Education and Culture
TEMPUS

Tempus JEP 41110-2006
Teacher Education - Innovation of Studies in Mathematics and IT
TEMIT

PROCEEDINGS

Part I

Завод за унапређивање образовања и васпитања
Institute for advancement of education

Belgrade
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Contents

Introduction

1. Educational system in Serbia

During its historical development during more than 200 years of progress of the modern Serbian state, teacher education in Serbia has undergone various changes. The present situation is the following. Elementary school's structure is 1+4+4, which means that children do have one year preparatory class (from age 6), four years lower elementary school (grades 1-4, age 7-11) and four years in the upper elementary or middle school (grades 5-8, age 11-15). Elementary education is compulsory by law but, since the introductory class has started only recently, there is a lack of facilities in this area and it covers approximately 40% of population. After finishing their elementary education, children may enroll (it is not compulsory) for the secondary or high schools, which may last either four years (grades 1-4, age 15-19) in the general type schools - gymnasia or in the vocational schools which do allow continuation to higher education, or three years (grades 1-3, age 15-18) in vocational schools which do provide possibility to work but not to continue to higher education level.

Teachers who teach in the first four grades of the elementary school (class teachers, colloquial *учитељ*) teach almost all school subjects. They obtain their degree at the university faculties for teacher training (*Учитељски факултет* or *Педагошки факултет*) in duration of 8 semesters. Teachers who teach in the second four grades of the elementary school (middle school subject teachers, colloquial *наставник*), as well as teachers in the secondary schools (high school subject teachers, colloquial *професор*) obtain their degree at the corresponding subject faculties of the university (philology, mathematics, chemistry, physics, biology, geography, history, etc.) also in duration of 8 semesters.

After obtaining their degrees and getting their first jobs at schools, and after one-year practical teaching supervised by experienced colleagues, they have to pass the state examination, covering three main topics: subject area, pedagogical and psychological skills, and legal framework. The system has been efficient for a long time. However, many problems did occur. Various reasons (low share of education in the GNP – now around 3%, developments in Serbia in the last 15 years etc.) have led to lowering of the overall education level. This is documented by relatively modest results of Serbia and Montenegro in TIMMS (http://timss.bc.edu) and PISA testing. Some reasons include disharmony between curricula and syllabi of mathematics and sciences. Others are due to recent legal changes: the new Law on Elementary and Secondary Education was adopted in May 2004, and the new Law on Higher Education was adopted in September 2005. These two laws impose a necessity for revision of the teacher training process. There is also a lack of student school practice in existing curricula, etc.

Yet more important are changes of the education requirements for future teachers, which are under discussion in various expert groups. The prevailing opinion is that the teacher education has to be on the master level (according to Bologna scheme) and that it has to include a certain fixed proportion of pedagogical and didactical subjects or credits (measured in the ECTS system adopted in Serbia).

2. The project

Having all this in mind, a group of university professors of mathematics and computer science at the University of Belgrade, Faculty of Mathematics, decided to apply for a Tempus grant in order to deal with abovementioned problems using relevant experience from various European countries. The effort ended in a successful project application Tempus JEP 41110-2006 called "Teacher Education - Innovation of Studies in Mathematics and IT" or "TEMIT" for short. In addition to Serbian institutions and experts, it includes higher education institutions from Bulgaria, Slovenia and Finland as partners and an expert from Germany. Its primary goal is to develop new curricula for teacher training in the areas of mathematics and information technology, in accordance with all requirements: harmonization between various subjects and adaptation to the new legal framework. Its secondary goal is the introduction of IT developments into curricula and syllabi of the education of mathematics and IT teachers. It also includes development of methodical and didactical courses for teacher training in all mentioned areas. Some broader perspective includes also production of new tools, including textbooks, program tools etc, and development of programs and courses for in-service teachers (life long learning concept).

The "TEMIT" project started on October 1st, 2007 and will last formally for 2 years, until September 30th, 2009. However, due to high motivation and energy of all participants, we expect it to last far more beyound that date, until the situation in the teacher education for mathematics and IT in Serbia (and, eventually, in other countries as well) would be really improved.

During this period, we had many study tours and meetings, to mention some of them:

Belgrade (Serbia), January 9th, 2008,
Borovetz (Bulgaria), April 11th-14th, 2008,
Jena (Germany), June 6nd-8th, 2008,
Ljubljana (Slovenia), July 2nd-5th, 2008,
Sv. Konstantin i Elena (Bulgaria), September 20th-23rd, 2008
Joensuu (Finland), October 24th-26th, 2008
Vrnjačka Banja (Serbia), November 28th-December 1st, 2008,
Belgrade (Serbia), March 27th, 2009.

You can find the official web site of the project on www.temit.matf.bg.ac.rs .

3. The Proceedings

The first volume of Proceedings of our "TEMIT" project contains contributions of various members of the project at the meetings mentioned above. All members of the project are experienced university professors, having done a lot of teaching and research in the area of preparation of future teachers in mathematics and information technology. Their conceptual and philosophical standpoints vary, and this increases the impact of the overall work. We are very lucky to have such prominent contributors.

The contributions cover a broad variety of subjects, including curricular development, philosophy of teaching, history of teaching, and more. We all share a feeling that the project is extremely important, we give the best of ourselves, and we shall succeed in our main goal: to improve the teaching of mathematics and IT in Serbia. We are grateful to the TEMPUS programme of the European Union Directorate for education and culture for financing the work which, to quote Coxeters' words, we would do anyway.

This is the first volume of the Proceedings. Enjoy reading this one, the next will be coming soon.

Belgrade, May 2009

Project coordinator

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Study of Mathematics Teacher Preparation

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Abstract. In recent years, many studies have highlighted an alarming decline in young people's interest for key science studies and mathematics. Despite the numerous projects and actions that are being implemented to reverse this trend, the signs of improvement are still modest.

The articulation between national activities and those funded at the European level must be improved and the opportunities for enhanced support through the instruments of the programmes in the area of education and culture.

Teachers are key players in the renewal of mathematics education. Among other methods, being part of a network allows them to improve the quality of their teaching and supports their motivation. (Report Rocard (2006))

Although setting curricula remains the prerogative of the relevant bodies and ministries much could be done that would have a substantive impact on the way that mathematics is taught:

- actions to promote the adoption of new teaching techniques;
- actions aimed at helping teachers present the subject in an exciting and relevant manner;
- actions that stimulate inquiry-based learning among young people.

Introduction

Interest in teachers' knowledge of the content of school subjects has a long history. Government officials and policy makers in many countries share a common concern that too many teachers are ill-equipped to teach mathematics well. Ever since the foundation works of Shulman (1986, 1987) researchers and those who prepare mathematics teachers have wondered about the nature of subject matter knowledge needed for teaching, and to what extent ideas such as pedagogical content knowledge interact with subject matter knowledge in the work of teaching. Many scholars around the world, e.g., Adler & Davis (in press), Ball & Bass (2000), Blum & Krause (2006), Even (1993), Fan & Cheong (2002), Hill & Ball (2004), Hill, Rowan & Ball (2005), Kunter et al. (in press), and Ma (1999), have been studying various aspects of mathematical knowledge for teaching. As reviews by Ball, Lubienski, & Mewborn (2001) and Hill, Sleep, Lewis & Ball (2007) indicate, these studies address matters of definition (e.g., What comprises mathematical knowledge for teaching?), measurement (e.g., How can we quantify mathematical knowledge for teaching?), and effects (e.g., How is mathematical knowledge for teaching related to pupil's achievement?). A volume on the *Professional education and development of teachers of mathematics,* edited by Ball & Even, based on the work of Study Group 15 of the International Commission on Mathematics Instruction is expected soon.

Subject-matter knowledge and pedagogical content knowledge

Recently, researchers have begun to study mathematical knowledge for teaching cross-nationally. The Mathematics Teaching for the 21st Century (MT21) project (Schmidt et al., 2008) is a cross-national study that investigated the preparation of middle school mathematics teachers in 34 institutions in six countries. The frameworks and item development work from MT21 have informed the work described in this paper. MT21 also provides preliminary evidence about the extent to which teacher preparation practices and achievement of future teachers vary in Bulgaria, Chinese Taipei, Germany, Korea, Mexico, and the USA.

For the past several years the Teacher Education and Development Study (TEDS-M 2008), a large-scale cross-national research project to study the preparation of teachers of mathematics at the primary and secondary level has been building upon this earlier work. Among other things, TEDS-M is examining the nature and extent of mathematical knowledge for teaching among future teachers enrolled in their last year of teacher preparation programs, and factors associated with that knowledge. TEDS-M 2008 is being conducted under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) with leadership provided by researchers at Michigan State University (MSU) and the Australian Council for Educational Research (ACER).

TEDS-M is the first cross-national research on mathematics teacher education to develop concepts, measurement strategies, indicators and instrumentation. It provides rigorous scientific evidence on the effectiveness of mathematics teacher preparation cross-nationally and builds capacity in participating countries to develop a program of rigorous research on mathematics teacher education that is: policy relevant, valid, replicable, and potentially self-sustaining (e.g., self-study in teacher education). TEDS-M collects data on the policies, curriculum, programs, processes and outcomes of teacher education from either censuses or national probability samples of institutions, teaching staff and students in these institutions. Results will enable policy makers, researchers and educators in participating countries to compare the teacher education in their country with the policies and programs and outcomes of other countries in relation to the preparation of future teachers of mathematics at the primary and lower secondary levels. In addition, the design of the study and planned analysis allow for the identification of any key factors (such as opportunity to learn, nature of route, length of program, etc.) that lead to desirable outcomes.

TEDS-M addresses several research questions:

- What are the characteristics of teacher education programs that prepare future teachers of primary and secondary mathematics effectively?
- How can the outcomes of teacher education programs for teachers of mathematics be measured in ways that are reliable and valid?
- What kinds of policies are effective in recruiting qualified teachers of mathematics from diverse backgrounds?

In the TEDS-M 2008 study, mathematical knowledge for teaching is assumed to have two dimensions: Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge. Each of these dimensions is assumed to be composed of various sub-domains.

Because one part of the TEDS-M study examines relations between teachers' content knowledge and the content of the curriculum they will be expected to teach, TEDS-M uses the mathematics content framework that has been used in the TIMSS studies of primary and lower secondary students. Thus, in the TEDS-M Field Trial, items were administered testing mathematical content knowledge in four content domains: Number, Algebra, Geometry, and Data. To ensure that various levels of cognitive demand were represented, a fairly uniform distribution of items testing Knowing, Applying, and Reasoning were selected across the content domains for the

TEDS-M instruments. (See Mullis et al. (2007) and Garden et al. (2006) for details on the TIMSS frameworks.) Much of the mathematics content assessed is at the level that the future teacher might teach, i.e. primary or lower secondary levels. However, some questions require knowledge of mathematics several years beyond that at which the future teachers will teach. For instance, the items assessing the mathematical knowledge of future teachers of lower secondary mathematics include some questions about calculus and linear algebra.

Due to psychometric constraints of reporting by sub-domain, (number of items required to report reliably per sub-domain) and practical constraints on available survey time, in the International Report for TEDS-M, sub-scores are expected to be reported for only three content domains: Number, Algebra, and Geometry. Some items about Data have been included on both primary and secondary instruments to increase the overall validity and reliability of the measures, but there are not enough questions to report a sub-score in this domain.

The TEDS-M framework for mathematics pedagogical content knowledge is hypothesized to have three sub-domains called Mathematics Curricular Knowledge, Knowledge of Planning Mathematics, and Knowledge of Enacting Mathematics. This framework pays attention to the temporal dimension of teaching, moving from knowing what mathematics to teach, to planning to teach it, to carrying out instruction.

Table 1: Sub-domains and aspects of the sub-domain of Mathematical Pedagogical Content Knowledge used in TEDS-M.

Mathematical curricular knowledge	Establishing appropriate learning goals Knowing different assessment formats Selecting possible pathways and seeing connections within the curriculum Identifying the key ideas in learning programs Knowledge of mathematics curriculum
Knowledge of planning for mathematics teaching and learning	Planning or selecting appropriate activities Choosing assessment formats Predicting typical students' responses, including misconceptions Planning appropriate methods for representing mathematical ideas Linking didactical methods and instructional designs Identifying different approaches for solving mathematical problems Planning mathematical lessons

Enacting mathematics for teaching and learning	Analyzing or evaluating students' mathematical solutions or arguments Analyzing the content of students' questions Diagnosing typical students' responses, including misconceptions Explaining or representing mathematical concepts or procedures Generating fruitful questions Responding to unexpected mathematical issues Providing appropriate feedback

Like others, TEDS-M researchers have faced a number of challenges related to measuring mathematical knowledge for teaching. One of them is motivating future teachers to participate in the research.

