MATHEMATICAL INSTITUTE
SANU - BELGRADE

Druga nacionalna konferencija "Verovatnosne logike i njihove primene"

Beograd, Srbija, 27. i 28. septembar 2012.

Knjiga apstrakata

ORGANIZATOR:

Matematički institut, SANU

KONFERENCIJU FINANSIRAJU:

Ministarstvo prosvete i nauke Republike Srbije

Projekat Razvoj novih informaciono-komunikacionih tehnologija, korišćenjem naprednih matematičkih metoda, sa primenama u medicini, telekomunikacijama, energetici, zaštiti nacionalne baštine i obrazovanju, III 044006

Projekat Reprezentacije logičkih struktura i formalnih jezika i njihove primene u računarstvu, ON 174026.

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Druga nacionalna konferencija
"Verovatnosne logike i njihove primene"

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TEME KONFERENCIJE:
- verovatnosne logike, problemi potpunosti, odlučivosti i složenosti,
- logičke osnove u zasnivanju verovatnoće,
- Bayes-ove mrrže i drugi srodni sistemi,
- programski sistemi za podršku odlučivanju u prisustvu neizvesnosti,
- primene verovatnosnog zaključivanja u medicini itd.

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Zoran Marković (Matematički institut SANU)
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ORGANIZATOR:
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Program konferencije:

Dan 1 – 27. 9. 2012.

10:00 – 10:15 *Otvaranje*, predsedava Miodrag Rašković

10:15 – 11:00 Zvonimir Šikić
Probability as relative frequency and countable additivity

11:00 – 11:30 *Pauza*

11:30 – 13:10 *Sesija 1*, predsedava Zvonimir Šikić

- Lazar Velimirović, Zoran Perić, Miomir Stanković, Jelena Nikolić
Design of Asymmetrical Scalar Quantizer with Extended Huffman Coding for Gaussian Source
- Miloš B. Djurić, Velimir M. Ilić, Miomir S. Stanković
The computation of mathematical expectation and covariance on junction trees
- Miroslav Ristić, Predrag Popović, Aleksandar Nastić
INAR Models and Application
- Radosav Djordević, Nebojša Ikodinović, Vladimir Ristić
Topological class logic $L_{\mathbb{A}}^{\text{cont}}(O^n, C^n)_{n\in\omega}$
- S. Ghilezan, J. Ivetić, P. Lescanne, S. Likavec
The resource control and strong normalisation

13:10 – 15:30 *Pauza*

15:30 – 16:30 *Sesija 2*, predsedava Zoran Marković

- Aleksandar Pejović, Žarko Mijajlović
Verification of Boolean Formulas by Use of Free Vectors
- Dragana Valjarević
Some generalization of granger's causality and orthogonality of martingales
- Miodrag Kapetanović
Another logical probability calculus

16:30 – 16:45 *Pauza*

(nastavak programa prvog dana je na narednoj strani)

16:45 – 18:05 *Sesija 3*, predsedava Miodrag Kapetanović

- Nataša Glišović
  A Fuzzy Regression Model Approach for Medical Research
- Velimir M. Ilić, Miomir S. Stanković
  Pseuddo-additive entropies as expected information content
- Velimir M. Ilić, Edin H. Mulalić, Miomir S. Stanković
  Two parameter deformation of information content
- Predrag Rajković, Miomir Stanković, Sladjana Marinković
  The q–exponential function like a limit of expectations

Dan 2 – 28. 9. 2012.

10:00 – 11:20 *Sesija 1*, predsedava Žarko Mijajlović

- Angelina Ilić Stepić, Zoran Ognjanović
  Complex valued probability logics
- Dragan Doder
  Probabilistic logics and measures of inconsistency
- Marija Milojević, Zoran Ognjanović, Nataša Glišović
  Application of Bayesian Network to Reliability Assessment of Mechanical System
- Milica Knežević
  Reasoning in Multi-Agent Systems

