right) inverse of X which is not a left (right) inverse for Y.

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ABSTRACT

Title: Mathematical and metamathematical applications of realizability

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AMS/AMOS Classifications: 02, 04

Unlike the realizability interpretations which feature so prominently in Beeson [1] and which apply directly to nonextensional intuitionistic set theory (IZF), there are innumerable versions of "number realizability" which model extensional IZF immediately. The most familiar of these arises as an extension both of Kleene's original notion and of the Kreisel–Troelstra realizability for intuitionistic second-order arithmetic. It is easy to see, working classically, that this structure, V(R), is sound with respect to IZF+CT(0)+AC(0,X)+DC+MP(0)+IP+UP(0). In V(R), the universe is unzerlegbar, every powerset is uncountable and the class of ordinals on which membership is decidable forms a set. V(R) also provides a countermodel for the Cantor–Bernstein theorem, Kripke's Schema and full Church's Thesis. The weak counterexamples of Brouwer which do not rely upon the theory of the creative subject and the set-theoretic weak counterexamples of Grayson [2] are easily transformable into strict falsehoods over V(R), and, hence, into independence results from the above extension of IZF.

What is more significant than the preceding from a mathematical or philosophical point of view is the provision by the model of a "transfer principle" like that sought by Kreisel [3] to make explicit the relations holding between the effective "set theories" of Myhill, Dekker et al. on the one hand and their constructive correlates in traditional formulation on the other. There is, for example, an translation A* of the classical theory of the recursive equivalence types <RET, +, x, => into a structure <P(N)-stable, +, x, => specified in purely set-theoretic terms over the stable subsets of N in V(R) such that A holds in <RET, +, x, => iff V(R) satisfies A*.

From this result and the character of A*, it follows that the study of the recursion-theoretic relations between RET's can be carried out without loss as pure cardinal arithmetic in an extension of IZF and that a simple characterization of those statements which hold constructively over RET is available. Quite similar results linking the isols to Dedekind-finite cardinals over V(R) are now easily obtained.

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Abstract

Title Concerning ultrafilter characterizations for huge cardinals.

AMS/AMOS Classification number 03E55

For a cardinal κ an extension of the elementary embedding characterizations of measurable and supercompact defines κ to be huge if there exists an elementary embedding $j: V \to M$ such that κ is the first ordinal moved while V and M have the same sequences of length $j(\kappa)$. In Zermelo Fraenkel set theory with the axiom of choice this definition is both equivalent to the existence of a κ-complete, fine, normal ultrafilter over $\{x \subset \lambda: |x| = \kappa\}$, and a κ-complete, fine, normal ultrafilter over $\{x \subset \lambda: \bar{x} = \kappa\}$, (where $|x|$ and $\bar{x}$ denote the cardinality and order type of x respectively).

It can be shown that every member of any κ-complete, fine, normal ultrafilter over $\{x \subset \lambda: |x| = \kappa\}$ must have a member of order type κ.

The existence of certain filters and ultrafilters over these sets are studied. Notions resembling that of closed and unbounded for subsets of $P_\kappa\lambda = \{x \subset \lambda: |x| < \kappa\}$ are investigated here with several analogous results.

Roman Murawski

ON "TRACE" EXPANSIONS OF INITIAL SEGMENTS

We consider the expandability of initial segments of models of Peano arithmetic PA to models of second order arithmeic A_2^- or similar theories. Recall the following

DEFINITION. A model $M \models PA$ is said to be A_2^--expandable iff there is a family $\mathcal{X}_M \subseteq \mathcal{P}(M)$ such that $(\mathcal{X}_M, M, \in) \models A_2^-$.

More information on expandability can be found e.g. in our survey paper [2] and on expandability of initial segments in [1].

We shall treat, for convenience, a model for A_2^- as a structure $(M,\tilde{S},E)$ where $M \models PA$, $\tilde{S} \in M$, $E \subseteq M \times \tilde{S}$. A_2^- can be also formalized in a language with one sort of variables and with predicates $\tilde{S}$, E. Denote such a theory by $\underset{\sim}{A}_2^-$. We shall consider models of $\underset{\sim}{A}_2^-$ with the following property:

(*) relations $\tilde{S}$ and E are inductive.

THEOREM 1. If a countable nonstandard model $M \models PA$ is expandable to a model $\mathfrak{A} = (\mathcal{X}_M, M, \in)$ of A_2^- such that it is isomorphic to a model $\mathfrak{A}_1 = (M,\tilde{S},E)$ of $\underset{\sim}{A}_2^-$ with the property (*) and such that $\tilde{S}$ is bounded in M then M has $2^{\aleph_0}$ initial segments $I \subseteq_e M$ such that $(\mathcal{X}_M \cap I, I, \in) \models A_2^-$ where $\mathcal{X}_M \cap I = \{X \cap I : X \in \mathcal{X}_M\}$.

THEOREM 2. If a countable nonstandard model $M \models PA$ is expandable to a recursively saturated model $\mathfrak{A} = (\mathcal{X}_M, M, \in)$ of A_2^- such that it is isomorphic to a model $\mathfrak{A}_1 = (M,\tilde{S},E)$ of $\underset{\sim}{A}_2^-$ with (*) and such that $\tilde{S}$ is bounded in M then M has $2^{\aleph_0}$ elementary initial segments $I \prec_e M$ such that

1° $(\mathcal{X}_M \cap I, I, \in) \models A_2^-$,

2° $(\mathcal{X}_M \cap I, I, \in) \prec (\mathcal{X}_M, M, \in)$.

We have also the following "negative"

THEOREM 3. Let M be a countable nonstandard model of PA which is expandable to a model $\mathfrak{A}$ of $A_2^* = A_2^-$ + CONSTR such that $\mathfrak{A}$ has a full substitutable satisfaction class S with the following properties: 1° $S(A_2^*)$ and 2° the comprehension scheme holds in $\mathfrak{A}$ also for atomic formulas containing S. Then there exist an extension M_1 of M such that $M_1 \models$ PA and an A_2^--expansion $\mathfrak{B} = (\mathcal{X}_{M_1}, M_1, \in)$ of M_1 such that $(\mathcal{X}_{M_1} \cap M, M, \in)$ is not a model of A_2^-.