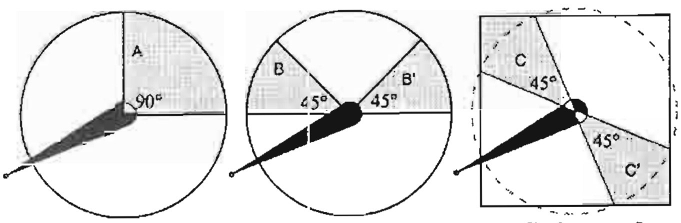
Because TEDS-M assesses adults who are university students, the TEDS-M research team was worried that future teachers might not be as keen as primary or secondary school students about completing an achievement test. There was a concern that a large proportion of "no-responses" would appear in the test items if the test looked like an "ordinary" achievement test. Something needed to be done to attract and motivate future teachers to respond to the items we wished to administer.

As has been done in other recent studies of teachers' knowledge, e.g. the Learning Mathematics for Teaching project (Hill & Ball, 2004) and the Mathematics Teaching for the 21st Century study (Schmidt et al, 2008), researchers developed items testing knowledge of mathematics content that are set into pedagogical contexts (see examples in section on Content Validity below). Similarly, pedagogical content knowledge, as described in Table 1, is assessed using references to mathematics problems appropriate to the level at which the future teacher hopes to teach. The assumption is that such items are not only appropriate for measuring both content knowledge and pedagogical content knowledge, but they are also interesting for the future teachers, because they are connected with their future profession.

This approach leads to another problem: the classification of the items, i.e. what exactly such an item measures. For the TEDS-M Field Trial these items contributed to both mathematics and mathematics pedagogy scale. For the Main Study reporting purposes each item will contribute to one of these scales only. This means that each item is classified either a mathematics or a mathematics pedagogy item.

Figures 1 and 2 represent items that were used in the TEDS-M Field Trial.

Your students are examining the three spinners shown below.
They are discussing the probability that the spinner stops over a shaded region.



Spinner 1 Spinner 2 Spinner 3

Please indicate whether the following statements of four students are Completely True, Partly True or Completely False. If one sentence is true, but the other is false, check Partly True.

		Check one box in each row.		
		Completely True	Partly True	Completely False
A.	Sherry says, "The probability is twice as large for spinners 2 and 3 compared to spinner 1 because they have two regions to stop on and spinner 1 has only one region."	☐	☐	☐
B.	George says, "Spinners 1 and 2 have the same probability since the shaded regions have the same area. Spinner 3, however, has a higher probability than spinner 2 because the shaded region is a larger area."	☐	☐	☐
C.	Paul says, "Spinners 1, 2 and 3 have the same probability because the angles of the shaded regions are the same size."	☐	☐	☐
D.	Rainey says, "The probabilities for spinners 1 and 2 are the same because those areas are the same proportion of the whole circle. For spinners 2 and 3, however, the probabilities are different because the shaded areas for spinner 3 are a bigger proportion of the whole square than they are of the circle."	☐	☐	☐

Figure 1. Complex multiple choice item testing mathematics content knowledge administered of future teachers of lower secondary schools in the TEDS-M Field Trial. Idea for item was provided by Prof. Walter Whitely, York University, Toronto, Ontario, Canada. Key: A, Completely false; B, Partly true; C, Completely true; D, Partly true.

A teacher gave the following problem to her class.

> The numbers in the sequence 7, 11, 15, 19, 23, ... increase by 4. The numbers in the sequence 1, 10, 19, 28, 37, ... increase by nine.
> The number 19 is in both sequences.
> If the two sequences are continued, what is the next number that is in BOTH the first and the second sequence?

(a) What is the correct answer to this problem? ___________

(b) A student gives the response 27 and 46 to the question above. What is the most likely reason for this response?

Figure 2. Two constructed response items administered to future primary teachers in the TEDS-M Field Trial. Part a) of the item, which tests mathematics content knowledge, is a released item from TIMSS Grade 8 2003. Part b) of the item, which tests mathematics pedagogical content knowledge, was developed by ACER.

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Looking for the Basis of a Mathematics Curriculum

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Abstract. This contribution discusses the conception of 'curriculum' from theoretical and practical point of view, making a clear distinction from 'syllabus'. The considerations begin by representing the curricular bindings. This allows a critical discussion also mistakes, which have been done in Finland during the last decades, even though the Finnish curriculum structure might be on the paper very modern one, allowing and even provoking all determinants in the educational system to the design process of an appropriate curriculum. The main focus of the contribution is, however, to emphasize how curriculum should offer research-based frameworks and concrete prototypes for teaching praxis so that teachers would be able to scaffold collaborative constructions process of viable knowledge through radical constructions among the learners. The considerations begin with a cavalcade of challenges for a modern curriculum representing their groundings and implementation with critical issues. One of the main foci is to represent the conceptions of instrumentation and instrumentalization, though which opportunities to develop and apply mathematics appear in a modern society. The first one means that technology shapes the actions of the users, whereas the latter refers to the fact that also the mathematical objects to be investigated are shaped by the users. Mathematicians, educators and administrators on all levels must be able to master and assess this genesis within instrumental orchestration.

Keywords: conceptual, curriculum, constructivism, mathematics education, minimalist instruction, modeling, instrumentation, instrumentalization, procedural, technology-based.

1 Introduction

Traditionally, a *curriculum* has been seen as a synonym to the concept 'Syllabus' or 'Lehrplan', meaning a set of courses, course work, and content offered at an institution, being partly or entirely determined by an external, authoritative body. This kind of characterization, however, does not fit the global trends in educational systems. Not long ago, in England and USA, for example, schools made their curriculum almost independently, whereas at the same time in Scandinavia the national bureaus of education set quite detailed description of course content. Later on, shifts in those countries have developed in opposite direction. In Finland, for example, today schools make their own curriculum based on the general guidelines of the National Board of Education. In England, schools make their curriculum based on the National Curriculum for England. In the USA, each state builds its curriculum with great participation of national academic subject groups selected by the United States Department of Education, e.g. National Council of Teachers of Mathematics (NCTM) for mathematical instruction. In Australia each state's Education Department establishes curricula. The same holds for Germany. So there are 16 different curricula of the 16 states (Bundesländer). Furthermore, there are different traditions in Western and Eastern Germany. While in Western Germany the state curriculum has the function of guidelines "Rahmenrichtlinien", along which the schools can develop their own curricula, in Eastern Germany the teachers are used to take curricula as laws which they should not think about but which are to be followed strictly. Since a couple of years there is something as a rough national curriculum, called "Bildungsstandards", which have been developed in case of mathematics along the "Standards" of the NCTM. The primary mission of UNESCO's International Bureau of Education is to study curricula and their implementation worldwide. Also European Union has a lot of strategy programmes concerning education on different levels. It seems solid to define Curriculum at different level of educational system as follows:

At *national level* curriculum means more or less meta-level goal setting, taking into account the nature of each subject as own discipline with its application in one hand, and on the other hand the subject as a component in general educational purpose. I will introduce later as an example the so-called Principles, developed within an expert group set by the National Board of Education at the beginning of 1990s to formulate the meta-goals for the Finnish mathematics education at comprehensive school. These principles are in accord with NCTM Standards, which also can be interpreted as meta-level goal setting for mathematics education. (http://www.nctm.org/standards/default.aspx?id=58)

At *local level*, the educational organizations can modify the national goal setting by taking into account special aspects of that particular district. However, usually local level curriculum means Curriculum at institutional level. This basically includes the following components:

(i) *Curriculum-spirit*: Description of goal setting, teaching, learning, and assessment in such a way that teachers, students and other interest groups involved in the educational process have realistic possibilities to understand and agree those terms.

(ii) *Syllabus-spirit* (Lehrplan): The content and range of courses from which students choose what subject matters to study for a specific learning program. Definition of the course objectives is usually expressed as learning outcomes and the assessment strategy, often grouped as units (or modules).

An implication for our TEMPUS work is that the working group aiming to formulate general pedagogical (didactical) aspects for Serbian Curriculum does not stress the Lehrplan aspects (ii) at all. Deep analysis of curriculum is required of us especially when trying to realize constructivist views on the nature of mathematical knowledge and the genesis of ideas. For example, history of mathematics in a broad sense with its abundance of problems, its richness of ideas and its power to develop intuition has to be redefined. Modern technology can be used for getting and realizing of these ideas. However, to build up a progressive curriculum is much more complicated than to make merely pedagogical variations within existing system. I will next point out the importance of philosophy of science beyond curriculum development.

2 Main determinants of curriculum

I will discuss the curricular issues by considering the following interest groups, spanning the first dimension of Figure 1: experts formulating the meta-goals, administrative people, designers of learning material, teaching personnel, and students. The second dimension is defined according to which components are emphasized in the learning and teaching of mathematics: experiences, building mathematical knowledge through meaningful situations, learning mathematics as cognitive facts, utilizing experiences and semantic meanings of the learner, stressing mathematical structures, and emphasizing general metacognitive thinking and learning skills. The third dimension is defined by the four aspects connected to problem solving, emphasized by Shoenfeld (1985): Resource, Strategies, Control, Beliefs. I will shortly discuss the role of each of those interest groups, which very often might wander through the "random house" of Figure 1 without ever meeting each other.

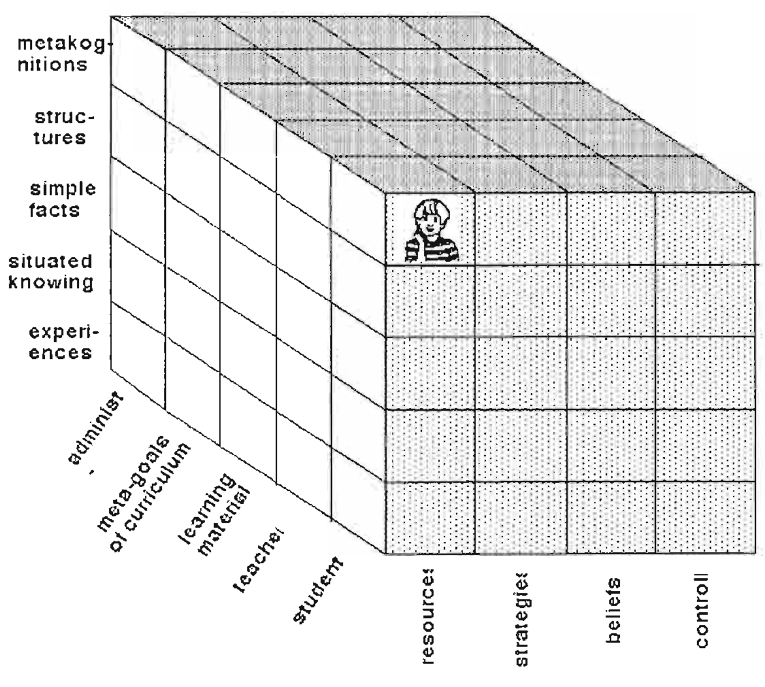

Fig. 1. Curricular determinants

2.1 Meta-goals

Mathematics curriculum should offer sufficient scaffolding for pedagogical measures to make the modeling processes of Figure 1 alive. I would like to recall the so-called *Principles*, which were formulated by an expert group set by the National Board of Education at the beginning of 1990s (cf. Kupari & Haapasalo 1993):

PRINCIPLE 1: Mathematics should be seen as a more extensive phenomenon than mere learning of certain computational skills. The acquisition of mathematics should be an integral part in pupil's mental growth, increasing the extent and consistency of his/her thinking and constructing his/her worldview. Problem solving must, in addition to mathematico-logical demands, have an essential role in the teaching.

PRINCIPLE 2: The goal of mathematics teaching should be above all to develop student's ability to classify, organize and model the situations he/she encounters in the surrounded world. This means that we must give up introducing only readily organized mathematical knowledge but instead laying stress on the heuristic processes with the aim of gradually organizing and specifying knowledge and skills into a structure, which is feasible from the point of the student.