11:20 – 11:40 *Pauza*

11:40 – 14:00 *Sesija 3*, predsedava Dragan Doder

- Ivan Čukić
  Algorithm for automatic clustering of resources based on usage statistics
- Marija Boričić
  Probabilistic logic as a labelled deductive system
- Nebojša Ikodinović
  Non-Archimedean probability measures
- Saša Rašuo
  Decidable theories
- Predrag Tanović
  Tipovi i mere
- Aleksandar Perović
  Dynamic probability logics
- Dragan Radojević
  Verovatnosna logika kao i realno-vrednosna logika se nalaze u istom algebarskom okviru kao i klasina logika

Apstrakti

Sadržaj

Probability as relative frequency and countable additivity

Zvonimir Šikić, Zagreb

As frequencies satisfy probability axioms it is tempting to think of them as probabilities. The problem is that frequencies change with a number of trials. A proposed solution is to define probabilities as limiting frequencies. By introducing this presupposed infinity of trials we face the problem that the limiting frequency of an infinite sequence of trials need not exist.

Richard von Mises proposed we restrict ourselves to random sequences which have limiting frequencies (he called them collectives). Church further clarified this notion by imposing a condition that limiting frequencies of all recursive subsequences of a given collective are equal to the limiting sequence of the original collective. Also, limiting frequencies should be stable under all recursive reorderings (we think this is not implied by Churchs condition).

But why should an infinite sequence, e.g. of coin tosses, be a collective? Or why should a finite sequence of tosses be an initial part of a collective?

Furthermore, there is an old problem (discussed by Kolmogorov and many others) that limiting sequences violate countable additivity. We prove this is not a problem of limiting frequencies but a problem of the definition of the real numbers (the same problem we face with Cantor- Lebesgue function not satisfying Newton-Leibniz formula).

Design of Asymmetrical Scalar Quantizer with Extended Huffman Coding for Gaussian Source

Lazar Velimirović *Zoran Perić* *Miomir Stanković*
Jelena Nikolić *†‡

In this paper we propose a novel class of asymmetrical two-level scalar quantizers with extended Huffman coding that are designed to provide the required quality of the quantized signal, measured by SQNR (Signal to Quantization Noise Ratio), and for the average bit rate to approach the source entropy as close as possible. The only constraint in the design of the novel quantizer is that the value of SQNR decreases no more than 1 dB from the optimal SQNR Lloyd-Max's quantizer value. The two-level Lloyd-Max's quantizer with zero decision threshold is a special case of our quantizer. Unlike the two-level Lloyd-Max's quantizer having the decision threshold settled in zero, the novel quantizer with the same number of quantization levels proposes that the determination of the variable decision threshold is performed in a way that it has a non-negative value, which is designed depending on which SQNR has to be achieved. The basic idea is that, unlike to the Lloyd-Max's quantizer, the asymmetry of representation levels is assumed such that to provide an unequal probability of representation levels for the symmetric Gaussian probability density function (PDF). This in turn provides the proper basis for the further implementation of a lossless compression techniques. Output levels of a quantizer can be considered as a discrete source of symbols and can be coded using fixed-length codewords. However, a more effective manner of coding is by using entropy code with variable-length codewords. The bit rate of any lossless code is always higher than the entropy, where the aim is to approach the entropy as close as possible. To achieve this, symbols with large probabilities are coded with short codewords and less-probable symbols are coded with longer codewords. Among many lossless compression techniques,

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such as Huffman code, arithmetic code and Golomb-Rice code, the most suitable for utilization is the extended Huffman coding technique that achieves the lowest average length of code words. The procedure of Huffman coding includes determining the optimal length of code words for a given probability of symbols. It is sometimes beneficial to additionally reduce the bit rate by blocking more than one symbol together. Extended Huffman coding is the procedure of determining the optimal length of code words for blocks of two or more symbols. For that reason, we propose a quantizer that has only two representation levels and we apply extended Huffman coding on the output levels of this quantizer. As with Lloyd-Max's quantizer, these representation levels are determined from the centroid condition. It is shown that by using the extended Huffman coding technique and the set of quantizers with variable decision thresholds, approaching of the average bit rate to the source entropy can be achieved.

The computation of mathematical expectation and covariance on junction trees

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We consider the algorithms for computation of mathematical expectation and covariance of decomposable vector random variables with probability density functions which factorizes according to junction tree. The algorithm runs using message passing schemes by Shafer-Shenoy and Lauritzen-Spiegenhalter. Previously, different message passing computational schemes has been developed.