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CYLINDRIC-NATURAL SET ALGEBRAS, THE NATURAL EXTENSION OF BOOLEAN OPERATIONS TO "FIRST-ORDER LOGIC OPERATIONS" DEFINED ON ARBITRARY SETS, by I. Németi.

AMS classification code: 03G15, 03C95, 03G25.

Let V and D be two arbitrary sets and $i,j \in D$. Sets is the class of all sets. Define

$$\tilde{}_V \overset{d}{=} \langle V \sim x \ : \ x \subseteq V \rangle \quad \text{and}$$

$$C_i^V \overset{d}{=} \langle \{y \in V \ : \ (\exists g \in X) y \sim (\{i\} \times Sets) = g \sim (\{i\} \times Sets)\} \ : \ X \subseteq V \rangle$$

$$D_{ij}^V \overset{d}{=} \{y \in V \ : \ y \cap (\{i\} \times Sets) = y \cap (\{j\} \times Sets)\} .$$

Clearly, $\tilde{}_V : \mathcal{P}V \to \mathcal{P}V$, $C_i^V : \mathcal{P}V \to \mathcal{P}V$ and $D_{ij}^V \in \mathcal{P}V$ for all $i,j \in D$. The full cylindric-natural set algebra of dimension D (from now on the full CNs_D) over V is defined to be

$$\mathfrak{P}V \overset{d}{=} \langle \mathcal{P}V \ ; \cap, \tilde{}_V , C_i^V , D_{ij}^V \rangle_{i,j \in D} .$$

$$CNs_D \overset{d}{=} \{ \mathfrak{A} \ : \ \exists V (\mathfrak{A} \subseteq \mathfrak{P}V)\}.$$

That is, a CNs_D is a subalgebra of some full CNs_D. For any class K of algebras we let $IK \overset{d}{=} \{ \mathfrak{A} \ : \ (\exists \mathfrak{B} \in K) \ \mathfrak{A} \cong \mathfrak{B} \}$.

<u>THEOREM 1</u> There exists a decidable set E_D of equations such that $ICNs_D = Mod(E_D)$.

E_D will be explicitly defined in the lecture. We show that CNs_D is the algebraic counterpart not only of classical first order logic but also that of many-sorted logic, modal (first-order) logic, intuitionistic logic, etc. Hence CNs_D theory is rather algebraic abstract model theory and not only algebraic first order logic. In [Henkin,L. Monk,J.D. Tarski, A. Andréka,H. Németi,I.: Cylindric Set Algebras, Lecture Notes in Mathematics 883, Springer-Verlag, 1982] the class Crs_α was defined for every ordinal α.

<u>THEOREM 2</u> $\alpha > 1$ iff $ICrs_\alpha = ICNs_\alpha$. CNs_0 = "Boolean set algebras" and CNs_1 is the variety of closure algebras of Tarski.

The logic of categories of partial functions and recursiveness

Adam Obtułowicz

Abstract:

The category of partial functions in an elementary topos with Natural Number Object is considered. It is presented the characterization of this category by giving the system of partial operations defined on its arrows and the system of equations valid in the category, in other words this category is characterized as a category with additional equational structure in the sense of J. Lambek [2]. This characterization gives rise to certain formal system which one can apply in the description of A. Grzegorczyk's recursive objects [1] and partial recursive objects, for instance certain partial recursive functions and functionals. This description emloys the concept of definability of a function in the system which is similar to the concept of lambda-definability of a function in lambda-calculi. The formal system is related to equational characterization of a cartesian closed category (cf. [2]). The presented characterization gives rise also to certain new construction of a free elementary topos in the sense of J. Lambek [2].

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[2] J. Lambek, From types to sets, Advances in Mathematics 36 (1980), pp. 113-164.

G.E.Minc. Simple proof of the coherence theorem for cartesian closed categories

This theorem, first proved in /1/, states in proof-theoretic terms that $A \to B$ has at most one derivation (up to equality of normal forms) in the intuitionistic $(\&, \supset)$-calculus if $A \to B$ is balanced, that is no variable occurs there more than twice. Applying familiar depth-reducing transformation /2/ we can restrict attention to balanced sequents $S = Y \to k$ where k is a variable, and Y is a list of formulas having one of the forms p, $p \supset q$, $p \supset (q \supset r)$, $(p \supset q) \supset r$, $p \supset (q \& r)$ for variables p,q,r. An S-sequent is one of the form $Y'V \to l$, where l is a variable having a positive occurrence in S, the list Y' is contained in Y, and V is a list (possibly empty) of variables such that for any $v \in V$ the list Y contains a member of the form $(v \supset a) \supset b$. An antecedent member F of some sequent $X \to v$ is redundant if F has one of the forms $p \supset (v \supset q)$, $v \supset A$ or $(p \supset v) \supset q$.

Our proof of the coherence theorem proceeds by induction on normal derivation of a balanced sequent S using the following pruning lemma.

No normal derivation of an S-sequent contains redundant members.

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AMS/MOS 80: 03F05 + 03 B40 + 18A15

Regularity of internal weak solutions of partial differential equations [1)]

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In his book "Non-Standard Analysis" Abraham Robinson proved that the standard part of a finite internal harmonic function is a standard harmonic function. His argument applied Poisson's integral. We generalize Robinson's result for strictly elliptic partial differential equations

$$L(u) = f \tag{1}$$

of second order in a bounded domain of $\mathbb{R}^n$. The coefficients are assumed to be in C^∞. Our method is a nonstandard approach to the regularity theory, and we prove among other things:

<u>Theorem 1:</u> If (1) has a norm-finite (in the L^2 sense) internal weak solution, then (1) has a standard C^∞ solution.

<u>Theorem 2:</u> If $u \in C^\infty$ is a norm-finite (in the L^2 sense) internal weak solution of (1), then ${}^\circ u$ is a standard C^∞ solution of (1).

1) AMS classification: 03H05 primary, 35J15 secondary.

2) The research is carried out in collaboration with prof. O. Martio.

Quantifier elimination in projectable L-groups.