PRINCIPLE 3: The student pupil must be seen as an active individual who constructs, collects, processes and stores information and for whom learning means correction and completion of earlier thinking and action models.

PRINCIPLE 4: The process objectives of learning will be emphasized in spite of the content-centered objectives. These objectives must be formulated, in the spirit, which supports pupil's mental growth, so that they also direct the teaching in practice and the preparation and use of teaching materials.

PRINCIPLE 5: Mathematics should also be contextually open for new knowledge, inventions, subject matter demanding special attention and current applications. Although the structure of mathematical contents should take the origins of mathematics into consideration, the traditional subject matter must be examined critically. Such knowledge that is not absolutely essential from the point of view of understanding the structure of mathematics should be left out.

PRINCIPLE 6: The integration of mathematics teaching into the teaching of other school subjects presupposes creating suitable application models and exploiting subject matter and methods from other fields of science. On the other hand knowledge is needed of the feasibility and limitations of models and computation methods provided by mathematics. No curricular obstacles should exits for integration.

PRINCIPLE 7: The assessment of learning and the principles applied strongly direct the emphasis of the various areas of teaching. When the emphasis of assessment is transferred to learning processes, new measuring and assessment methods and feedback to students must be developed.

PRINCIPLE 8: The principles of the curriculum as well as other guidelines should be formulated so that they make the flexible use of new equipments and new knowledge possible in teaching. Because of the fast development of technical equipments (e.g. calculators and computers) it is not possible to foresee all possible uses with their consequences, the guidelines should be sufficiently general.

PRINCIPLE 9: The principles of the curriculum should offer options both for municipalities and schools. The curriculum of the municipality and school should on the other hand offer also the pupil enough options.

These principles include several varieties of constructivism, offering also instructional assumptions for using technology. They are in accord with NCTM Standards, published about at that time. It is my position that these principles could still be taken as comprehensive basis when re-formulating them to fit the latest viable views stressed in our international scientific community. I will come back to this issue in my closing words.

2.2 Administration

Even though the Ministry of Education in Finland has highlighted and supported technological development in education in many ways, it has, likewise the whole administration throughout, neglected the utilizing of educational research especially as regards curriculum (see. Haapasalo & Silfverberg 2007, and Silfverberg & Haapasalo 2008). The Finnish National Board of Education terminated the work of the above-mentioned expert group just before its real work to write concrete prototypes should begin. The administrative people never took those Principles as a part of curriculum but formulated text like "The aim of mathematics teaching is that the students appreciates mathematics". When going closer to mathematical content, in the very same curriculum you find the following text for grade three: "To students will be thought, how to denote a part of a whole and how to read and write this denotation". Those examples show that irrelevant meta-goals never imply reasonable measures on the operational level and those problems became even more serious when related to technology.

As regards the assessment, the Finnish tradition does not support the implementation of ICT. This signal is reinforced by the fact that the general upper secondary school ends with the matriculation exam, which is the final assessment drawn up each year by the Matriculation Examination Board. In mathematics, the tests are arranged at two different levels, according to difficulty and the candidate is allowed to use just calculators, which have been approved by the Board. Programmable calculators or those with CAS are not allowed. The results of the study of Berry et *al.* (2004) suggest that the nature of the assessment system, which is tightly controlled through the national framework curriculum system, influences the attitude towards the rich usage of the technology.

2.3 Learning materials

It is not a surprise that design of learning materials are guided by the curricular goal setting. Textbooks are more a collection of stereotypic tasks than written within an appropriate pedagogical framework. They have been written by teachers to each other rather as task sources than to scaffold students' learning within modern views of teaching and learning. Few places where technology is involved, it is used for trivial instrumentation, actually for simple calculations. A closer analysis revealed that textbooks do not support to make links between mathematical representations. Identification tasks (see Figure 2) are totally missing and production tasks are very degenerated type consisting mainly manipulations within symbolic forms. This is against utilizing the ideology of educational instrumentalization and multiple forms of representation as ClassPad, for example, would allow.

2. 4 Teaching and teacher education

Concerning teachers' pedagogical philosophy, studies like Niederhauser & Stoddart's (2001) have indicated that there are coherent patterns between teacher's instructional approach and the quality of technological applications used in the classroom. Computer-oriented teachers prefer to use clearly skill-based applications. More generally, there seems to be a relationship between teachers' epistemological and pedagogical perspectives and their way to use educational software. These concerns are even more crucial, when students use technological applications in a more informal way, as on their free time. Only when educators accept the challenge of learning how to integrate computers into learning, a base of effective professional practice can be developed. This must be done through thoughtful software selection and creative utilization. Educators can advance colleagues' experiences by documenting and sharing their own successful technology practices like we do here in these proceedings. Teachers who have got modern teacher education can act as "agents" on their working places. Therefore teacher education probably has many implicit impacts, which should be studied carefully. A recent query study revealed that young students during their teacher education seem to prefer searching learning tasks and pedagogical support (e.g. interactive applets) from Internet and not from textbooks, whereas teachers in school answered that they prefer trusting on service-in-training and information given by their Teacher Union. According to may survey studies, most of the Finnish teachers still follow the order and content of the textbook quite strictly in their teaching, and on the other hand most mathematics textbooks are still basing strongly on the paper and pencil approach (Kupari 1993; Perkkilä 2002). On the other hand, active use of ICT in school does not seem to support learning mathematics in the same way as active use of ICT at home.

2.5 Student

As research-based examples in this article will show later, it is solid to assume that students might be the most flexible part in Figure 1. It is appropriate to underline that students very often learn in most effective way when they are not told to do it – especially when using modern ICT in their free time.

3 Main challenges for modern curriculum

Most of the expensive measures for development of mathematics curriculum in different countries have been more or less administrative, concentrating on irrelevant areas of education without any scientific background, even less based on the philosophy of science. For many universities - as for the majority of schools - mathematics consists of a number of discrete courses, which are frequently studied with little interdependence. An organized body of knowledge leaves students largely passive, practicing old, clearly formulated, unambiguous questions. For the students the large body of theory looks abstract and depends on an unfamiliar language. These features leave many students unmotivated and bored. It is not unusual for students to complete a three years undergraduate study of a subject called 'Mathematics' and yet to have little idea of what the subject really is about. A curriculum based on an *objectivist view of knowledge* seems to be without any success, maybe it is the main reason for the continuous crisis in (mathematics) education.

The science of mathematics education is, however, a complicated discipline. To develop research-based frameworks for teaching praxis we need to combine knowledge of mathematics and its history, philosophy, psychology, sociology, physiology and ICT, for example. Technology has caused a holistic change in our "mental art", i.e. in the way we think, plan, work and evaluate in a modern society. This implies that educators are forced to shift their views of mathematics and learning of it. Even though the development of a viable and sustainable praxis theories might be taught, on the other hand the situation is analogous with industrial production: viable and sustainable products are very often beautiful, simple and easy to use - otherwise nobody would buy them. This has been the starting point of author's MODEM –project[1], having been a basis for teacher education about two decades. The idea to use a quasi-systematic model for the planning of learning environments but on the other hand to allow free architecture of learning in the authentic learning situations sounds contradictory. However, this is the way our everyday life runs. We pick our food quite randomly from buffet, for example, even though the kitchen personnel have used all its expertise to plan and construct things for us very systematically. This article emphasizes challenges arising from this meta-text and discusses at the examples of pedagogical implementation with critical issues.

3.1 Challenge 1 - Solid framework theories for collaborative social constructions

Mathematics education research should produce sustainable, viable and communicable frameworks for teaching praxis so that teachers would be able to scaffold collaborative constructions process of viable knowledge through radical constructions among the students.

Groundings: The slogan of constructivism in our scientific community started about twenty years ago. Later on this discussion extended to include collaborative social group processes. Still, there are quite few empirically tested models how teachers can plan and realize learning environments within that paradigm. Concerning *group development*, the famous classification of Tuckman (1965) gives a nice framework: *forming, storming, norming,* and *performing*[2]. Collaboration means that student teams define their own goals for their work and maintain active processes towards this goal, profiting from a diversity of perspectives and opinions, and a free flow of information. With respect to our socio-constructivist framework, the *pragmatic theory of truth* emphasized by the famous philosopher Charles Peirce would give a solid basis, making the debate between radical and weak constructivism sound unnecessary and even naive. When an open problem is given, namely, the teams work in causal interaction with this problem under collaboration. After testing the viability of radical ideas among the teams and between the teams, only those ideas finally remain which are viable for the whole social group consisting of those teams (see Figure 2).

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[1] see Haapasalo 2003, 2007 or http://www.joensuu.fi/lenni/modemeng.html

[2] When typing-in 'Tuckman group process', Google offered more than 72000 hits with interesting links in July 2008.

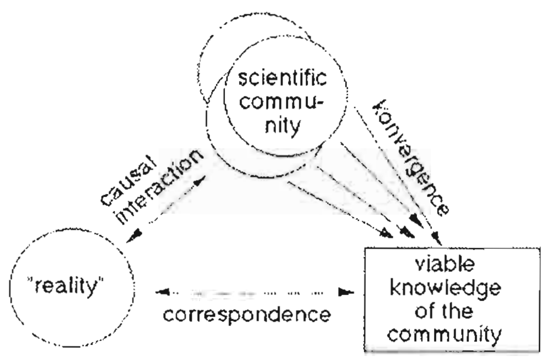

Fig. 2. Viable knowledge as a result of radical constructions (Eskelinen & Haapasalo 2007).

Example of implementation: Based on a recent dissertation, Eskelinen & Haapasalo (2006; 2007) describe features of group dynamics when applying the Learning by Design principle within collaborative socio-constructivist approach for two student groups in teacher education. The objective of the elementary student teachers' design process was a hypermedia, whereas the mathematics student teachers planned and realized microteaching lessons within their peer group. Apart from other studies of design processes the study stresses the impact of knowledge structure, pedagogical philosophy and support for reflective communication. The results revealed that working within socio-constructivist collaborative ICT-based design processes for the production of a hypermedia-based learning environment, even during a short period of time, changed student's conceptions of teaching and learning from an objectivist-behaviorist viewpoint to a constructivist view, and decreased students' interest in having support for computer routines. The group dynamics varied in the design process as anticipated in the light of earlier studies: in the planning phase the amount of stress and aggression increased, whereas in the phase of implementation the self confidence increased, which was especially the case within the educational approach. Design of technology-based learning environments within an adequate constructivist theory linked to the knowledge structure might be a proper framework to respond to the main challenge of teacher education: to get students understand which are the basic components of modern constructivist theories on teaching and maintaining the learning through cognitive conflicts. The developmental approach based on spontaneous procedural knowledge seems to be appropriate in relation to both cognitive and affective variables. In order to apply the educational approach so as to stress the importance of conceptual knowledge, an educator needs to be sensitive to the cognitive and emotional variables present in the learning process.

Critical issues of implementation: To analyze very complicated studying environments several aspects should be considered at the same time. The research tradition in mathematics education community has, unfortunately, concentrated on focusing on one or few components at a time. There are only seldom studies, which refer explicitly to the complexity of mathematics instruction. Our ongoing research is about to reveal that teacher education is full of challenges that we never expected. For example, students need many kind of tutoring and continuous support for reflective communication to make the philosophy of Figure 2 work.

3.2 Challenge 2 - Solid framework theories for linking conceptual and procedural knowledge

The following two dilemmas have been neglected in our scientific community, even though they might appear throughout our mental life: (1) Do we have to understand being able to do, or vice versa? and (2) Should mathematical objects be emphasized as objects or as processes when learning them?

Groundings: Literature analysis reveals the dominance of procedural knowledge[3] over conceptual one in the development of scientific and individual knowledge (Haapasalo & Kadijevich 2000, Zimmermann 2003). We know from the basics from cognitive psychology that our world is a world of meanings, not a world of stimuli. The investigation problems (labeled "reality" in Figure 1) should be psychologically meaningful for the students. This implies the need to apply the so-called developmental approach in the instructional design: students should have opportunities to go for their more or less spontaneous procedural knowledge. On the other hand, perhaps the most important educational goal in a modern society is – especially if we trust on mathematics' power to trigger general educational goals - to scaffold citizens' abilities to identify and construct links within complicated multi-causal and multi-disciplined knowledge networks. This means investing on conceptual knowledge in the sense of footnote 3, even in such a way, that students also learn appropriate procedural skills. This so-called educational approach causes the following conflict: Does a student have to understand being able to do, or vice versa (Haapasalo 2003). This dilemma is forced by the fact that for too many students, one of the basic difficulties for the learning of mathematics is that very often entities appear as objects as well as processes.