Mauá et al. and Ilić et al. give an algorithm which operate as order pair message passing over junction tree known as *EMP*. On the other side, Lauritzen and Nilsson develop an algorithm which can be considered as normalized version of *EMP*. As the consequence of normalization, Lauritzen and Nilsson is computationally more efficient in comparison to *EMP*.

Covariance computation is previously considered by Kulesza and Taskar. They consider four tuple order pair message passing algorithm which can be considered as an four tuple extension of *EMP*. In this paper we present normalized version of Kulesza-Taskar algorithm. Proposed algorithm is computationally more efficient in comparison to Kulesza-Taskar algorithm and it can be seen as a four tuple extension of Lauritzen-Nilsson algorithm.

INAR Models and Application

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This paper is about integer-valued autoregressive models. It gives insight in one and two dimensional models and their application on a real data. The models are based on binomial and negative binomial thinning operators. The structure and stationarity of models are discussed. The conditional and unconditional first and second moments are derived. Least square, Yule-Walker and Maximum likelihood methods for parameters estimation are given.

Topological class logic $L_{\mathbb{A}}^{\text{cont}}(O^n, C^n)_{n\in\omega}$

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The logic $L_{\mathbb{A}}^{\text{cont}}(O^n, C^n)_{n\in\omega}$ is an infinitary logic formed by combining the admissible fragment $L_{\mathbb{A}} = L_{\omega_1\omega} \cap \mathbb{A}$, where $\mathbb{A}$ is a countable admissible set, with the quantifier symbols O^n and C^n, $n \in \omega$.

The meaning of a formula closed by quantifier O^n is that the set defined by the formula is open in the nth product topology. Similarly, a formula closed by quantifier C^n means that the set is closed. It is proved that the system of axioms of this logic (in which we describe property of topological class spaces as well as property of topological products and continuous functions) is complete with respect to topological class-models.

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The resource control and strong normalisation

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We present the *resource control cube*, a system consisting of eight intuitionistic lambda calculi with either implicit or explicit control of resources and with either natural deduction or sequent calculus. The four calculi of the cube that correspond to natural deduction have been proposed by Kesner and Renaud in [4], while we introduce their sequent counterparts, starting from Espírito Santo's λ^{Gtz}-calculus [1] where structural rules are implicit, and finishing with $\ell\lambda^{\mathsf{Gtz}}$-calculus of [3], in which both contraction and weakening are explicit. The simply typed resource control cube represents the computational interpretation of intuitionistic natural deduction and intuitionistic sequent logic with implicit or explicit structural rules.

We propose a general type system that assigns a particular form of intersection types, namely strict types, to the resource control cube. This system extends the results from [2], where intersection types were introduced only to the two calculi of the cube that contain both contraction and weakening explicit. This proposed system increases the expressiveness both of types and terms, which ensures that more precise properties can be checked and more features can be encoded. By instantiating it to each of the eight calculi of the cube, we obtain type systems that are syntax-directed, satisfy preservation of types under reductions and equivalences and assign types to all strongly normalising terms of the corresponding resource control calculus.

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Verification of Boolean Formulas by Use of Free Vectors

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Let $f(x_1, x_2, \dots, x_n)$ be a boolean expression in n variables $x_1, x_2, \dots, x_n$. A method for checking if the identity $f(x_1, x_2, \dots, x_n) = 1$ is valid for all boolean values of $x_1, x_2, \dots, x_n$ is proposed, based on the bitwise parallelism inherent in computer hardware. We give a construction (see [1]) of n boolean vectors $b_1, b_2, \dots, b_n$ of the size 2^n with the following property:

If $f(b_1, b_2, \dots, b_n) = 1$, then $f(x_1, x_2, \dots, x_n)$ is identically equal to one.

It appears that the vectors $b_1, b_2, \dots, b_n$ are exactly the free generators of a free boolean algebra (see [2]) having n free generators. If m is the size of this algebra, then $\log_2 m = 2^n$. We also consider related combinatorial problems, for example the number of different Boolean algebras on a set having 2^n elements.

We also give an OpenCL implementation that can be run on multi-core CPUs as well as on highly parallel GPUs such as Nvidia Tesla C2075 with 448 CUDA cores. We use this implementation to demonstrate gains in reduction of computing steps by fully utilizing underlying hardware compared to the number of 2^n computing steps in the usual table-checking procedure.