A L-group is a L-structure which is an abelian group and such that the language L contains only function symbols and among them $\{+,-,0\}$.
A projectable simple L-group is a L-group where a binary function $p(\ ,\ )$ is defined as follows : $p(a,b) = 0$ if $b \neq 0$ and $p(a,b) = a$ if $b = 0$.
A projectable L-group is a subdirect product of projectable simple L-groups.

An example of such structures is the projectable ℓ-groups i.e. the lattice ordered groups which satisfy the following axiom :

$$\forall a \ \forall b \ \exists h \ (|h| \wedge |b| = 0 \ \& \ \forall f \ (|f| \wedge |b| = 0 \rightarrow |f| \wedge |a-h| = 0)).$$

Representation theorem :

A projectable L-group A is isomorphic to the structure $\Gamma_c(X, \bigcup_{x\in X} A_x)$ of all sections with compact clopen supports of a locally boolean sheaf of projectable simple L-groups. (Theorem 6.12 of [K] about representation of projectable L-groups is thus a particular case of this theorem).

Now we classify the projectable L-groups which have q.e. in L, in term of their base space and the projectable simple L-groups of their sheaf space.

Theorem :

A projectable L-group A admits q.e. in L iff

(1) the base space X of A is the topological sum of a space X_o without isolated points and finitely many discrete

spaces X_i, $i \geq 1$, with one or two points.
Moreover (i) there exists terms t_i s.t. for any element r of A : $t_i(r)$ equals r over X_i and equals 0 elsewhere ;

(ii) there are closed L-terms whose supports are the X_i which are compact and contain at least two points.

(2) the class of projectable simple ℓ-groups $\{A_x | x \in X\}$ admits positive q.e. and for any term $t(.)$, there exists a term $s(.)$ s.t. the following equivalence holds in this class :
$\forall z \ (z = 0 \vee (\exists y \ \ t(y,\bar{x}) \neq 0 \leftrightarrow s(z,\bar{x}) \neq 0))$.

Applications :

(1) The projectable ℓ-groups which admit q.e. in $\{+,-,0,\wedge,p\}$ are distributed in the two following classes (having q.e.) :

- the class of all divisible totally ordered ℓ-groups,
- the class of all divisible projectable ℓ-groups satisfying :

(i) $\forall f \ \exists g \ \ (g \neq 0 \ \ \& \ \ |f| \wedge |g| = 0)$ and

(ii) $\forall f \ \exists f_1 \ \exists f_2 \ (f \neq 0 \rightarrow (f_1 \neq 0 \ \& \ f_2 \neq 0 \ \& \ |f_1| \wedge |f_2| = 0 \ \ \& \ \ f = f_1 - f_2))$.

Weispfenning extends the language of projectable ℓ-groups by the following unary function symbols : $./n$ with $n \in \omega - \{0\}$, defined as follows :

$p(x/n, n.(x/n) - x) = x/n$ and $\forall z \ (x = n.z \rightarrow z = x/n)$.

Weispfenning has shown that any non trivial totally ordered abelian group which admits q.e. in $\{+,-,0,\wedge,./n;\ n \in \omega-\{0\}\}$ is dense, regular (see [W]).
Thus, we get the second application :

(2) The projectable ℓ-groups which admit q.e. in $\{+,-,0,\wedge,p,./n;\ n \in \omega-\{0\}\}$ are distributed in the two following classes (having q.e.) :

- the class of all dense regular totally ordered ℓ-groups,
- the class of all projectable ℓ-groups satisfying :

  (i), (ii) and the following axiom :

  $\forall f\ \forall g\ \exists h(|h|\wedge|f|=|h|\ \&\ h\neq 0\ \&\ |h|\neq|f|\ \&\ |h|-(|g|/n).n=|h|-|g|)$

[K] K. Keimel, the representation of lattice-ordered groups and rings by sections in sheaves, Lecture Notes 248, A. Dold, Heidelberg and B. Echmann, Zürich, 1971, p. 55.

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Model Theory and Representation of Artinian Rings.

Mike Prest

A ring is right pure semisimple if each of its modules is a direct sum of, necessarily finitely generated, indecomposable submodules. It is equivalent to require that each of its modules be totally transcendental. Such rings are right artinian, but it is not known whether such a ring need be left artinian.

A right pure semisimple ring is said to be of finite representation type if it has, up to isomorphism, only finitely many indecomposable modules. It is equivalent to require that each of its modules has finite Morley rank. It is unknown whether this apparently stronger condition is equivalent to being right pure semisimple: it is known to be equivalent to right and left pure semisimplicity.

I show how techniques from the model theory of modules may be used to show that the properties are equivalent in some special cases. In particular I use the connection between positive primitive types, and certain sets of matrices over the ring.

I show, further, how use of model theory illuminates some of the category-theoretic methods used in the area of representation theory, and how it provides quick and conceptual proofs of a number of results.

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AMS/AMOS 1980 03C60
16A64

EXTENSIONAL REALIZABILITY (abstract)

We introduce APEX, an extensional theory based on intuitionistic partial logic and on application as term-building device. APEX is inspired on Feferman's applicative theories (see [F]); it can be seen as a conservative extension of HA (intuitionistic arithmetic).

In APEX we define extensional realizability $\underline{e}$ and show that it can be characterized by EAC, an extensional axiom of choice. Using forcing it is proved (as in [B]) that $\mathrm{APEX} \vdash \exists x\, x\, \underline{e}\, A$ implies $\mathrm{APEX} \vdash A$ for arithmetical A, hence APEX + EAC is conservative over HA.

Finally we interpret Martin-Löf's basic theory ML_0 (without well-orderings and universes; see [M-L]) in APEX + EAC, which yields:

ML_0 is conservative over HA.

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AMS/AMOS 1980 classification: 03 F 50.

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PRINCIPAL TYPE SCHEMES FOR AN EXTENDED TYPE THEORY

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ABSTRACT.