Example of implementation: The framework of author's MODEM project (see footnote 1) can be highlighted with Figure 3. To plan a constructivist approach to the mathematical concepts under consideration, the focus is on the left-hand side of Figure 2, whereas offering students opportunities to construct links between representation forms of the concept, the emphasis is to scaffold mathematical concept building within the stages of the right-hand box.

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[3] I adopt the knowledge type characterization of Haapasalo & Kadijevich (2000):
• *Procedural knowledge* denotes dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation forms. This usually requires not only the knowledge of the objects being utilized, but also the knowledge of format and syntax for the representational system(s) expressing them.
• *Conceptual knowledge* denotes knowledge of and a skilful "drive" along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representation forms.

Concerning the logical relation between these knowledge types, four views can be found in literature. Developmental and educational approaches are based on those.

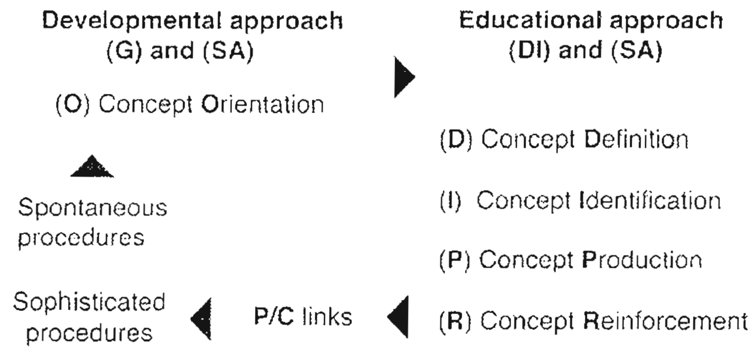

Fig. 3. MODEM framework as sophisticated interplay between developmental and educational approach.

The framework has been successfully tested for planning and assessment within many conceptual fields of school mathematics (see footnote 1). It is also a solid substantial framework in the design studies mentioned above. Examples later will show how this quasi-systematic model can be combined with minimalist instruction philosophy[4].

Critical issues of implementation: Many teachers and researchers seem to interpret Figure 3 as a strict systematic model so as to contradict the starting point of constructivism. However, as my examples later will so, this model is more quasi-systematic than systematic. Furthermore, quite poor metacognitive abilities among teachers and students can prevent utilizing the task types even though they would be tailored to learning with very simple actions (see Haapasalo 2003; 2007). Hence, the design of such tasks would be even more difficult to teachers.

3.3 Challenge 3 – Dilemma between systematic models and minimalist instruction

To emphasize the genesis of heuristic processes and students' ability to develop intuition and mathematical ideas within constructivist or minimalist approach a systematic planning of the learning environments (i.e instrumental orchestration, to be explained later) is needed. In learning situations, however, students must have freedom to choose the problems that they want to solve within continuous self-evaluation instead of relying on guidance by the teacher.

Groundings: Zimmermann's (2003) study of the history of mathematics reveals eight main activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years: *order, find, play, construct, apply, argue, evaluate, and calculate.* Especially the five first activities very often run optimally without any external instruction or demand. Students frequently neglect teacher's tutoring or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these

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[4] For interactive applets: http://www.joensuu.fi/mathematics/MathDistEdu/index.html

tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term minimalist instruction, introduced by Carroll (1990), is crucial not only for teachers but also for those who write manuals and help menus for the software. The features of *minimalism* include several varieties of constructivism, offering also instructional assumptions (see Haapasalo 2007):

- Learning is modelled and coached for students with unscripted teacher responses.
- Learning goals are determined from real tasks stressing doing and exploring.
- Errors cannot be avoided and should be used for instruction
- Learners construct multiple perspectives or solutions through discussion and collaboration.
- Learning focuses on reflexive awareness of the process of knowledge construction.
- Criterion for success is the transfer of learning and a change in students' action potential.
- Assessment is ongoing and based on learners' needs.

This challenge is even more sever, because there might be different types of learners: *Conceptually-orientated* learners who aim to learn things by advancing from conceptual knowledge towards procedural knowledge, *Procedurally-orientated* learners who act in the opposite way - they advance from procedural knowledge towards conceptual knowledge, or *Procedurally-bounded* learners who concentrate only on procedural knowledge. To solve the dilemma above, empirically tested more or less systematic pedagogical models, as MODEM, can be helpful. When planning a constructivist approach to the mathematical concepts under consideration, the focus is on developmental approach. On the other hand, when offering students opportunities to construct links between representation forms of a specific concept, the focus is on educational approach, enhancing links between mathematical representations.

Example of implementation: The following example of the ClassPad project (Eronen & Haapasalo 2006) shows that the challenge can be responded by organizing different task types into a "problem buffet". To go for linear function, one student team, for example, initially selected quite a complicated problem series on optimizing mobile phone costs. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set, which happened to consist of identification tasks – the first and lowest level of the concept building within the systematic MODEM framework, which was on the basis of the planning of the learning environments. This example shows that a sophisticated interplay between a systematic approach and minimalism can be achieved even by simple pedagogical solutions.

Critical issues of implementation: To be able to compose theoretically grounded approaches in appropriate way, teachers must be deeply involved in problem-solving culture, which is not usually the case. Even though the end product might be simple, cheap and easy to use (cf. metaphor in the introduction), teachers might have difficulties to combine those many aspects that are needed for this kind of task design.

3.4 Challenge 4 – Shaping the instrumental genesis through instrumental orchestration

The constructivist viewpoint states that both making of mathematics and teaching of mathematics must relate to the instrumental genesis in modern society.

Groundings: Opportunities to develop and apply mathematical ideas appear in modern society through *instrumentation* and *instrumentalization*. The first one means that technology shapes the actions of the users, whereas the latter refers to the fact that also the mathematical objects to be investigated are shaped by the users. Today it sounds crazy to make paper-and-pencil manipulation of $1/\sqrt{2}$ to $\sqrt{2}/2$, for example, because every-man's instrumentation allows to make sophisticated use of SQR and 1/X keys. Instead of old-fashioned "papermedia", we can enrich curriculum by revitalizing curves, being topic for famous mathematicians and physicists of 17th and 18th century. Today they can, namely, be visualized even with pocket computer almost by any layman. Educators on all levels must have know-how for scaffolding this genesis, this process being called instrumental orchestration (Trouche 2004). This genesis has already changed - and it will change even more radically - our views on making and teaching mathematics. The fact that most part of students' instrumentation and instrumentalization very often happens on their free time, implies that educators should shift the focus from well-prepared classroom lessons on minimalism. Instead of acting like a pace car in a race, institutions should be types of pit stops to scaffold students' "race" outside the classroom. I will support this view with concrete examples below. I would also emphasize that governments and schools probably never have enough money to equip schools with the newest technology. Most students in well-fare countries will always have a little bit more sophisticated technology at home – perhaps even in their pocket. Therefore, by looking the relationship between technology and mathematics education from five perspectives, I suggest that instead of speaking about 'implementing modern technology into classroom' it might be more appropriate to speak about 'adapting mathematics teaching to the needs of information technology in modern society' (Haapasalo (2007). This means emphasizing more the making of informal than formal mathematics within the framework of the above-mentioned eight main activities and motives, which have proved to be sustainable in the history of human thinking processes and making of mathematics (see Zimmermann 2003). The fact that ordinary people can realize outstanding examples of simple and powerful ideas from the history of mathematics implies that also organizing the content of the curriculum should be made in a meaningful way instead of treating the same idea in several disguised forms under the guise of "spiral curriculum".

Examples of implementation: Examples of revitalizing geometric ideas from the history of mathematics can be found and downloaded at http://www.math.jyu.fi/~kahanpaa/TUBerlin/home.html, the website referring to material production within our Joint European project (see http://www.joensuu.fi/lenni/modem.html). Those visualizations can be utilized in many ways almost on any level of mathematics teaching. The first level of modeling could be just to watch the beautiful simulation and try to explain in own mother tongue what happens on the screen. The highest level of modeling would be to make an own computer-based model, which makes the same simulation or perhaps improves it.

The following example illustrates a successful instrumentalization within the simultaneous activation of conceptual and procedural knowledge with the ClassPad calculator (see http//www.classpad.org). This tool allows, namely, manipulation between the algebraic and geometric windows with drag-and-drop maneuvre or Geometry Link operation. By applying minimalism, Eronen & Haapasalo (2006) gave students at 8th class opportunity to study voluntarily 9th class mathematics with this device during their summer holiday. This totally new tool was shortly represented to them just a few days before their summer holiday. The only duty was to write a portfolio if they worked with the tool. The following sample, taken from the portfolio of a quite average student, shows that she was able to utilize modern technology in sophisticated way on her free time. She moved from instrumentation to instrumentalization without any tutoring from teacher's side. Note that when varying the constant, the lines does not go up and down (as been taught in school) but moving horizontally. Thinking graphic solution of an equation, for example, this is even more relevant interpretation showing that even a quite poor student could through her instrumentation get us to shift conventional views among educators!

6th session on 15th of July 2005. Time 00.27

· I draw a line (cf. a in Figure 4). When drag-dropping, the equation of the line is y= 1.613x-0.5992 (b).
· By changing the equation to y=2x-0.5992 the angle between the line and y-axis is getting smaller (c).
· By changing the equation to y=1x-0.5992, the angle between the line and y-axis is getting bigger.
· I change the equation to y= 1.613x-0.4. I don't see any changes.
· I change the equation to y= 1.613x-4, the line moves to the same direction away from origin (d).
· When changing the equation to y= 1.613x+4, the line moves in the same way, but to another direction on x-axis with equal distance from the origin.
· I will continue in the morning. Time is now 01.42. I worked 1 h 15 min.

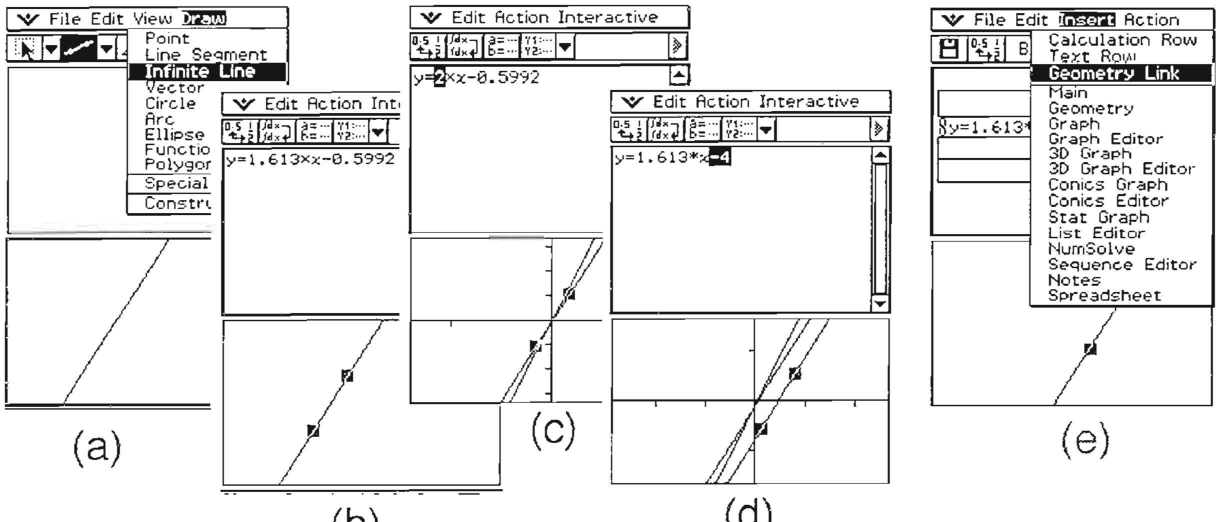

Fig. 4. Utilizing SA-method through drag-and-drop technology (a-d) or Geometric Link (e).

ClassPad project (see Haapasalo and Eronen 2006) also used Zimmermann's idea to model mathematical activities as an octagon and to quantify each activity as follows: the distance of the activity from the centre tells us how strong a student thinks this activity is represented by each of the following three profiles (see Figure 2 later):

- *Math-profile:* How strong each activity appears to the student when using the term 'mathematics';
- *Identity-profile:* How good the student thinks he or she is performing each of the activities;
- *Techno- profile:* How suitable a computer is in performing each of the activities.