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Some generalization of granger's causality and orthogonality of martingales

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The paper considers a statistical concept of causality in continuous time between filtered probability spaces which is based on Granger's definition of causality. As shown in *Lj.Petrović and S.Dimitrijević*: Statistical causality and adapted distribution, (Czech.Math.J, 61, 136 (2011), pp.827–843), the given causality concept is equivalent with the concept of adapted distributions (introduced in *D.Hoover, J.Keisler*: Adapted probability distributions, Trans.Amer.Math.Soc. 286, (1984), 159–201). Their thesis is that two processes with the same adapted distribution share the same probabilistic properties. The given causality concept is closely connected with the notion of synonymity between two processes (introduced in *D.Aldous*: Weak convergence and the general theory of processes, preprint (1981)), which is a weaker notion then adapted distribution.

Causality is, in any case, a prediction property and the central question is: is it possible to reduce available information in order to predict a given filtration. Let $\mathbf{F}, \mathbf{G}$ and $\mathbf{H}$ be arbitrary filtrations on the same probability space. We can say "$\mathbf{G}$ entirely causes $\mathbf{H}$ within $\mathbf{F}$" if $(\mathcal{H}_\infty)$ and $(\mathcal{F}_t)$ are conditionally independent when $(\mathcal{G}_t)$ is given, written as $\mathcal{G}_\infty \perp \mathcal{F}_t \mid \mathcal{G}_t$. In other words $(\mathcal{G}_t)$ contains all information from the $(\mathcal{F}_t)$, needed for predicting $(\mathcal{H}_\infty)$. These definitions can be applied to stochastic processes if we are talking about the corresponding induced filtration.

The given causality concept can be applied to the strongly orthogonal martingales and local martingales which have an important application in financial mathematics (in the theory of option trading). This connection is cosidered for stopped local martingales, too.

Another logical probability calculus

Miodrag Kapetanović

Based on the propositional probability calculus LPP2 from [...] and similar theories a first order quantifier free theory is presented in which propositional formulas naturally appear as constant terms. A probability semantics is based on lattice ordered Abelian groups with the usual finite additivity law. An associated tableau system is sketched.

A Fuzzy Regression Model Approach for Medical Research

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Regression methods are essential to any data analysis which attempts to describe the relationship between a response variable and any number of predictor variables. Frequently, situations involving discrete variables arise. For example, in a medical setting, an outcome might be presence/absence of disease. Regression analysis is one of the most popular methods to estimate the functional dependence between the dependent and independent variables. Fuzzy regression analysis is a kind of extension of the statistical regression analysis in terms of individual model parameters fuzzification. In real life, there is a large number of problems that can be modeled by phase regression analysis. In recent years it has gained quite significant and it is not only is a method that can possibly only supplement the results obtained by classical techniques but as an independent method of assessment. Confirmation of this is the significant scientific attention, which is in recent years given stage regression analysis, the number of papers and a variety of implementation. The aim of this paper is to present these models in medical research.

KEYWORDS: Fuzzy regression analysis, selection of independent variables, medical research.

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Pseuddo-additive entropies as expected information content

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Pseudo-additive information content $I_q(p)$ is generalization of standard information content [2]. It is defined function of two variables $q \in \mathbb{R}^+$ and $p \in [0,1]$, which satisfies the following axioms

[T0] $I_1(p) = -k \ln p$, $k \neq 0$

[T1] I_q is continuous with respect to $p \in (0,1)$ and $q \in \mathbb{R}^+$,

[T2] $I_q(p)$ is convex with respect to $p \in [0,1]$ for any fixed $q \in \mathbb{R}^+$,

[T3] there exists a function $\varphi : R \to R$ such that

$$\frac{I_q(p_1 p_2)}{k} = \frac{I_q(p_1)}{k} + \frac{I_q(p_2)}{k} + \varphi(q) \cdot \frac{I_q(p_1)}{k} \cdot \frac{I_q(p_2)}{k} \tag{1}$$

for any $p_1, p_2 \in [0,1]$, $\varphi(q) \neq 0$ for $q \neq 1$ and $\varphi(q)$ is continuous.