In [Barendregt et al.,1981] the basic functionality theory of Curry is extended, generalizing both type structure and assignment rules, so that types are preserved under convertibility and only unsolvable terms have a trivial functional characterization. An important feature of this system is that it is semantically complete and it induces a λ-model.
It is desirable to show an internal coherence between the type schemes (in general infinite) which are deducible for a given term. To this aim , the authors prove the existence, for every term X, of a "principal type scheme" (p.t.s.) , from which all and only the type schemes deducible for X can be generated, by suitable operations. As the type schemes deducible for a term X are all and only the type schemes deducible for the approximants of X, also the p.t.s. of X is build from the p.t.s.s of it's approximants.
The p.t.s. of an approximant A is defined inductively on the structure of A, and it is the simplest type scheme deducible for A. In fact it can be assigned to A by means of a normalized deduction tree D, whose structure corresponds to the structure of A itself. All type schemes deducible for A can be generated from the p.t.s. of A by means of repeated applications of three operations, which reflect the relationship between the associated deductions and the deduction D.
The p.t.s. of a term X is a single type scheme in the case that X has a finite set of approximants, it is an infinite set of type schemes otherwise.
It is easy to prove that the unification (i.e.,the problem to find, given two typed terms, a type scheme for their application) is not decidable.

But it is possible to study "unification procedures" and the feature that the given operations generate deducible type schemes also starting from arbitrary deducible type schemes (not only p.t.s.s) may be useful to this aim.

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ABOUT FRAMES AND FILTERS

by Giuseppe Rosolini

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We have given a simple construction of the pushout of frame morphisms $f:C \to A$ and $g:C \to B$ as the frame of Galois connections over C.

We study lifting of filters of A or B to $A \otimes B$ and iterations of reduced frame extensions of algebraic structures by means of a sort of Beck condtion on a commutative square of toposes.

AMS MOS Classification: 06D20, 18B25.

Abstract

An arithmetic model for modal logic*

(Feys T system and Von Wright M system)

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Neuchâtel and Basque Country Universities

Each variable and each well-formed formula of two-valued propositional calculus can take in our model a numerical value (expressing its modal value in any system whatsoever) within the set of natural numbers between 0 and a number $f=2g-1$, in which $g=2^n$, n being any natural number previously fixed.

The arithmetic conditions expressing the truth of a well formed formula of an elementary modal calculus (such as the Feys T systen or the Von Wright M system) will then be the following:

the formula	which states	is true if and only if	example for n=3
□p	it is necessary that p	p=0	p=0
p	p	p<g	p<8
◇p	it is possible that p	p<f	p<15
~□p	it is not necessary that p	p>0	p>0
~p	not p	p>(g-1)	p>7
~◇p	it is impossible that p	p=f	p=15
◇p&~□p	it is contingent that p	0<p<f	0<p<15
p&q	p and q	[p, q]<g	[p, q]<8
pvq	p or q	(p, q)<g	(p, q)<8
p→q	p materially implies q	(f-p, q)<g	(15-p, q)<8
p o q	p is compatible with q	[p, q]<f	[p, q]<15
p ⥽ q	p strictly implies q	(f-p, q)=0 p!q	(15-p, q)=0 p!q
p=q	p is strictly equivalent to q	[(f-p, q), (f-q, p)]=0 p=q p-q=0	[(15-p, q), (15-q; p)]=0 p=q p-q=0

in which the arithmetic functions f-p, [p, q] (least common binary compound** of p and q) and (p, q) (greatest common binary component** p and q) are respectively associated with the logic functions ~p, p&q and pvq, whilst the arithmetic relation p!q is the realtion p is a binary compound** of q.

The paper shows that by means of that arithmetic model it is possible, from a theoretical point of view, to prove by numerical calculation all the theorems of such modal systems as T Feys system and M Von Wright system and, on another hand, from a practical point of of view, that it is possible to apply arithmetic procedures to any scientific theory that involves the use of modalities.

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*AMS Classification: C10/M10.

**p is a binary component of q if and only if any power of 2 which is part of the binary expression of p is also part of the binary expresdion of q; p is a binary compound of q if and only if q is a binary component of p.

Abstract

Finitely generic abelian lattice-ordered groups

by Dan Saracino and Carol Wood

Work of Glass and Pierce [1] is extended to give a characterization of the finitely generic abelian lattice-ordered groups, producing easy axioms for these among all hyperarchimedean ℓ-groups. Examples relating representations as real-valued functions and model-theoretic properties are alco given.

[1] Glass and Pierce, Trans. A.M.S. 26 (1980), 255-270

MULTIPLE FORMS OF GENTZEN'S RULES AND SOME INTERMEDIATE LOGICS

Z. Šikić

(Abstract)

Gentzen's sequential system is a formalization of classical or intuitionistic logic depending on whether we take its rules in multiple or singular form. Indeed, in the singular system extended by the initial sequents of the form $\rightarrow A \vee \neg A$, it is possible to prove at once the permissibility of the multiple forms of all the inference rules, [1].

An analysis of each rule separately shows that the multiple form of the introduction of negation or implication in the succedent is sufficient for the formalization of classical logic. The multiple form of the introduction of universal quantifier in the succedent is not sufficient for the formalization of classical logic, and at the same time it is too strong for the formalization of intuitionistic logic [2]. The multiple forms of the other rules do not extend the intuitionistic system. The extension of the singular intuitionistic system by the multiple form of the introduction of universal quantifier in the succedent is therefore the formalization of an intermediate logic. We call this extension L_2.

We want to show that the system L_2 is related to Gödel's completeness theorem. Namely, Kleene's detailed analysis of the proof of the theorem [3], reveals that the only non-intuitionistic assertion used in the proof is of the form $\forall x A(x) \vee \exists x \neg A(x)$. Therefore, we will compare our system L_2 to the singular intuitionistic system extended by the initial sequents of the form $\rightarrow \forall x A(x) \vee \exists x \neg A(x)$. We call this extension L_3.

Moreover, Kleene's analysis shows that the predicate A(x) is decidable. Therefore, we will also compare our system L_2 to the singular intuitionistic system extended by the initial sequents of the form $\forall x (A(x) \vee \neg A(x)) \rightarrow \forall x A(x) \vee \exists x \neg A(x)$. We call this extension L_1.