The results suggest that doing mathematics with ClassPad, even during a short period of time outside the classroom, enlarged student's mathematical identity within these motives and activities. As we see from the example above, student's portfolios reveal sophisticated metacognitive skills[5] on students' way to instrumentalization. The learning of mathematics at 9th grade only with ClassPad without any textbooks within interaction between minimalism and systematization was successful concerning students' cognitive development. Students scored in all test items significantly better after the ClassPad working than in the pre-test. They also showed remarkable procedural skills not only connected to the linear function but to other function types. A postponed test, after 5 months, revealed that this scoring level remained consistent, and for many students it even improved. Students liked the feeling that they had reached action potential, which was described to be one of the main aspects in assessment within minimalism. They also liked the learning without any pre-set goals or tutoring from teacher's side.

[5] Haapasalo (2007) reveals degenerated metacognitions among teachers and students when working with a CBL – program in school.

Critical issues of implementation: Perhaps the most difficult obstacle in the instrumental orchestration is that educational policy makers in most countries are against allowing the use of modern technology in examinations. This implies that teachers are even less voluntary to learn to develop their own instrumental genesis, and even less ready to make the same concerning instrumental orchestration. As another problem I would like to mention the fact that technological tools are made for those who apply mathematics, and not for learning purpose. So, it is a very difficult task for us educators to shift this development in educational direction. Manuals, as for ClassPad for example, consist often several hundredths of pages containing huge amount of conceptual mathematical knowledge. This causes a contradiction between the versatility of the tool and minimalist instruction. Furthermore: The example above demonstrates, that there are students can gain profit in the way they worked and played independently. To what extent this effect is attainable for the majority of (nearly) all students?

Concerning the educational policy, the problem of "math dropouts" has increased now that "mathematics for all" has come into fashion as a slogan. 'Mathematics' is normally presented as a meaningless collection of knowledge - unrelated to the experience of the students and totally uninteresting. Sterile "logical connections" seldom lead to understanding or appreciating. This has given rise to a flourishing enterprise - empirical research - which studies and characterizes the symptoms without producing a cure. We need a new approach to the teaching of mathematics but there is little hope it will emanate from this psychological perspective. Epistemological perspectives and historical sources offer much more hope. Besides, they must not be forgotten when planning curriculum or constructivist learning environments for students´ productive activity.

3.5 Challenge 4 – Modeling through technology

It might be appropriate to discuss the role of technology in modeling processes. Some researchers like Treffers (1987) speak about 'horizontal modeling' when a real-world situation is interpreted through mathematical structures, whereas 'vertical modeling' refers to manipulating structures inside mathematics. The often-used term 'mathematization' would mean rather the first one. I would, however, like to avoid using those complicated terms, which might be irrelevant from pedagogical point of view. Very seldom, namely, a real world situation is directly interpretable through mathematical structures before making a simplification – sometimes even very progressive one. A virtual model on computer screen, for example, can very often offer for the learner, even for a child, a more appropriate *investigation space* (Entdeckungsraum) than a conventional "real-world situation". Hence, from constructivist viewpoint I prefer to make the following generalization, seeing modeling almost synonym to the term 'mental model':

• *Modeling* means mathematical interpretations made by the student when interpreting a situation, which is psychologically meaningful for him or her.

From constructivist viewpoint learning and applying mathematics are basically triggered by the same kinds of modeling processes, general aspects of problem solving in the sense of Polya playing a central role within those activities. The interpretations can appear in different forms of representation (*V*erbal, *S*ymbolic, *G*raphic), and on different cognitive levels (***I***dentification, ***P***roduction). Drag-and-Drop activity with *ClassPad*, for example, allows Production from Graphic to Symbolic form (***PGS***) showing how opportunities to develop and apply mathematical ideas appear in modern society through instrumentation and instrumentalization. The aim of Figure 7 below is to illustrate that technology cannot only be an interface on a one-way street from real world to mathematics but to enhance and empower the interpretations in different kinds of modeling processes. I would stress that even more important than to solve a given problem is to promote students to find and pose new own problems related to a situation. Instead of speaking about "real-world" problems, nowadays it might be more appropriate to speak about "situations, which are psychologically meaningful for the student". Very often an animation on computer screen, for example, is more "real" than what happens in physical world. Furthermore, instrumentation releases the student from cognitive overload whereas instrumentalization scaffolds him/her to make own mental models (which might not always be viable ones when considering the objectivity of mathematical models). At the very moment when the task for the student is to try to find a meaningful situation where the animation of Figure 5 could exist, the student begins to make his or her mental modeling of the simulation through instrumentation and instrumentalization with ClassPad.

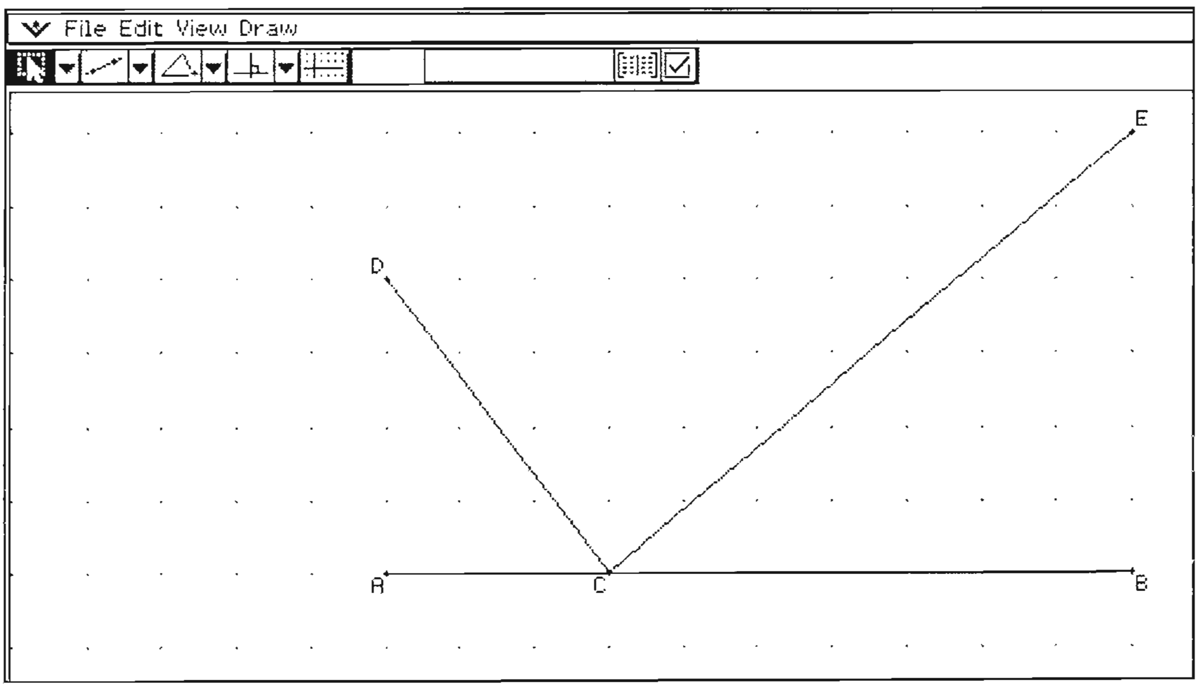

Fig. 5. ClassPad animation of a classical problem.

In Finland, for example, where many people spend their summer at lakes, students could construct the following authentic situation: "*I and my friend living on two different islands in our summer houses would like to have a nice weekend with our classmates. This means we have to make a lot of rowing to carry our friends and a lot of food and drinks. However, even an empty boat is extremely hard to row for us novices. So, a relevant question is where on the beach to ask our friend to have a meeting point.*" This kind of psychologically meaningful interpretation can very often trigger students to see analogous situations in physics (cf. Figure 5; e.g. which route would "light" choose, or in geometry utilizing symmetry, for example).

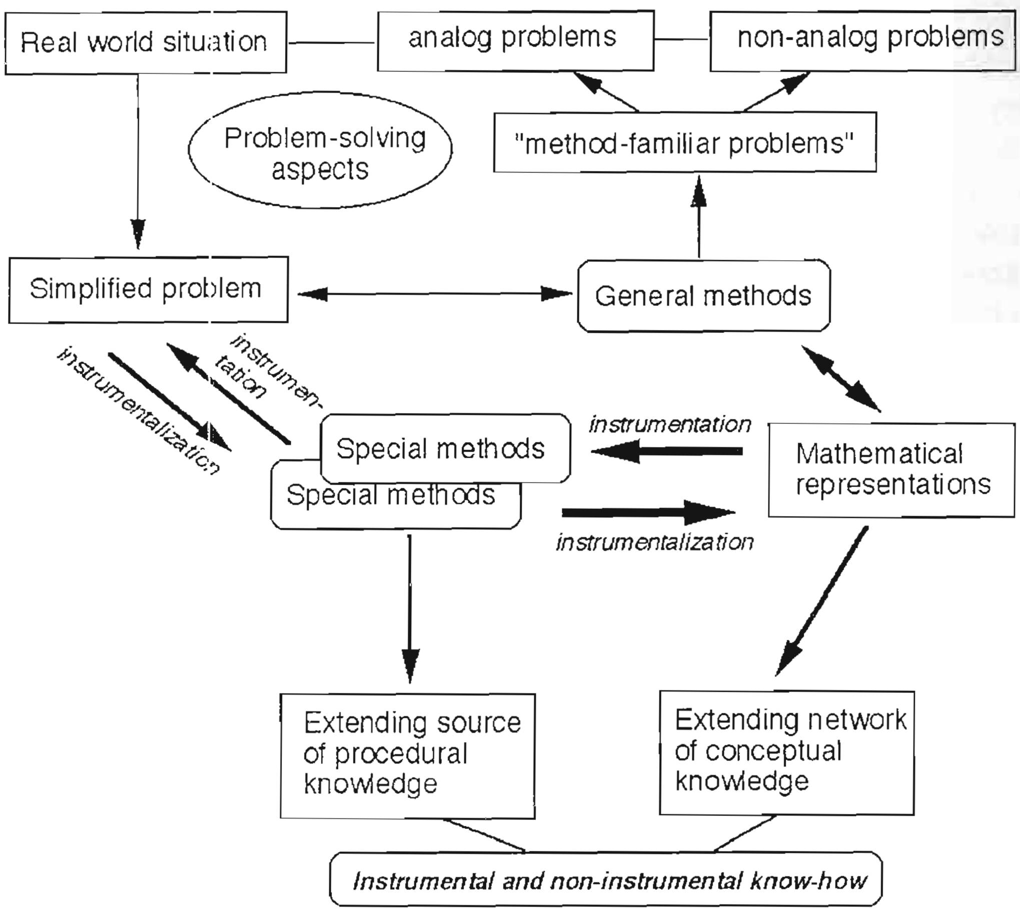

Fig. 6. Implementing technology in modeling processes.

4 Closing words

Professor Bert K. Waits, one of the pioneers of technology enriched mathematics education, has condensed the relation of technology and mathematics into three slogans (see Kokol-Volj 2000, 7). By adding the term [education] in parenthesis I would like to modify those slogans to refer not only to mathematics but to mathematics education:

"Some mathematics [education] becomes more important because technology requires it."

"Some mathematics [education] becomes less important because technology replaces it."

"Some mathematics [education] becomes possible because technology allows it."

These slogans imply strict demands for teaching and assessment. Recalling Figure 1, the matriculation exam, for example, should allow students to get maximum marks by showing different kinds of abilities. At the same time when a student is able to show his or her skills and mathematical understanding by using technology, another student perhaps would like to show his or her huge conceptual understanding, methodological richness, and ability to pose new relevant problems.

Even though my aim is not to try to formulate curricular text in this article, I would like to recall the above-mentioned Principles and give a sample, in which direction I would like to develop them now, almost 20 years later, if I would be a responsible person to write meta-text for a curriculum:

Mathematics means dynamic network of eighth main components of human thinking, being viable and sustainable for more than 5000 years: order, find, play, construct, apply, argue, evaluate, and calculate. For a citizen in modern society, those activities appear and come into action through instrumental genesis, which should be a vital part of students' mental growth, increasing the extent and consistency of his/her thinking and problem solving abilities and constructing his/her worldview. This curriculum (prototypes should be given in the document, of course) will offer examples, how mathematics education should scaffold this development inside and outside the classroom, giving options both for municipalities and schools to plan and realize instrumental orchestration including assessment.

