The last Fermat's theorem

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Abstract. In this paper we in a simple way prove the last Fermat's theorem.

The last Fermat's theorem says that there are no $a, b, c \in N$ such that the equality

$$(Fermat) \qquad a^n + b^n = c^n \ , \quad \text{where } \ n > 2$$

is satisfied.

We can write $(Fermat)$ ih the following way

$$(*_1) \qquad a^n + (a+\alpha)^n = (a+\alpha+\beta)^n \ , \quad \text{where } \ n > 2$$

where $\alpha, \ \beta > 1$ are some integers. To prove that $(*_1)$ is impossible we shall use the following polynomial Φ_n:

$$\Phi_n(x) \ = \ (x+\alpha+\beta)^n \ - \ (x+\alpha)^n \ - x^n$$

where n denotes its degreee. For the first derivative we have

$$\Phi_n'(x) \ = \ n(x+\alpha+\beta)^{(n-1)} \ - \ n(x+\alpha)^{n-1} \ - nx^{n-1}$$

The polynomials $\Phi_n(x), \ \Phi_n'(x)$ are relatively prime. In the contrary case for some $c \in N$ we must have the equality $\Phi_n'(c) = 0$. Continuing such reasoning we can conclude that the polynomials

$$(3 \ - \ 2) \qquad (x+\alpha+\beta)^3 \ - \ (x+\alpha)^3 \ - x^3, \quad (x+\alpha+\beta)^2 \ - \ (x+\alpha)^2 \ - x^2$$

have the factor $x - c$. In an elementar way we can see that it is impossible. So, we have proved that polynomials $\Phi_n(x), \ \Phi_n'(x)$ are relatively prime.

Consequently for some polynomials $P(x)$, $Q(x)$, whose degrees are less than n, we have the following equality

$$(*_2) \qquad P(x)\Phi_n(x) + Q(x)\ \Phi_n'(x) = 1$$

Suppose that $(*_1)$ has at least one solution, and that x obtains some value c. We may suppose that the degrees of polynomials $P(x)\Phi(x)$, $Q(x)\ \Phi'(x)$ are less than $n-1$. Then from $(*_2)$ we obtain the following equality

$$(*_3) \qquad Q(c)\ \Phi_n'(c) = 1$$

Denote by $\Phi_{n-1}(x)$ the polynomial $Q(x)\ \Phi_n'(x) - 1$. So we have the following step:

(*Step from n to* $n-1$) If c satifies the equation $\Phi_n(c) = 0$, then c satisfies the equation $\Phi_{n-1}(c) = 0$.

In this step we passed from $\Phi_n(c) = 0$ to $\Phi_{n-1}(c) = 0$.

Of course, in a similar way we can introduce polynomials $\Phi_{n-1}(x)$, $\Phi_{n-2}(x)$, $\cdots$, $\Phi_3(x)$ such that the following equalities

$$\Phi_{n-1}(c) = 0, \quad \Phi_{n-1}(c) = 0, \quad \cdots, \quad \Phi_3(c) = 0$$

are satisfied. But the equality $\Phi_3(c) = 0$ is impossible. That means that we have obtained a contradiction. Thus, the equation $(*_1)$, i. e. (*Fermat*) is impossible. The last Fermat's theorem is proved.