LOGIC

Abstract of talks given at the
joint conference of the AMS and the IMU
held at the Hebrew University of Jerusalem on 24-26 May, 1995

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Tomek Bartoszyński, Department of Mathematics, Boise State University, Boise, ID 83725, USA. *Cardinal invariants and sets of reals.*

We investigate some old and new properties of sets of reals and related cardinal invariants.

Aleksander Błaszczyk, Instytut Matematyki, Uniwersytet Śląski, Katowice, Poland, and **Andrzej Szymański,** Department of Mathematics, Slippery Rock University of Pennsylvania, Slippery Rock, PA 16057, U.S.A. *Regular subalgebras of complete Boolean algebras.*

Under the assumption that there exists a P-point there is contructed an atomless, complete, sigma-centered Boolean algebra that does not have any regular free subalgebras.

Jacek Cichoń, Institute of Mathematics, University of Wrocław, pl. Grunwaldzki 2/4, 50-384 Wrocław, Poland, and **A. Kharazishvili,** Institute of Applied Mathematics, University of Tbilisi, Unicersity Str. 2, Tbilisi, Republic of Georgia. *On ideals with Borel Base.*

The family of all Borel subsets of a topological space X is denoted by $B(X)$. If S is any field of subsets of X and I is any ideal of subsets of X then $S(I)$ denotes the least field generated by S and I, i.e.

$$S(I) = \{(A \setminus B) \cup (B \setminus A) : A \in S \wedge B \in I\}.$$

Let X be a Polish space and let I be an ideal of subsets of the space X. We say that I has a Borel base if for every $A \in I$ there exists $B \in I \cap B(X)$ such that $A \subseteq B$.

Let S be any family of sets. We say that S is point-finite if

$$\{A \in S : x \in A\}$$

is finite for each point x.

THEOREM 1: *Suppose that X is a Polish topological space, I is a σ-ideal of subsets of X with a Borel base, E is any metric space and that $f : X \to E$ is $B(X)(I)$-measurable. Then there exists $A \in I$ such that $f(X \setminus A)$ is a separable subspace of E.*

If E is a metric space then by $Comp(E)$ we denote the family of all compact non-empty subsets equipped with the Vietoris topology. If S is a field of subsets of X, E is an arbitrary metric space and

$$\Phi : X \to Comp(E),$$

then Φ is upper S-measurable, if for every closed set $Z \subseteq E$ we have

$$\{x \in X : \Phi(x) \cap Z \neq \emptyset\} \in S.$$

THEOREM 2: *Suppose that X is a Polish topological space, I is a σ-ideal of subsets of X with a Borel base. Suppose that E is a metric space and let $\Phi : X \to Comp(E)$ be upper $B(X)(I)$-measurable. Then there exists a $B(X)(I)$-measurable selector of Φ.*

COROLLARY: *Suppose that X is a Polish topological space, I is a σ-ideal of subsets of X with a Borel base. Suppose that E is a metric space and let $\Phi : X \to Comp(E)$ be upper $B(X)(I)$-measurable. Then there exists $Z \subseteq X$ and a separable closed subspace $F \subseteq E$ such that $X \setminus Z \in I$ and*

$$(\forall x \in Z)(\Phi(x) \subseteq F).$$

James Cummings, Hebrew University. *Reflection.*

Some consistency results about stationary reflection.

Patrick Dehornoy, Mathematiques, Univ., 14032 CAEN, France. *An application of set theory to the topology of braids.*

We shall give a survey of recent work that connects set theory with algebra and low dimensional topology. Using the critical ordinals R. Laver has derived from the existence of an elementary embedding of a rank into itself a purely algebraic order statement about the selfdistributivity identity. We shall sketch how this statement can be proved directly using some group that describes the geometry of this identity, and how the latter proof gives as a corollary the existence of an order on the braids, and, ultimately, a new very efficient method for solving the word problem of Artin's braid groups. The latter result is therefore a direct application of set theory and large cardinal axioms, yet its style is made different from the classical examples by the fact that the logical assumption is NOT necessary.

Mirna Džamonja, Institute of Mathematics., Hebrew University of Jerusalem, Israel. *Some results on sucessors of singulars.*

We present results on various topics from cardinal arithmetic, like reflection principles, club guessing, ideal $I[\lambda]$ etc. The results come from two joint papers with S. Shelah. We also mention some consistency results from a joint paper with J. Cummings and S. Shelah.

Sy D. Friedman, M.I.T. and Universite' de Paris 7. *Fine structure and class forcing.*

In this talk I will discuss aspects of my forthcoming book on the fine structure theory for L and the theory of class forcing. A modified Levy hierarchy can be used to give a smoother treatment of fine structure theory and we use it to establish square, morass with square, fine scales and gap-2 morasses in L. Class forcings do not in general preserve ZF and we isolate the condition of Tameness required for this to be the case. The four basic examples of tame class forcings are: Easton, Long Easton, Reverse Easton and Amenable forcing.

We then explore the question of generic class existence. L is rigid in class forcing extensions of L and we introduce $L[0^\#]$ as the least inner model in which L is not rigid. A class forcing is Relevant if it has a generic definable over $L[0^\#]$. Of the basic examples, Easton and Long Easton are relevant only if one avoids inaccessibles whereas Reverse Easton is relevant without this restriction.