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Partnership between faculty and schools: mathematics teachers as researchers of interdisciplinary connections.

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Abstract. The paper presents the idea of teachers as researchers of their own teaching practice. We invited teachers to join the MODEL IV project: the Partnership Between Faculties and Schools, Research of Educational Practice and Direct Application of Results in Education: the Teacher-Researcher and Interdisciplinary Connections made possible with the co-funding of the European Social Fund of the European Union and the Ministry of Education and Sport of the Republic of Slovenia. Since the main research problem of the project was interdisciplinary connections, the teachers involved in the project were asked to develop a teaching method based on joining different school subjects. The aim of developing such a method was to deepen the pupils' mathematical knowledge and improve their skills of using and applying mathematics. Most of the teachers used the methodology of action research based on three action steps. The paper presents some theoretical issues of interdisciplinary connections, some examples of action research methods which were developed and performed by the teachers, and the analysis of the research findings.

1. Introduction

The teaching of mathematics at any level is a very demanding task. First of all, mathematics is one of the most difficult school subjects, secondly, pupils do not view mathematics as useful for everyday life and, thirdly, many pupils have difficulties in understanding mathematical concepts. There has been a lot of research done on teaching and learning mathematics, but it seems that the results (in many cases very encouraging from the researchers' perspective) do not come to life in the mathematics classrooms. We believe that one of the most significant reasons for this development is the fact that teachers are unwilling to change their subjective theories on teaching and learning mathematics, and therefore usually do not accept the research findings as accurate or useful. This was the main reason which encouraged us to invite teachers to research by themselves. The content of their research was interdisciplinary connections as a teaching method based on joining different school subjects.

2. Interdisciplinary connections

One of the important principles of revitalizing the primary school curriculum is the horizontal integration and intertwinement of knowledge, which can best be achieved by means of interdisciplinary and inter-subject connections. Interdisciplinary connections represent a didactic approach, a teaching method, where the teacher tries to present or treat a certain content or problem as comprehensively as possible. Such work demands that the teacher clearly defines the objectives and the content of various subjects and combines them. It also requires careful planning and good content-related and organizational implementation based on cognitive constructivist teaching methods. Interdisciplinary connections should be adjusted to the pupils' grade level and their previous knowledge. Sometimes, the cooperation of teachers of various subjects is required as well. Interdisciplinary connections should be implemented when this course of action is logical, when there are reasons for such connections and when this enables us to improve the pupils' creativity and motivation.

There has not been much research done on the interdisciplinary connections in the Slovenian context. The term interdisciplinary connections itself is relatively new. Shoemaker (Lake, 2002) defines the interdisciplinary connections as education organized in such a way that it combines the common characteristics of various disciplines into a coherent whole. Martin-Kneip, Fiege and Soodak (1995) define the interdisciplinary connections as an example of holistic learning and teaching which reflects the real interactive world and its complexity, bridges the boundaries between the disciplines, and fosters the principle that all knowledge is linked.

When we talk about interdisciplinary connections, we do not refer only to conceptual knowledge, but rather emphasize generic skills, for the learning of which content is important but not fundamental. Generic skills could be defined as skills which are independent of content, transferable, and useful in various situations. Let us enumerate a few: critical thinking, problem solving, data processing, use of IKT, execution of project assignments, active learning, reading, writing, listening etc. In theory, interdisciplinary integration is related to the real life, which gives the pupil a reason and a strong motive for learning.

Besides the importance of discipline connections and generic skills development for interdisciplinary connections we have to mention the importance of the interdisciplinary connections learning process. In many practical cases, integration does not prove to be as logical as we would have expected, but the process of integration is much more significant than the content of interdisciplinary integration. With this we wish to emphasize that a child learns from the process of integration itself, recognizing that contents can be integrated, searching through the subjects connected, and, not least, establishing stronger relations between concepts. Inevitably, the logical integrations will be more effective in the adoption of the integration process that the less sensible ones.

Cromwell (Lake, 2002), for example, explains that the human brain is organized in such a way that it can process many facts at the same time and that information gained by holistic experiences is recalled easily and quickly. We should also mention the Caine and Caine (1997) research findings, which prove that the brain of every person is universal and that every person has their own way of learning. Their research revealed that every person's learning style and use of knowledge depend on the integration of new information with previous knowlědge and his/her experience. This leads to the conclusion that we have to approach learning and teaching very carefully and that we must not favour one principle which might seem the most logical and sensible. Naturally, this stands for interdisciplinary integration as well. Sylwester (1995) summarized a number of research studies and, focusing on brain activity, developed a list of teaching advice for the teacher. We will enumerate only a few (Sylwester, 1995):

- Organize cooperative learning (emphasize the social aspect of the experience)
- The emotional element of learning is important.
- Employ passive as well as active methods of work.

- Teach what is important to know or understand.
- Employ technology.
- Work in a team.
- When treating problems take into consideration all intelligences. (Gradner (1983) suggests that each person has at least 8 intelligences and claims that schools develop and measure mainly the logical-mathematical and the linguistic intelligence.) When teachers are stimulating all of their pupils' intelligences, the curriculum becomes interdisciplinary, which, among other things, enables the pupils to employ various learning styles (Drake, 1998).

Regarding the curriculum we should emphasise that we can distinguish at least three types of curriculums according to different ways of interdisciplinary integration/connections (Erickson, 1995): the multidisciplinary curriculum, the interdisciplinary curriculum, and the transdisciplinary curriculum. The multidisciplinary curriculum is organized in such a way that a certain theme is treated by various school subjects, which can encourage the pupils to search for connections between the subjects on their own. With the interdisciplinary curriculum the emphasis lies on the subject integration, which is presented to pupils explicitly; certain themes or concepts which are common to all disciplines, are taught together. The transdisciplinary curriculum, as the highest level of integration, is based on the realistic context. The results of the learning process are more often assessed from the perspective of social responsibility development and personal growth than from the perspective of discipline integration.

In the Slovenia context, we practically cannot find a curriculum based entirely on the concept of interdisciplinary integration, even though some possibilities for content integration have been indicated within the curriculums of particular subjects; more in the sense of content correlations. This is mostly the case with lower grades, where all subjects are taught by one teacher and such connections are easier to implement from the organizational perspective. On the other hand, the implementation of interdisciplinary integration in higher grades requires much teamwork and coordination. Some related experiments in school practices will be presented in the empirical part of the paper.

In the paper we will not be discussing the curriculum, but rather the interdisciplinary integration/connections as a teaching method, as already mentioned in the introduction.

In school practices in Slovenia, the interdisciplinary connections are best achieved in the first trimester, when all subjects are taught by one teacher who knows the curriculum and is quick to find the possibilities for connections. In the second trimester we encounter the interdisciplinary connections mostly in the organizational forms such as summer camps, sports days, project work, sports weekends and months, research assignments, first aid classes, cycling proficiency tests, and competitions.

Kovač, Jurak, Starc (2004) claim that the following criteria have to be met for the success of interdisciplinary connections:

a) the teacher has to know precisely what objectives he/she wishes to achieve in certain subjects by means of interdisciplinary integration, and consequently he/she has to be familiar with the objectives and content of various subjects;
b) joint cooperation of teachers of different subjects is necessary;
c) the choice of contents, the transfer of knowledge, and the organization of lessons have to be adjusted to the children's level of development and education;
d) each interdisciplinary connection has to be planned carefully and has to include individual work of pupils;
e) at the end, the teacher analyzes the realization of the objectives defined.

We wish to emphasize that in Slovenia there are contrasting examples of integration practices and various reasons for integration: transferable knowledge, save time, learning for life, motivation, supplementing the disciplines. As mentioned before, with transferable knowledge we mostly refer to generic skills, which are presently underdeveloped in Slovenia, but some changes in this field have been indicated as well. When supplementing the disciplines in practice, we encounter various forms or ways of implementation. Let us look at some examples of integration between mathematics and other disciplines.

1. Example

Mathematics: The pupil can arrange objects according to their characteristics.

Music: The pupil recognizes different instruments and plays them, recognizes different tones (the difference in pitch) and arranges them.

<u>One possible lesson plan</u>: The pupil is arranging tones according to their height. He/She is learning how to arrange objects and how to recognize different tones.

<u>Possible issues, traps</u>: When both themes are new to the pupil,

learning is made more difficult and more complex, because he/she has to arrange things that he/she is still learning about.

2. Example

Music: The pupil understands the concept of pitch, tones of different height (introducing a new lesson)

Mathematics: The pupil can arrange objects, ideas (to reinforce the knowledge acquired)

One possible lesson plan: The pupil plays different instruments, sings and arranges the tones according to their height. The process of arranging objects is familiar to him/her, while tones of different heights represent a new concept.

3. Example

Language: The pupil is familiar with "puzzle" words with "lj", "nj" (spelling and pronunciation differ) and regular words with "lj", "nj" (introducing a new teaching unit).

Mathematics: The pupil fills in the Carroll diagram of two sets (testing the knowledge gained).

One possible lesson plan: The pupil uses the Carroll diagram to arrange words. The diagram is a tool the pupil is familiar with, used for the practical demonstration of newly acquired knowledge.

By means of interdisciplinary integration we can also increase the pupils' motivation (in this case we refer to correlations and not the actual integration of contents), for example, pairs of socks and the multiplication table for the number 2.

It is clearly evident from the research (Filipič, Hodnik Čadež, 2005) that integration is most commonly achieved by supplementing the disciplines and by correlations, which are in most cases, as already mentioned, motivating examples. Integration of this kind can hardly be called a teaching method or approach, because these are actually situations where we wish to thematically link two related disciplines, and where the integration of concepts from different disciplines and the development of generic skills and procedural knowledge are not emphasized.

Consequently, we used the MODEL IV project: the Partnership Between Faculties and Schools, Research of Educational Practice and Direct Application of Results in Education: the Teacher-Researcher and Interdisciplinary Connections (leader Dr. Janez Krek), made possible with the co-funding of the European Social Fund of the European Union and the Ministry of Education and Sport of the Republic of Slovenia, (hereinafter called "Model IV") to encourage the teachers to develop a teaching method which would be based on the interdisciplinary integration and with the help of which they would try to improve the pupils' and students' level of useful knowledge. The teachers acted as researchers of their own teaching practice, which included mastering the skills of interdisciplinary connections, planning, conducting and evaluating a practical case of interdisciplinary connections, and teamwork. The idea of teachers conducting research on educational practice came from the work of the 1973-1976 Ford Teaching Project in the United Kingdom, under the direction of John Elliott and Clem Adelman. This project involved teachers in collaborative action research into their own practices. Its notion of the "self-monitoring teacher" was based on Lawrence Stenhouse's (1975) views of the teacher as a researcher and as an "extended professional". Stenhouse's view of educational research implies doing research as an integral part of the role of the teacher, just as a teacher who uses research into their subject as a basis for teaching implies that s(he) does research into the subject through their teaching. Another great impetus for the development of the "teacher as a researcher" movement was the work of David Schön, in particular his books The Reflective Practitioner: How Professionals Think in Action (1983) and Educating the Reflective Practitioner (1991). The discourse of the reflective practitioner emphasizes the particular skills needed to reflect constructively upon ongoing experience as a way of improving the quality and effectiveness of one's work. The discourse encourages teachers to take into account the whole picture – analysing the effectiveness of a lesson or series of lessons through an attempt to evaluate what was learned, by whom, and how more effective learning might take place in the future. As such, it involves careful evaluation by teachers of their own classroom performance, planning, assessment and so on, in addition to and in conjunction with evaluations of pupils´ behaviour and achievement. It also implies a sound understanding on the teacher's part of relevant educational theory and research (Moore 2007). According to Schön (1991), practitioners should: (1) participate in research of their own practice and (2) develop educational theories that directly reflect actual educational practice. Action research, as presented in below, provides an appropriate means for realising these objectives.