We prove constructively the following theorem:

<u>Theorem</u>: L_3 extends L_2 and L_2 extends L_1. (It is plain that L_3 properly extends L_1, [2]).

The question remains: Is the system L_2 equivalent to L_1 or possibly to L_3?

[1] G.Gentzen: Untersuchungen über das logische Schliessen, Math. Zeitschr. 39, 1935.

[2] S.C.Kleene: Introduction to Metamathematics, Van Nostrand, 1952.

[3] S.C.Kleene: Mathematical Logic, J.Willey, 1968.

AMS/AMOS 1980 classification 03B55, also of interest for 03F99.

CONDITIONS STRONGER THAN HOMOGENEITY OF BOOLEAN ALGEBRAS

Petr Štěpánek

We call a Boolean algebra B homogeneous if for every two nonzero elements u, v < $\mathbb{1}$, there is an automorphism of B such that $\varphi(u) = v$.

Proposition For every Boolean algebra, the following conditions are equivalent (i) B is homogeneous,
(ii) for every finite subalgebra C of B, every automorphism of C extends to an automorphism of B.

The following result due to M. Weese shows that we cannot get a stronger notion of homogeneity by extending (ii) to all subalgebras.

Theorem (Weese) Every infinite homogeneous Boolean algebra B contains a subalgebra with an automorphism which does not extend to an automorphism of B.

We can prove the following

Theorem (i) There is an infinite complete homogeneous Boolean algebra B such that for every complete subalgebra C of B, there is an embedding e : C ⟶ B such that every automorphism of e[C] extends to an automorphism of B.

(ii) For every infinite cardinal κ, there is a complete homogeneous Boolean algebra B with a dense subset of power κ such that the conlusion of (i) holds for every subalgebra of B of power at most κ.

Problem 1. Does there exist a homogeneous Boolean algebra B such that the conclusion of (i) in the above theorem holds for every subalgebra of B ?

Problem 2. Do there exist two complete homogeneous Boolean algebras B, C such that C is a complete subalgebra of B and no nontrivial automorphism of C extends to an automorphism of B ?

03C60

SIMON THOMAS. The classification of the stable simple locally finite groups

Cherlin conjectured in [2] that an ω-stable simple locally finite group is a Chevalley group over an algebraically closed field. We have been able to reduce the conjecture to an identification problem using results from [1] and [3] and the classification of the finite simple groups. This problem is solved by the following result, which is of some interest in its own right. It answers many of the questions raised in [3].

Theorem 1 [4]

Let $G = \bigcup_{i\in\omega} G_i$, where each G_i is a Chevalley group of Lie type L over a finite field. Then G is a Chevalley group of type L over a locally finite field.

Using this result, we obtain:

Theorem 2

A stable simple locally finite group is a Chevalley group over an algebraically closed field.

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[1] R.M. Bryant, Groups with the Minimal Condition on Centralizers, J. Algebra. 60 (1979) 371-383.

[2] G. Cherlin, Groups of small Morley rank, Ann. Math. Logic 17 (1979) 1-28.

[3] O. Kegel and B. Wehrfritz, Locally Finite Groups (North-Holland, Amsterdam, 1973).

[4] S. Thomas, An identification for the locally finite nontwisted Chevalley groups, to appear in Arch. Math.

DECIDABILITY OR UNDECIDABILITY OF SOME THEORIES OF MEASURE SPACES.

(Sauro Tulipani, Univ. of Camerino)

Let B be a Boolean algebra and F be an ordered real-closed field. A finitely additive measure (shortly a "mass") is a non-negative function $m: B \longrightarrow F$ such that $m(x+y)=m(x)+m(y)$ whenever $xy=0_B$. The triple (B,F,m) will be called here general measure space (g.m.s.). Moreover, a mass m will be called strongly non-atomic if for every $x \in B$ the restriction of m to x is onto the interval $[0, m(x)]$ of F; m will be called strictly positive if $m(x)=0_F$ implies $x=0_B$.

Let H, K be classes of general measure spaces. Denote by $H \otimes K$ the class whose members are isomorphic to g.m.s. of the form $(A \times B, F, m_1 \oplus m_2)$ for some $(A,F,m_1) \in H$ and $(B,F,m_2) \in K$; $m_1 \oplus m_2$ is defined by $m_1 \oplus m_2(a,b) = m_1(a)+m_2(b)$. Define, now, the following classes. H_1 is the class of g.m.s. (A,F,m) where A is an atomic algebra and m a strongly non-atomic mass. H_2 is the class where A is atomless and m strongly non-atomic and strictly positive. H_3 is the class where A is atomless and m strongly non-atomic mass such that for every non-zero $z \in A$ the restriction of m to z is never strictly positive. K_n is the class of g.m.s. where A is a finite Boolean algebra with at most n atoms.

THEOREM 1. ([1])- The theory Th(H) of the class $H = H_1 \otimes H_2 \otimes H_3$ is decidable.-

The proof is by the method of elimination of quantifiers.

THEOREM 2. ([2])- The theory T_n of the class $H \otimes K_n$ is interpretable by parameters in Th(H). Hence, T_n is decidable.-

However, the following theorem shows that the decidability of the theory of all g.m.s. fails.

THEOREM 3. ([2])- The class K^a of all g.m.s. $(A, \mathbb{R}, m)$, where A is an infinite atomic algebra and $\mathbb{R}$ is the field of real numbers, has an hereditarily undecidable theory.-

The proof is obtained by interpreting the theory of finite graphs into the theory of K^a.

[1] S. TULIPANI, "Invarianti per l'equivalenza elementare per una classe assiomatica di spazi generali di misura", Suppl. Boll. U.M.I. Vol. 2 (1980), 107-118.

[2] ———————— ,"A use of the method of interpretations for decidability or undecidability of measure spaces", to appear in Algebra Universalis (issue dedicated to the 80th birthday of A. Tarski).

For presentation at Logic Colloquium '82,
Florence, Italy, 23-28 August 1982

DOLPH ULRICH, Some extensions of implicational S5 not complete with respect to any class of frames.