Pseudo-additive entropy of distribution p, $S_q(p)$ is defined as the appropriate expectation value of $I_q(p)$,

$$S_q(p) \equiv E_{q,p}[I_q(p)], \tag{2}$$

and the Kullback-Leibler (KL) divergence between distributions p^A and p^B is defined by means of the information contents difference,

$$K_q(p^A \parallel p^B) = E_{q,p^A}\left[I_q(p^B) - I_q(p^A)\right]. \tag{3}$$

In this paper we determine the expectation operators corresponding to our information content, using the condition that Kullback-Leibler divergence induced by pseudo-additive information content is nonnegative [1]. We define two types of operators: generalized unnormalized and normalized expectations. It is shown that generalized unnormalized expectation does not preserve the form invariance of pseudo-additivity between entropy and its information content, while generalized normalized expectation preserve it.

The entropy measure derived in this paper generalizes well known Tsallis and Havrda-Charvát entropies [3], [4]. It also represents a generalization of pseudo-additive entropy derived in [1].

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Two parameter deformation of information content

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In recent years, research of many natural phenomena showed necessity for various kinds of statistical mechanics formalism – non-extensive statistical mechanics. Similarly to the definition of Shannon entropy in classical Boltzmann-Gibbs-Shannon (BGS) theory, non-extensive entropy is defined as the appropriate expectation value of non-extensive information content. In Shannon's case information content is represented by Neper's logarithm and it can be characterized in axiomatic way [6] as additive function. In nonextextensive situation, additivity property is generalized to pseudoadditivity, and Neper's logarithm is generalized to deformed logarithm [3]. In this paper we consider one-parametric and two-parametric logarithm deformation of Kaniadakis type [1], [2], [4], [5].

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The q–exponential function like a limit of expectations *

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The well known number $e \approx 2.71818\cdots$ appeared like a expectation value in an trial problem by Putnam[1]. It has attracted important attention and the solutions appear in the Shultz paper [2]. The last consideration about it, can be found in the paper of Ćurgus [3].

Here is our generalization of this problem in q–domain.

Let $q > 1$ be a real parameter. To any random point

$$\mathbf{x} = (x_1, x_2, \ldots, x_n) \in [0,1]^n \qquad (n \in \mathbb{N}), \tag{1}$$

we can join the finite sequence of deformed partial sums by

$$S_k(\mathbf{x}, q) = x_1 + \frac{1+q}{2}\, x_2 + \cdots + \frac{1+q+\cdots+q^{k-1}}{k}\, x_k \quad (1 \le k \le n). \tag{2}$$

Let us define the next procedure.

Algorithm.

Step 1. Choose two real numbers q and t.

Step 2. Take two random numbers x_1 and x_2 from the interval $[0,1]$ and denote $n = 2$.

Step 3. If $S_n((x_1,\ldots,x_n),q) > t$, then memorize the value n and stop.

Step 4. If $S_n((x_1,\ldots,x_n),q) \le t$, then increase n into $n+1$, take the next random number x_n from $[0,1]$ and return to Step 3.

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Repeat this procedure a lot of times. What is the average value for n? We will show that the expectation for n is

$$e_1(q) = e_q((1-q)t), \quad \text{where} \quad e_q(z) = \sum_{n=0}^{\infty} \frac{z^n}{(q;q)_n},$$

is the small q-exponential function very important in the theory of basic hypergeometric functions [4].

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Complex valued probability logics

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In this article we present a two complex valued valued probabilistic logics, $COMP_B$ and $COMP_S$ which are complete and decidable extensions of classical propositional logic. Namely, $COMP_B$ is designed in such a way that the elementary probability sentences $B_{z,\rho}\alpha$ actually do have their intended meaning - the probability of propositional formula α is in the complex ball with the center z and the radius ρ. Similarly, in the logic $COMP_S$ we make statements such as $S_{z,\rho}\alpha$ with the intended meaning - the probability of propositional formula α is in the complex square with the center z and the side $2 \cdot \rho$. The key difference between those logics is in the complexity of decision-making procedures.