3. The Characteristics of Action Research

Action research is a form of self-reflective enquiry undertaken by participants in social (including educational) situations in order to improve the rationality and justice of (a) their own social or educational practices, (b) their understanding of these practices, and (c) situations in which the practices are carried out (Kemmis 2007, p. 168). The idea of action research originates from a work written by social psychologist, Lewin (1946), who described research as a set of steps in a spiral, each containing planning, action, and assessment of the achieved result. One of the initiators of action research in education was Corey (1953, p. 70), who was convinced that "the disposition to study /.../ the consequences of our own teaching is more likely to change and improve our practices than is reading about what someone else has discovered of his teaching." Educational action research is a form of educational research which places control over processes of educational reform in the hands of those involved in the action. Educational practitioners must play a central role in carrying out action research if its relevance is to be assured. The role of outsiders, such as university academics and school counselling service, can only be as collaborators, providing assistance. On Stenhouse´s account ″In action research real classrooms have to be our laboratories, and they are in the command of teachers, not of researchers″ (Stenhouse et al. 1979, p. 20). For teachers who wish to perform action research it is assumed that in addition to their willingness and motivation to undertake research they also have the possibility or professional autonomy to make the decisions necessary for the research (e.g., implementation of changes in the educational and training process) (cf. Fraenkel & Wallen 2006, p. 568).

In action research the researcher prepares a flexible indicative research plan which has to be updated throughout the entire research. The plan of the entire action research divides individual realisable action steps whereby each step is oriented towards activity with specific objectives. The action researcher will embark on a course of action strategically (deliberately experimenting with practice while aiming simultaneously for improvement in the practice, understanding of the practice and the situation in which the practice occurs); monitor the action, the circumstances under which it occurs, and its consequences; and then retrospectively reconstruct an interpretation of the action in context as a basis for future action. Knowledge achieved in this way informs and refines both specific planning in relation to the practice being considered and the practitioner's general practical theory (Kemmis 2007, p. 173). Action research is not distinguished by the use of a particular set of research techniques. It is true, however, that in general the techniques for generating and accumulating evidence about practices, and the techniques for analysing and interpreting this evidence more closely resemble the techniques employed by qualitative researchers than empirical-analytic researchers. These methods place the practitioner at centre stage in the educational research process: actors' understandings are crucial in understanding educational action. One of the central techniques recommended in the reflective practitioner discourse is the keeping of diaries or journals by teachers (Moore 2007, p. 122) in which they reflect systematically on their experiences as they perceive them, keeping a record that can be returned to and re-examined in the light of subsequent experiences and providing scope for the self-setting of targets and goals.

Action research is usually carried out in a single school or class. It is important that the description and analysis of the course of the action research, as well as the results achieved, are published and made publicly accessible. With the proper description of the execution of action research the reader obtains a model of how the participants studied a specific situation, solved dilemmas, and improved the quality of pedagogical practice, as well as influencing the circumstances that ensure a higher quality educational process. While remaining cognisant of their own circumstances, the reader can transfer the results of the action research to their educational practice by taking that which makes sense and acting according to it or by adapting the findings to the characteristics of their own specific situation. The results of the teachers' work are presented in the following sections.

4. Teacher researcher. Analysis of examples of good practice of interdisciplinary connections.

4.1 Background

In Slovenia it is rarely emphasized that research is one of the teacher's roles. Teachers are not trained well enough in the field of practical research and their ongoing professional training is focused on course content and contemporary teaching methods. After the reform of the previous higher education system to university education (1987–1988) or the reorganisation of the previous Academy of Education to the Faculty of Education (1990) all teachers are required to complete a four-year university study programme where they acquire knowledge in the fundamentals of educational methodology and statistics.

In Finland, for example, the guiding principle of teacher training is research, the importance of which is based on the following: teachers need to be familiar with the latest research in the field of teaching and learning, they need to know how to implement the research findings in practical situations, and they need to possess the necessary academic and professional competence for research, which enables them to systematically plan the instruction, to help develop social and ethical dimensions of pedagogical work, and to assume a more responsible role in the society (Niemi, Jakku-Sihvonen, 2006).

We are convinced that Slovene teachers should play a more active role in the society. Systematic research would enable teachers to participate actively in the discussions and argumentations about the curriculum, its contents and goals, about the learning process, instruction, pupils' development, and, not least, the ethical questions of their profession.

Training the teachers in the field of research was the main goal of our Model IV project. The guiding principle of the Model IV project was that the teachers must actively participate in the research on the way their pupils are gaining knowledge, how they are learning. When teachers play an active role in the research process and when they enjoy the support of their partners form the Faculty of Education, the probability that they will consider their new role in the classroom and even transform it on the basis of the research findings, is far greater than when they are only informed of the research findings, or when researchers conduct a research study in their groups of pupils. The teachers involved in the project conducted a research on interdisciplinary connections. The teachers played the role of researchers on interdisciplinary integration in classes they teach, and in most cases they worked in teacher teams. We wish to emphasize that we organized a number of training classes in the field of interdisciplinary integration and research for the teachers who wished to participate in the project. The teachers participated in the project for one school year (2006/2007) and they conducted their research during regular classes in close cooperation with their mentors from the Faculty of Education. The teachers chose various disciplines for interdisciplinary integration. We played the role of mentors to those teachers who chose to integrate mathematics with other disciplines. In total, we cooperated with 33 teachers from 11 schools and kindergartens (2 kindergartens, 1 grammar school, and 8 elementary schools). At the completion of the project we received 9 reports on interdisciplinary connections which included the field of mathematics. One report was not complete, but 8 were satisfactory. A total number of 25 teachers cooperated on the eight reports which are the subject of our analysis.

4.2 Teacher researchers' reports analysis

4.2.1 Statement of the problem

At the completion of the year-long research, the teacher researchers who participated in the Model IV project wrote their reports, consisting of the key structural elements of the research report: summary, keywords, theoretical and empirical part, significance of the research for practice and conclusion, and appendixes. All reports have two themes in common: action research and interdisciplinary integration including the field of mathematics.

The main objective of this teacher researchers' reports analysis is to combine their findings, present examples of good practice and make recommendations for the implementation of interdisciplinary integration and action research for teachers in school practice.

4.2.2 Research questions

When analyzing the teacher researchers' reports we were trying to answer the following questions:

Which research problems the research was focused on?
Which research method was used?
How did they implement the research in practice?
What were their research findings and recommendations for implementation in practice?

4.2.3 Method

We employed the method of descriptive analysis.

4.2.4 Sample

The sample includes 8 teacher researchers' reports (in total, 25 teachers and 420 pupils who were included in the research). The research was conducted during the 2006/2007 school year. The teachers involved work in different Slovenian regions. This is a purposive sample.

4.2.5 Reports analysis

The teachers' research reports are quite extensive, 20 to 40 pages each (without appendixes), or 30 to 60 pages with appendixes. All contributions are structured as reports, consisting of: summary, theoretical part, empirical part (problem definition, methodology, and findings analysis), conclusion and recommendations for practical use, and appendixes (lesson plans, tests, questionnaires).

Coding of reports

When analysing the reports we chose a few key categories which are important for the analysis and interpretation of each teacher's research on interdisciplinary integration. These categories are: the number of teachers per team, grade level and number of pupils included in the research, research problem, research methodology, and research findings.

Reports analysis according to key categories

1. Number of teachers

As already mentioned, 25 teachers handed in their reports (18 class teachers, 2 single subject teachers, and 5 grammar school teachers). They all worked in a team of 2 to 5 teachers, namely 4 pairs of teachers, one team of three teachers, one team of four teachers, and two teams of five teachers. Three pairs, the team of three teachers, and the team of four teachers consisted of class teachers, while one of the five-member teams consisted of class teachers and the other of grammar school teachers. One pair consisted of subject teachers. It turned out that all teachers involved chose to work in teams.

2. Number of pupils

The total number of students involved in the analyzed sample of teacher researchers' reports was 420, namely 30 first-grade pupils, 28 second-grade pupils, 9 third-grade pupils, 151 fourth-grade pupils, 83 fifth-grade pupils, and 34 eight-grade pupils from elementary schools as well as 20 first-year pupils and 25 third-year pupils from grammar schools. The number of fourth-grade elementary school pupils stands out, since the largest number of teachers-researchers teach fourth grade in elementary schools (the total number is 8 class teachers).

3. Research problem, methodology, findings

We established that the research problems of all the reports relate to interdisciplinary connections or mathematics. The reports will be divided into two groups: 1 report dealing with mathematics, action research method (I. group) and 7 reports dealing with action research interdisciplinary connections (II. group). We will present the analyses of these two groups of reports separately.

I. group of reports (the common theme is mathematics, the method is action research)

This group comprises of the report dealing with the mathematical word problem solving in the first five grades of elementary school. This research was conducted by 5 teachers (one teacher of each of the first five elementary school grades) and included 47 pupils (6 first-grade pupils, 8 second-grade pupils, 9 third-grade pupils, 15 fourth-grade pupils, and 9 fifth-grade pupils).

The report presents the research on mathematical word problem solving in the first five grades of elementary school and sets forth the following research problem: "How to employ an effective teaching method to improve the mathematical word problem solving?« The research team employed the method of action research in 3 steps: assessing the existing knowledge (how successful the pupils are in word problem solving), developing a teaching method/approach to word problem solving, and assessing the pupils' progress in mathematical word problem solving. The instruments of the first and third steps are tests in mathematical word problem solving. The researchers presented the results of these test by means of tables and diagrams. The teaching method they developed emphasizes the strategies of mathematical word problem solving (read the text, describe the problem in your own words, underline key information, write down the calculation, check the solving process, check the logic of the result, write down the answer). It turned out that the pupils did follow the process of problem solving, but their results or accurate answers were not much improved from the 1st to the 2nd tests (for example, the fifth-grade pupils had a 53% success rate in the 1st test and a 56% success rate in the 2nd test, third-grade pupils' results were similar: a 65% success rate in the 1st test and a 67% success rate in the 2nd test). The forth-grade pupils' results are somewhat different: their success rate in the 2nd test displayed an average increase of 26%.

It turned out that it is useful to know how to use the mathematic word problem solving process, but that the basic tools remain the knowledge of calculations and the understanding of problematic situations. The latter is undoubtedly complex and not necessarily in direct proportion to the knowledge of the word problem solving processes. The researchers concluded their report by stating that they have observed a visible improvement in the knowledge of the word problem solving processes, but no visible progress in problem solving. This is an interesting observation which leads to the search for new solutions for the successful instruction of mathematical word problem solving, which causes problems for a number of pupils.

II. group of reports (the common subject is interdisciplinary connections, the methodology is action research)

This group comprises of the 7 reports dealing with interdisciplinary integration, namely how the mathematical content can be combined with other contents of the curriculum, and action research. These research studies were conducted by 20 teachers (1 first-grade teacher, 5 fifth-grade teachers, 2 eight-grade teachers and 5 grammar school teachers) and included 373 pupils (Diagram 1).

All reports have two main research questions in common:

How to improve the pupils' or students' useful knowledge of data processing?

How to develop a teaching method/approach that would enable the teachers to improve the useful knowledge of their pupils or students?

The methodology that was used for all research studies is the method of action research. The three action steps can be summarized in the following way:

1. Assess the level of basic and useful knowledge of data processing (test 1).
2. Based on the results of the test 1, develop a teaching method based on interdisciplinary connections, namely on combining the data processing contents with various other contents (developing the pupils' useful knowledge).
3. Assess the pupils' progress in useful knowledge and determine the success of the teaching method developed (use test 2).

In step 1, the teachers/researchers determined that the pupils were familiar with the data processing concepts when these were strictly mathematical, but that they ran into problems when asked to apply or use this knowledge on other contents (especially natural sciences and social sciences). Let us look at some of the results of the first and second tests, which clearly show that the progress in the pupils' knowledge following the teacher's intervention.

a. Two teacher researchers, 4th grade, total number of students: 47, theme: integration of data processing contents with language, natural sciences, physical education, social sciences, and music. The pupils had a 24% success rate in the first test, and a 77% success rate in the second test. The progress in the pupils' useful knowledge is obvious (Figure 1).
b. Two teacher researchers, 8th grade, total number of pupils: 34, theme: integration of mathematics, geography, history, chemistry, and biology. The arithmetic mean of the success rate in the first test was 6.96 in the useful knowledge tasks and 16.84 in the basic knowledge tasks, while the arithmetic mean of the success rate in the second test was 12.02 in the useful knowledge tasks and 18.76 in the basic knowledge tasks. It is obvious that the pupils' knowledge has improved, enabling them to use their knowledge in new situations (Figure 2).
c. Five teacher researchers, 2nd and 3rd year of grammar school, total number of students: 85, theme: integration of mathematics, geography, history, German language, and Slovene language. The increase in the success rate of second-year students was 17 % from the first to the second test, while the increase in the success rate of third-year students was 13.3 %. The teacher researchers ascribe the success to teamwork, integration of contents, and students' motivation (Figure 3).
d. Three teacher researchers, 5th grade, total number of pupils: 54, theme: integration of language and mathematics, mathematics and physical education, and mathematics and social sciences. The pupils' success rate when combining language and mathematics was 70.2 % in the first test and 94.9 % in the second test. When combining mathematics and PE, their success rate was 61.4 % in the first and 95.0 % in the second test. Their success rate when combining mathematics and social sciences was 90.6 % in the first test, which is why they were not tested for the second time (Figure 4).