With wffs built from letters and the binary connective *C*, C5 is the sentential calculus with axioms *Cpp*, *CCpqCrCpq*, *CCpCqrCCpqCpr* and *CCCCprqCprCpr* and rule detachment. Its theorems (cf. the author's *Strict implication in a sequence of extensions of S4*, Zeitschrift für mathematische Logik und Grundlagen der Mathematik, vol. 27 (1981), pp. 201-212, from which comes also terminology unexplained below) are the wffs valid in each frame $\langle W,R \rangle$ in which R is an equivalence relation, so C5 axiomatizes the strict-implicational fragment of S5. Where $\hat{A}\alpha\beta$ abbreviates $CC\alpha\beta\beta$, let C5.ω come by adding $\tau_\omega = \hat{A}CpqCqp$ to C5's axiom set and C5.n by adding $\tau_n =$ $\hat{A}\ldots\hat{A}Cp_1p_2\ldots Cp_1p_{n+1}Cp_2p_3\ldots Cp_np_{n+1}$ as well.

For n in $\{2, 3, \ldots, \omega\}$, let S_n be the matrix with values ω and the integers less than n, 1 designated, and the operation c defined so that $c(i,j) = 1$ if $i \geq j$ and $c(i,j) = \omega$ otherwise. Then $\vdash_{C5.\omega} \alpha$ iff $\models_{S_\omega} \alpha$ and $\vdash_{C5.n} \alpha$ iff $\models_{S_n} \alpha$.

C5.ω properly extends C5. Contrary to the well-known result that all proper extensions of full S5 have finite characteristic matrices, however, C5.ω has none, τ_n failing in S_{n+1} when letters are valued by their subscripts but valid in any matrix with n or fewer elements whose tautologies include those of S_ω. Of course the proper extensions of C5.ω do all have finite characteristic matrices: the S_n's.

Finally, C5.ω through C5.3 form an ascending chain of proper extensions of C5 none of which is complete with respect to any class of frames: if $\langle W,R \rangle$ is a frame for which there exist x and y in W with xRy and $x \neq y$, then τ_ω fails in any implicational model $\langle W,R,v \rangle$ based on that frame wherein $v(p,x) = v(q,y) = T$ but $v(q,x) = v(p,y) = F$; thus any frame for C5.ω must satisfy the condition $\forall x,y$, xRy *only if* $x = y$ and so validate the nonthesis *CpCqp*.

[Primary classification: 02C10 Modal logic, etc.
Secondary classification: 02C05 Many-valued logic]

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[Primary classification: 02C10 Modal logic, etc.
Secondary classification: 02C05 Many-valued logic]

A sequent calculus for the modal logic of provability

Silvio Valentini

During the last years, some researcher studied the modal logic of provability GL (see /B/) by proof-theory means.
Most of the interest in GL is due to the wellknown Solovay's theorem that estabilishes its completeness with Peano Arithmetic /S/.
A sequent calculus for GL can be obtained adding the following rule

$$\text{GLR:}\quad \frac{X, \Box X, \Box A \vdash A}{\Box X \vdash \Box A} \qquad \Box X = \{\Box B : B \in X\}$$

to the usual Gentzen-like propositional rules.
The first proof of cut-elimination for this calculus was obtained, using Kripke models, by G.Sambin and me /S-V 1/.
We then succeeded in deriving from it most of the wellknown results on GL: completeness with transitive terminal Kripke frames, finite model property, effective decidability, interpolation theorem and the fixed point theorem.
The problem to give a purely syntactic proof of cut-elimination revealed itself harder. At the matter of fact it was hard enough that the first attempt by D.Leivant /L/ to solve it came out with a wrong proof /S-V 2/.
Only few months ago G.Bellin /Bel/ gave a very complicated proof of normalization for the natural deduction version of GL.
Now, it is possible to obtain a simpler proof of cut-elimination for the sequent calculus using induction up to ω^3 and then to have a syntactic proof of consistency for GL.
The proof goes like an usual Gentzen-style proof of cut-elimination but the reduction proposed to solve the problem differs a lot from the usual ones and the structure of the cut-free proof obtained at the end of the process of reduction is usually very different

White and Black - a Boolean game

Peter Vojtáš

We examine the following transfinite game on Boolean algebras introduced by T.Jech in [J]. Given two players White and Black and given a Boolean algebra B , let White and Black define a decreasing sequence

$$w_0 \geq b_0 \geq w_1 \geq \dots \geq w_n \geq b_n \geq \dots \tag{1}$$

of nonzero elements of B of length ω by taking turns defining its entries: that is, White chooses w_0 , then Black chooses $b_0 \leq w_0$, then White chooses $w_1 \leq b_0$, and so forth. The play is won by Black if the sequence (1) has a nonzero lower bound, and by White if the intersection of (1) is zero.

The main problem concerned is the one of [J] : "Whether the existence of the winning strategy for Black implies that B has a σ-closed dense subset". Our goal is to build up a combinatorial structure on the set of all strategies and study its relationship to cardinal characteristics of Boolean algebras. Besides, improvements of some results from [F], [G], [J] and [V] are obtained.

AMS(MOS) subject classification (1980): Primary 04A20, Secondary 06E99

References:

[F] M. Foreman: Games played on boolean algebras. Preprint.

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[V] P. Vojtáš: A transfinite Boolean game and a generalization of Kripke's embedding theorem. (To appear)

Logic Colloquium '82 ; Abstract presented by title.

AMS Classification: 03C10, 03C60, 03B25, 06D05, 06E05.

Quantifier Elimination for Distributive Lattices.

Volker Weispfenning, Univ. of Heidelberg, W. Germany.

Let $L = \{\sqcap, \sqcup\}$ be the language of lattices and let L_C be L together with an arbitrary set C of constant-symbols. We characterize all classes K of distributive lattices in L_C that allow quantifier elimination (q.e.) in L_C. Any such K allows primitive recursive q.e. in L_C ; moreover for finite C, K is primitive recursively decidable. The number of different complete, elementary classes K allowing q.e. in L_C is finite for finite C, but rapidly growing with card C. Let q(n) (q'(n)) be the number of complete, elementary classes K of (relatively complemented) distributive lattices in L_C allowing q.e. in L_C , where C is fixed as a chain of length n. Then $q'(0)= 2$, $q'(n)= 3^{n+1}$ for $n \geq 1$; $q(0)= 3$, $q(1)= 41$, $q(n) > 2^{4n-5} \cdot 3^4$ for $n \geq 2$. As a corollary, we have a complete list of all classes of boolean algebras and relatively complemented, distributive lattices that allow q.e. in their respective languages with an arbitrary set of additional constants.