Probabilistic logics and measures of inconsistency

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The need for handling inconsistent knowledge bases has been recognized by the Artificial Intelligence community in recent years. Many logical formalisms are developed for reasoning under inconsistency, like paraconsistent logics, default reasoning, possibility theory, belief revision and formal argumentation [3, 4, 5, 6, 7, 8, 9, 10, 11]. Unlike classical logic, they enable inferring non-trivial conclusions from inconsistent knowledge bases, so that two different inconsistent sets can lead to different sets of conclusions.

Development of those techniques points out to the need for analyzing and comparing inconsistent sets [12]. In several papers, the measure of inconsistency depends on the proportion of the language that is affected by the inconsistency in a theory [14, 15, 16]. The limitation of this approach is the fact that it doesn't consider the distribution of contradiction among the formulas. The second approach considers the number of formulas needed to produce a contradiction: an inconsistent set A is better than an inconsistent set B, if the shortest derivation of a contradiction requires more members of A than B [17]. This idea implies that the set of formulas $\{\phi_1, \ldots, \phi_n\}$ is not equivalent to the singleton $\{\phi_1 \wedge \ldots \wedge \phi_n\}$. This property is valid in non-adjunctive logics [18, 19], special class of paraconsistent logics, and it can be meaningful in some practical applications. This approach turns to be closely related to probabilistic measure on formulas. In [20, 21], this idea was compared with semantic notion of existence of a probability measure which assigns a high probability to each formula of the theory.

Besides existence of probability-based measures of inconsistency propositional theories, some papers investigate degrees of inconsistency of sets of probabilistic formulas. In [22, 23], the level of inconsistency of a set of probabilistic formulas is proportional to distance from the function on formulas (given by the conditions of theory) to the closest probabilistic measure on formulas.

In this talk, we give a short overview of the results on the topic, and propose some directions for the further research.

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Application of Bayesian Network to Reliability Assessment of Mechanical System

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The application of Bayesian Network to semi-automatic mechanical system plant for painting metal products is shown. When diagnosing a failure of the machine it is necessary to make the right decision whether it is justified to continue the process of manufacturing or to stop it in order to eliminate failures. The main advantage of probabilistic reasoning over logic-based is the permission to make a rational decisions even when there is no enough information for proving that any action will be executed. The support system for decision-making and its specifically application will be shown in this presentation. The purpose of system is the prediction of quality of manufacturing products when some failure in the mechanical system occurs. In a broader sense, a computer system which receives the information of some failure as input value and gives the prediction of product quality as output value with the certain probability it is developed. Based on this, the user of the computer system can make decisions about whether to abort the process and eliminate failure occurred on the machine or it will continue the manufacturing process.

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Reasoning in Multi-Agent Systems

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Multi-Agent Systems (MASs) evolved from the field of Distributed Artificial Intelligence (DAI). MAS focuses on studying actions and interactions among agents which are driven by their goals and believes. In order to facilitate development of MASs and to make it possible to reason about expected and manifested behavior, the need for formal theory of MASs appeared. Logic based methods for analysis of concurrency, uncertainty, and non-determinism in MASs will be presented.

Algorithm for automatic clustering of resources based on usage statistics

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Most modern resource (documents, images, music, learning materials etc.) managers are able to group the resources into groups based on the topic. For example, book stores group the books into genres and sub-genres, history archives group them based on the age or geographical location etc.

While these clusters are useful for finding a specific document, it is unable to help detect the relations between documents of different groups.

These relations are not necessarily deductible from the resource content, but they might be deductible from the resource access logs. It is possible to create links between resources based on the usage statistics. Essentially, if two resources are accessed at the same time, there might be a possibility that they are related. If the same resources are accessed more times together, the probability of them being related grows.

Once we get a graph of resources and connections between them, we can separate them into different clusters, or topics, regardless of their contents. Using any common algorithm for graph clustering would be slow. Fitting for non-changing graphs, but not for the live ones. We are proposing relying on the implicitly built hierarchy of resources for efficient, almost linear clustering algorithm.