Next comes the method of Jensen Coding which utilizes the fine structure theory introduced earlier. There are 2 proofs given, one assuming $-0^{\#}$, and the other more intricate but without the $-0^{\#}$ hypothesis. Jensen Coding is of the Long Easton variety and is thereby shown to be relevant.

We use class forcing to answer 3 questions of Solovay: If a real does not construct $0^{\#}$ then must it be generic over L? Is there a Π^1_2-Singleton strictly between 0 and $0^{\#}$ in L-degree? Is there a real R such that the R-admissibles are exactly the recursively inaccessible ordinals? The first question is answered immediately by Jensen Coding if generic is taken to mean set-generic; otherwise hyperclass forcing is used. The second question requires a mixture of Reverse Easton and Jensen Coding techniques. The third question uses a refinement of Jensen Coding called Strong Coding.

Finally, there are other applications of class forcing to descriptive set theory and to other notions of set-theoretic definability.

Moti Gitik, Department of Mathematics, Tel Aviv University, Tel Aviv, Israel. *Some results on ω_1 saturated ideals and realvalued measurable cardinals.*

We are planning to present some new results related to the density of the forcing with ideals. In particular, if $2^{\aleph_0}$ is realvalued measurable then the density of the measure (or just the number of random reals needed) is $2^{2^{\aleph_0}}$.

Zarko Mijajlović, Department of Mathematics, Belgrade University, Belgrade, Yugoslavia. *Continuous quotients in nonstandard analysis.*

This paper deals with continuous quotients from the nonstandard point of view. Using infinitesimal aproximations of irrationals by hyperrationals, the classical theorem on the representation of irrationals by continuous quotients is proved. Also, the nonstandard construction of a homeomorphism between the space of irrationals and the Baire space $\mathcal{B}$ is given. It is studied in some details the nonstandard space $^*\mathcal{X}$ by use of hiperfinite expansions of nonstandard rationals into continuous quotients.

Mariusz Rabus, Department of Mathematics, Hebrew University, 91904 Jerusalem, Israel. *Forcing Boolean Algebras of cardinality $\aleph_2$.*

We present a technique for forcing Boolean algebras of cardinality $\aleph_2$ by a c.c.c. forcing. As an application we construct a special kind of minimally generated Boolean algebra whose Stone space has some interesting topological properties.

O. Finkeland and J.P. Ressayre, Equipe de Logique mathematique, URA 753, CNRS and Universite Paris 7. *Stretchings.*

A structure is **locally finite** if every finitely generated substructure is finite; **local sentences** are universal sentences all the models of which are locally finite. The **stretching theorem** for local sentences asserts a remarkable reflection phenomenon between the finite and the infinite models of these sentences. It was a result proved using strong axioms. We show that this is not superfluous: the theorem implies the existence of inaccessible cardinals. Moreover its consistency strength is precisely that of Mahlo cardinals of finite orders.

Zoran Spasojevic, Institute of Mathematics, Hebrew University of Jerusalem, Givat Ram, 91904 Jerusalem, Israel. *(ω_1, ω_1)-gaps in $(\mathcal{P}(\omega), \subset^*)$ and $(\omega^\omega, \leq^*)$.*

I will prove that the followinf statements are equivalent

1. $\mathfrak{t} > \omega_1$
2. Every increasing ω_1-sequence in $(\mathcal{P}(\omega), \subset^*)$ is a lower half of some (ω_1, ω_1)-gap
3. Every increasing ω_1-sequence in $(\omega^\omega, \leq^*)$ is a lower half of some (ω_1, ω_1)-gap

   I will also show that 3. implies 4. but 4. does not imply 3. where
4. $\mathfrak{b} > \omega_1$

Andrzej Szymański, Department of Mathematics, Slippery Rock University of Pennsylvania, Slippery Rock, PA 16057, U.S.A. *The metrizability number of compact spaces and related invariants.*

We present results of the research done jointly with M.Ismail over the period of the last few years. *The metrizability number of a space X, m(X),* is the smallest cardinal κ such that X can be represented as a union of κ metrizable subspaces. Sample results:

(1) If X is compact Hausdorff and $m(X) \leq \kappa$, then $t(X) \leq \kappa$ and $|X|^{c(X)\cdot\kappa}$.

(2) Let D^κ be the Cantor set of weight κ. Then

   (i) $cf(\kappa) > \omega \to m(D^\kappa) = 2^\kappa$;

   (ii) $cf(\kappa) = \omega + SCH \to m(D^\kappa) = 2^\kappa$;

(3) If X is a compact LOTS and $m(X) \leq \omega$, then X is metrizable.

Linus Kramer, Mathematisches Institut, Am Hubland, 97074 Würzburg, Germany, and **Katrin Tent,** Institute of Mathematics, Hebrew University, Jerusalem, Israel. *Algebraic Polygons.*

We prove the following: Over each algebraically closed field K of characteristic 0 there exist precisely three algebraic polygons (up to duality), namely the projective plane, the symplectic quadrangle, and the split Cayley hexagon over K. As a corollary we prove that every algebraic Tits system over K is Moufang and obtain the following classification:

THEOREM: *Let (G, B, N, S) be an irreducible effective spherical Tits system of rank ≥ 2. If G is a connected algebraic group over an algebraically closed field of characteristic 0, and if B is closed in G, then G is simple and B is a standard Borel subgroup of G.*