EDUARD W. WETTE, 1977 Wrocław abstract renewed: Computation of the anisotropic proportions in an anti-Einsteinian representation of all motions II. (68)

For the complete title cf. XLIV, p. 476, [12]; pp. 441 – 468 contained $64 = 37+8-1-2+22$ abstracts. (ii), (iii), (vi) refer to §2; see Part I later on. §2$^{>}$. Hypertorus recursion. $^{>}$ indicates that, as concerns numerical determination (and the direction of geodesics' projection), ν will precede $\nu-1$ in ϱ_ν (and β_ν), vs. hypertorus-generation.

(ii) $r_0 : .. : r_n$ determines $\mathfrak{T}(=n+1$ -torus). "∞" $\Leftrightarrow$ self-intersection; $\neg$"∞" $\to \Sigma_{\mu<\nu} r_\mu < r_\nu\ [\leftrightarrow \varrho_{\nu-1} < \varrho_\nu/(2-\varrho_\nu)]$, $\nu = 0..n$; $(r_0+..+r_\nu) : r_n = (1-\varrho_{n-1})\varrho_\nu/(1-\varrho_\nu)$, $\nu = n..-1$, solves $\varrho_\nu = (r_0+..+r_\nu):(r_n-r_{n-1}-..-r_{\nu+1}) \wedge \varrho_n = (1+\varrho_{n-1})/2 \wedge \varrho_{-1} = 0$, $n>\nu\geqslant 0$. $\mathfrak{T}$ embedded: ${}^{n+1}x_\nu = {}^{\nu+1}x_\nu = P_\nu \sin\beta_\nu\ (\nu\leqslant n)$, ${}^{n+1}x_{n+1} = P_n\cos\beta_n$; $P_l = \Sigma_{m\leqslant l} r_m \Pi^l_m$, $\Pi^l_m = \prod_{m}^{l-1}{}_\mu \cos\beta_\mu$. $r = P_n/r_n$, cosmic 'time' $t_r = r\beta_n$; $n-3=0$ results from §2 (v). $\Sigma_{j\leqslant k+1}(\mathrm{d}({}^{k+1}x_j))^2 = \Sigma_{j\leqslant k} P_j^2(\mathrm{d}\beta_j)^2 = (\mathrm{d}s_k)^2$. (iii) Components $\neq 0$: $g_{kk} = P_k^2\ (0\leqslant k\leqslant n)$; $\Gamma_{j.kk} = P_jP_k\sin\beta_j\cdot\Pi^k_{j+1} = -\Gamma_{k.jk} = -\Gamma_{k.kj}$, $R_{jkjk} = P_jP_k\Pi^j_0\Pi^k_0 = -R_{jkkj} = R_{kjkj} = -R_{kjjk}$ $(0\leqslant j<k\leqslant n)$; $R_{jkjk;l} = \sin\beta_l\cdot\Pi^j_0\Pi^k_0(P_l(P_j\Pi^k_{l+1} + (1-0^{j\dot{-}l})P_k\Pi^j_{l+1}) - (2-0^{j\dot{-}l})P_jP_k/\cos\beta_l) = -R_{jkkj;l} = R_{kjkj;l} = -R_{kjjk;l}$ $(l<k\wedge j<k,\ 0<k\leqslant n)$; $R_{jkjl:kl} = P_j\Pi^j_0\Pi^k_0\Pi^l_0(P_k\Pi^l_0 - P_l\Pi^k_0) = -R_{jkjl:lk} = -R_{jklj:kl} = R_{jklj:lk} = R_{kjlj:kl} = -R_{kjlj:lk} = -R_{kjjl:kl} = R_{kjjl:lk}$ $[= -R_{jljk:lk} = ..$ skew-symmetric in $k,l]$ $(j\neq k\wedge j\neq l\wedge k\neq l$, $g^{jk} = \delta_{jk}/g_{kk})$, curvature's curvature $R^j_{klm:pq} = R^\nu_{klm}R^j_{\nu pq} - R^j_{\nu lm}R^\nu_{kpq} - R^j_{k\nu m}R^\nu_{lpq} - R^j_{kl\nu}R^\nu_{mpq}$ $\leftrightarrow \underset{43}{\Delta}\underset{21}{\Delta}\xi^j + R^j_{\nu\varrho\sigma}\underset{21}{\Delta}\xi^\nu \underset{3}{\mathrm{d}}u^\varrho\underset{4}{\mathrm{d}}u^\sigma = -R^j_{\kappa\lambda\mu:\varrho\sigma}\xi^\kappa\underset{1}{\mathrm{d}}u^\lambda\underset{2}{\mathrm{d}}u^\mu\underset{3}{\mathrm{d}}u^\varrho\underset{4}{\mathrm{d}}u^\sigma$ $(\underset{21}{\Delta} = \underset{21}{\mathrm{dd}} - \underset{12}{\mathrm{dd}}$, $\mathrm{d}\xi^j = -\Gamma^j_{\kappa\lambda}\xi^\kappa\mathrm{d}u^\lambda)$.