Probabilistic logic as a labelled deductive system

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There are two essentially different approaches to the language of propositional probability logic. When the propositional language is extended by a finite list of operators, then, in the first case, each propositional formula can be prefixed by one operator only (see [5]), or, in the second case, the operators can be applied inductively finitely many times on each formula (see [4]). In both of these cases the obtained system presents a kind of polymodal logic using the probability operators on the object–level of language. On the other side, the tradition of probability theory does not consider iterated probabilities like the following one $P\{P(A) \geq r\} \leq s$, where A is an event and $r, s \in [0, 1]$. Informal logical tradition treats probabilities sometimes on the meta–level (see [3], [6] and [8]) and sometimes on the object–level (see [1], [4] and [5]). The concept of labelled deductive systems (see [2] and [7]) makes it possible to formalize and unify the concepts concerning non–iterated probabilities present in [3], [5], [6] and [8] through a unique sequent calculus. The basic form of this calculus is a sequent of the form $\Gamma \vdash^{[a,b]} \Delta$ expressing the following statement: "the probability of truthfulness of a sequent $\Gamma \vdash \Delta$ is in the interval $[a, b]$", with its particular case $\vdash^{[a,b]} A$ meaning that "the probability of truthfulness of a formula A is in the interval $[a, b]$". In this case the interval $[a, b]$ presents a label of the corresponding sequent. The main problem is how to weaken both the original Gentzen's sequent calculus and algebra of labels in order to obtain a reasonable prof–theoretic and probability–theoretic system. We propose one version of such a system.

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Non-Archimedean probability measures

Nebojša Ikodinović

The presentation deals with probability measures with values in a non-Archimedean field. Some advantages of such measures compared to the classical (as well as Popper's, lexicographic, etc.) probability measures will be presented. In addition, several applications (in Economics, Game theory, AI etc.) will be discussed.

Decidable theories

Saša Rašuo

Though they are not designed as such, over the course of the twentieth century first order theories have become mathematical models and formal framework for description and study of numerous natural phenomena and manmade (synthetic) objects. The emergence of easily available computational machines (most notably personal computers) have established automatization and digitization as very important and thriving research areas. Consequently, decidability of first order theories has rather rapidly evolved from mathematical abstraction to contemporary engineering and science.

In this talk I will give an overview of some important decidable first order theories, and also mention some important undecidable first order theories.

Tipovi i mere

Predrag Tanović

Svaki tip teorije prvog reda mozemo posmatrati i kao konacno aditivnu meru koja formuli koja pripada tipu daje vrednost 1, a njenoj negaciji vrednost 0. Kakve veze ovo ima sa teorijom dimenzije u matematici?

Dynamic probability logics

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In this talk I will discuss some axiomatization issues in dynamic logics and its extensions such as dynamic probability logic. To briefly describe the problem, the standard modal semantics imposes serious, probably unbridgeable difficulty in standard completion techniques for infinitary modal logics - up to my knowledge it is an open problem whether the "monster model" (the set of states consists of all complete theories; for dynamic operator $[\alpha]$ the corresponding accessibility relation $\xrightarrow{\alpha}$ is defined by $u \xrightarrow{\alpha} v$ iff , for all ϕ, $u \vdash [\alpha]\phi$ implies $v \vdash \phi$) is a model at all.

Verovatnosna logika kao i realno-vrednosna logika se nalaze u istom algebarskom okviru kao i klasična logika

Dragan Radojević

Konvencionalne više-vrednosne logike kao i konvencionalne fazi logike počivaju na principu istinitosne funkcionalnosti. Pokazuje se da je istinitosna funkcionalnost, kao jednoznačna slika strukturne funkcionalnosti na vrednosnom nivou, valjana samo u slučaju klasične dvo-vrednosne realizacije. Sve generalizacije u smislu više-vrednosnosti koje počivaju na principu istinitosne funkcionalnosti nisu Bulovski konzistentne.

Klasična logika počiva na Bulovoj algebri. Logika aditivne verovatnoće se svodi na teoriju klasičnih skupova čija je algebra Bulovska. Neaditivna verovatnoća, Šokeov (Choquet) integral npr., je zasnovana na principu strukturne funkcionalnosti, koji je imanentan Bulovoj algebri. Princip strukturne funkcionalnosti je osnova realno-vrednosnoj Bulovski konzistentnoj logici. Bulovski konzistentne generalizacije klasične logike u više-vrednosne i realno-vrednosne i/ili fazi logiku počivaju takoe na strukturnoj funckonalnosti.