(vi) $\mathfrak{T}$-geodesics $\leftrightarrow P_j^2\,\mathrm{d}\beta_j/\mathrm{d}s_j = C_{(j)} = \mathrm{const}$. $\mathrm{d}\beta_j = 0 \leftrightarrow j\neq j_\lambda \dot{\leftarrow} 0\leqslant j_1<..<j_l\leqslant n$, $\Pi^{j_1}_0 \neq \pm 1$ envelopes only; $a = j_\lambda \wedge b = j_{\lambda+1}\ (l>\lambda>0) \vdash r^*_{(ab)0} = P_a\Pi^b_{a+1} \wedge r^*_{(ab)1} = \sum_{a+1}^{b}{}_m r_m\Pi^b_m$. Integration: $\mp\beta_b = G(r^*_{(ab)0} : C_{(b)} : r^*_{(ab)1}; \beta_a)\cdot P_a/(C_{(a)}\Pi^b_{a+1}) - \gamma_b \leftarrow \Pi^b_{a+1}\neq 0$; $\mp\beta_b = -\gamma_b + \beta_a\cdot(1-(C_{(b)}/r^*_{(ab)1})^2)^{-1/2}\cdot(P_a/r^*_{(ab)1})^2\cdot C_{(b)}/C_{(a)} \leftarrow \Pi^b_{a+1} = 0$. Finiteness $\supset$ closed $\mathfrak{T}$-geodesics: $Q(\varrho, C_1) = u(\varrho, C_1; 0) = \langle p_1^*, m_1^*/|m_0^*|\rangle\cdot\pi \leftarrow \langle 1-\varrho, 0\rangle \leqslant C_1 \leqslant \langle 1+\varrho, 1-\varrho\rangle \wedge m_0^*\langle =, \neq\rangle 0$, $C_1 = (1+\varrho)\sin\alpha_{min}$; p_1^* fraction (not semi-irrational), $m^*_\delta\ \beta^*_\delta$-turns on $\mathfrak{T}^*(\varrho, 1)$, C_1 Clairaut constant of a $\mathfrak{T}^*$-geodesic, α geodesic's azimuth. $\neg$"∞" $\to \langle p_1$ Egyptian, $m_1 = 1 \vee m_0 = 1\rangle$, if no $*$-projection; no constriction $\to p_1 = 0$; no "Hohlkosmos-Seitenwechsel" $\to m_1 = 1 \vee m_0 > 2$.

Computation. $t_0(\varrho, C_1; \beta_0^*) = \tau_0^{\langle +,-\rangle 1}$; $t_{1,2,3,4}(\varrho, C_1) = \tau_{1,2,3}^{\langle +,-\rangle 1}$, $\tau_4^{\langle +,-\rangle}$; $t_5(\varrho) = 2\varrho/(1-\varrho^2)^{1/2}$. $\tau_0^{-1} = \frac{1}{2}\tau_7\tau_6/\tau_5$, $\tau_1^{-1} = 4\varrho C_1/(\tau_7\tau_8)$, $\tau_2^{-1} = \frac{1}{2}(1+\tau_3^{-1})$, $\tau_3^{-1} = C_1/(1-\varrho) = \sin\alpha_{max}$, $\tau_4^- = 2\varrho/(\tau_7\tau_8)^{1/2}$. $\tau_4^+ = (\varrho C_1)^{1/2}/(1-\varrho)$; $\tau_5 = 1+\varrho\cos\beta_0^* - C_1$, $\tau_6 = 1+\cos\beta_0^*$, $\tau_7 = 1+\varrho-C_1$, $\tau_8 = 1-\varrho+C_1$. Practicable result: $Q(\varrho, C_1) = t_3t_4\mathrm{K}(k) - t_5(\mathrm{K}(k)E(k,\Phi) - \mathrm{E}(k)F(k,\Phi)) \leftarrow k^2 = t_1 \wedge \sin^2\Phi = t_2$; $G(\varrho : C_1 : 1; \beta_0^*) = t_4(F(k,\varphi) - (1-t_3)\Pi(N,k,\varphi)) \leftarrow \sin^2\varphi = t_0 \wedge -N = t_1t_2 = (k\sin\Phi)^2$, $t_4(1-t_3)\Pi(N,k,\varphi) = t_5(u\vartheta_1'(U)/\vartheta_1(U) - \frac{1}{2}\ln(\vartheta(U+u)/\vartheta(U-u))) \leftarrow u : U : \pi/2 = F(k,\varphi) : F(k,\Phi) : \mathrm{K}(k) \wedge q = \exp(-\pi\mathrm{K}'/\mathrm{K})$.

ADDENDA. (ii)(iii) $P_0 = r_0$, $P_{n+1} = r_{n+1} + P_n\cos\beta_n$; see l.c. [5, 3.], XLII p. 478. (vi) Computation. As to 2nd $\langle\ \rangle$-case (with $^{-1}$, $^{-}$) see l.c. [8, 4.1.1], XLVI p. 444/5. $u = \lim\varphi_\nu$: $\varphi_0 = \varphi$, $k_0 = k$, $k_{\nu+1} = (1-k'_\nu)/(1+k'_\nu)$, $\sin\varphi_{\nu+1} = (1-\Delta_\nu)/((1-k'_\nu)\sin\varphi_\nu)$ or $\tan^2\varphi_{\nu+1} = \tan^2\varphi_\nu\cdot\tau_9(k'_\nu, \Delta_\nu)/\tau_9(1, \Delta_\nu)$, where $k_\nu^2 + k_\nu'^2 = 1$, $\Delta_\nu^2 = 1 - k_\nu^2\sin^2\varphi_\nu$, and $\tau_9(k,c) = (k+c)/(1+c)$. $\vartheta(z) = \overset{+}{\underset{-}{\Sigma}}{}_\nu T(\nu; z, q)$, $T(\nu) = (-1)^\nu q^{\nu\nu}\exp(2\nu iz)$, $i^2 = -1$; $\overset{+}{\underset{-}{\Sigma}}{}_\nu T(\nu+\frac{1}{2}; z, q) = iq^{1/4}\exp(iz)\vartheta(z+\frac{1}{2}i\ln q^{-1}) = -\vartheta_1(z)$, $\vartheta_1'(z) = \mathrm{d}\vartheta_1/\mathrm{d}z$. $\mathrm{K}' = \mathrm{K}(k')$, $\mathrm{K} = \mathrm{K}(k) = F(k, \pi/2)$; $(F, E)(k,\varphi) = \int_0^\varphi \Delta(k,\psi)^{(-1,+1)}\mathrm{d}\psi$, $\mathrm{E}(k) = E(k, \pi/2)$. $\mathrm{K}' : \mathrm{K} : \pi/2 = 1/M(1,k) : 1/M(1,k') : 1$, where M arithmetico-geometric mean.