Following the results of the first test (with all teacher researchers this test showed that the pupils' useful knowledge was not as good as their basic knowledge) the teachers developed a teaching method or approach based on interdisciplinary connections. They combined mathematics, namely the contents of data processing, with other disciplines of the curriculum. The class teacher researchers mostly integrated the fields of natural sciences and social sciences, followed by language, PE, and music. Regarding the subject teachers and grammar school teacher researchers we can set forth the integration of mathematics with geography and history, followed by the integration of mathematics with chemistry, biology, and language.

We can conclude that the teachers planned their teaching methods according to the objectives and contents of specific disciplines. Schematically, we could present the concept of integration in the following way (Diagram 2).

Let us look at some examples of the integration planning process from the reports:

1. Example

Subject: mathematics, natural sciences, and technical science

Grade: 4

Theme: The animal kingdom

Contents and objectives: The pupil can distinguish between the living beings according to their physical appearance, habitat, and diet. He/She acquires the new concepts: herbivore, carnivore, omnivore, vertebrates, invertebrates. He/She knows what the animals eat. He/she chooses a demonstration of one of the ways the animals eat. He/She then explains his/her choice of diagram.

2. Example

Subject: Slovene, Hungarian, German language, mathematics

Year: 3

Theme: Media

Contents and objectives: To introduce the media (television, radio, internet etc.). Use of media among students. Data processing. Advantages and disadvantages of computers and internet.

3. Example

Subject: mathematics, social sciences, natural sciences and technical science

Grade: 4

Theme: Space

Contents and objectives: Creating a mock-up (scale model) of a house. Definition of the living space. Developing measuring and planning skills, and using them in everyday situations. Creating a floor plan.

In every report the teachers illustrated the method of work with at least six descriptions of lessons. The teachers analysed each lesson from the perspective of achieving the objectives, the pupils' motivation, and other criteria. Some reports also contain pictorial matter: photographs of pupils working, pupils' products.

We find that all teachers in their reports described the research they had conducted as a pleasant experience, because the research work had the proper expert support provided by mentors and training, and because the research was though through, systematically organized and executed, and appropriately evaluated.

5. Conclusion

Based on the reports of teacher researchers involved in the Model IV project (in total, 25 teachers and 420 pupils) we can claim that the research and research findings represented a giant step forward in the pedagogic practice, in each and every case. A step forward in the following areas:

The pupils were acquiring knowledge throughout the process (none of the teachers reported a deterioration in their pupils' knowledge).

- This improvement in the pupils' knowledge is the result of the teacher's active intervention in the teaching practice.
- The teachers have gained new dimensions of treating the teaching contents. The disciplines are not separated from one another and they have common objectives and contents.
- The teachers have gained experience in research work and teamwork.
- The teachers were satisfied with the research findings.

The mentors of teacher researchers have observed that teachers need even more training in the field of practical research and that they need to upgrade their professional writing skills. The teacher researchers encountered numerous problems when writing their reports, especially when writing a clear interpretation of their findings.

We are convinced that we can influence the implementation of the teaching process and the teachers' research only through the close cooperation between the Faculty and the school teachers. We believe that teachers need the research knowledge and that they need to be informed of the latest research in the field of teaching. To achieve progress in the research we need to integrate the expert and educational contents. Teachers who conduct research gain an analytical insight into their own work, based on which they can draw certain conclusions that help them systematically develop the teaching and learning process, and enable them to play an active part in discussions and decisions related to education.

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Diagram 1 *Number of pupils involed in the research*

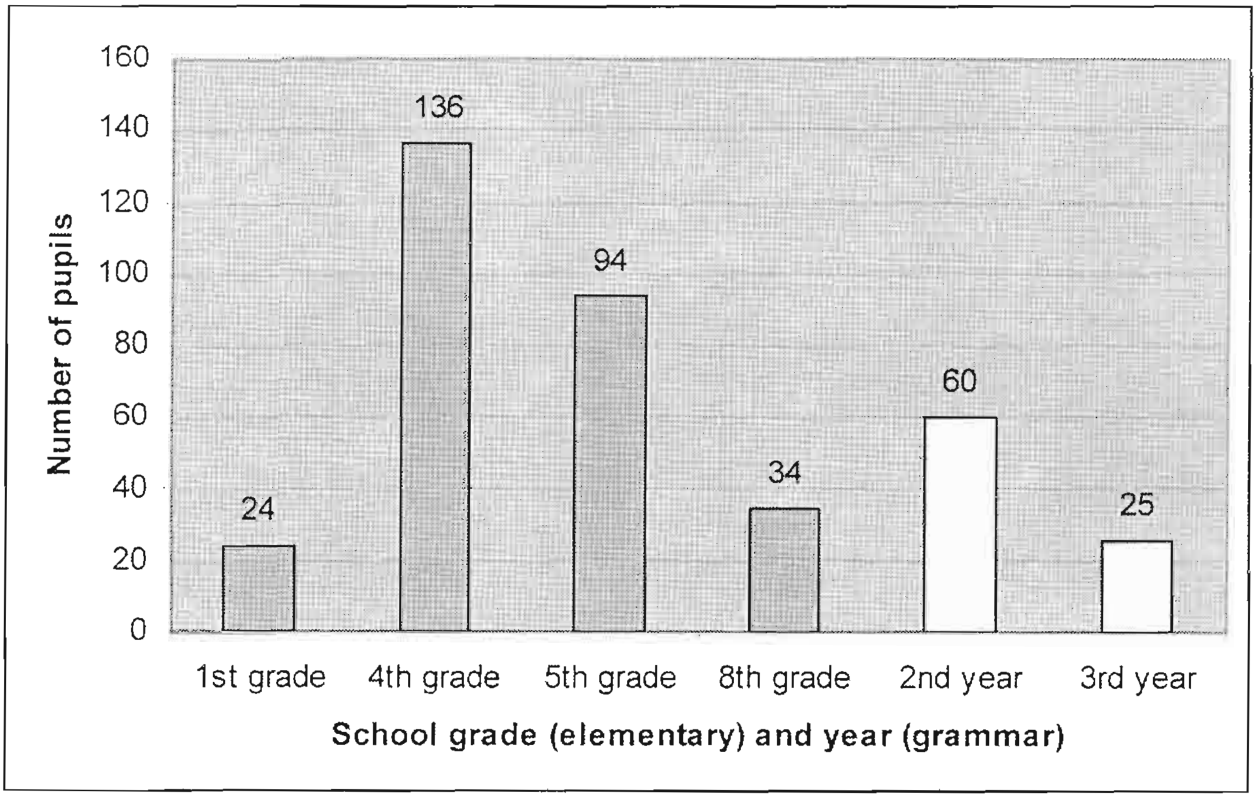
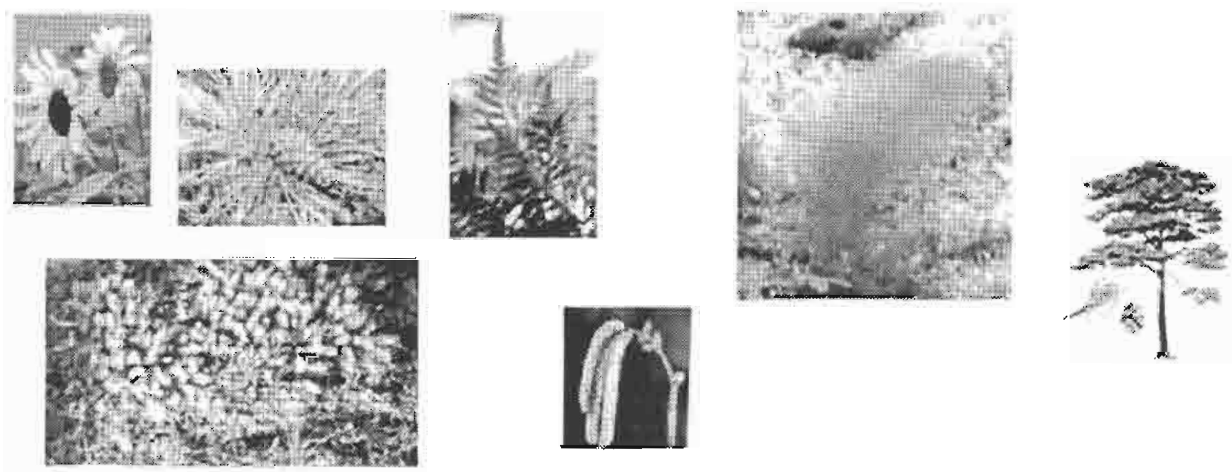


Figure 1 *Action research report (an example of a task from test 2: Zakotnik, A., Žibert, N. (2007))*

Observe, name the plants on the photographs, and fill in the table.



	IS A PLANT WITH SEEDS	IS NOT....
HAS FLOWERS		
DOES NOT HAVE		

How many plants with seeds have flowers?
The plant that has seeds, but not flowers is .....

Figure 2 *Action research report (an example of a task from test 1: Mohorič, L., Kalan., K. (2007))*

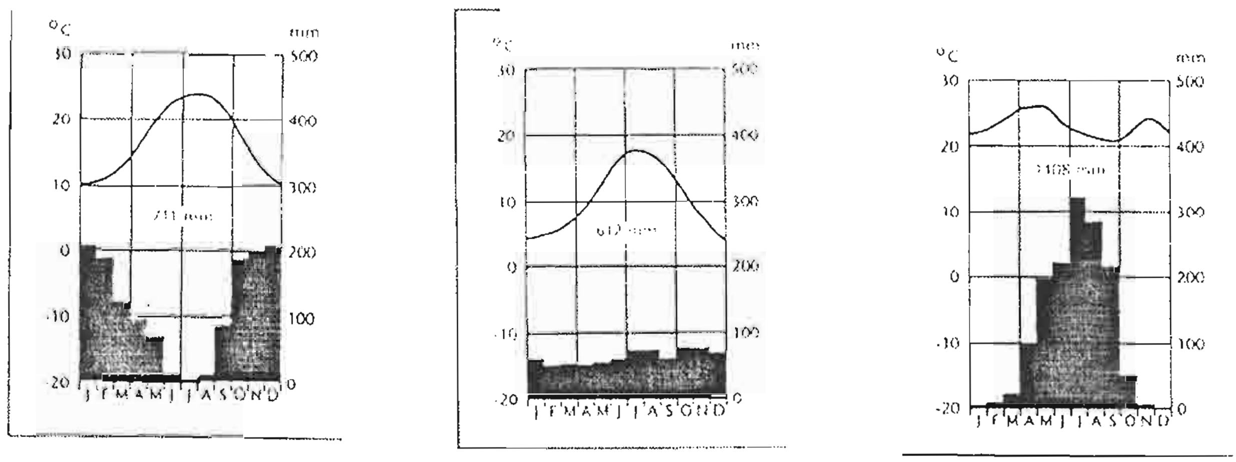
The countries of East Asia, with the exception of Mongolia, are known for rice production. This table shows the rice production of individual countries of East Asia.

Country	Rice production in million tons	Rice production in %
China	185,1	88, 7
Japan	11,4	5, 5
South Korea	6,35	3, 0
North Korea	2,57	1, 2
Taiwan	3,26	1, 6
Combined		

How much rice was produced in all the countries combined?
Use a pie chart to represent the rice % per country.

Figure 3 *Action research report (an example of a task from test 1: Ogrin, D., Majcen, J., Vignjevič, P., Kokelj Žerovnik, I., Golob, A. (2007))*

Mark a chart with Mediterranean climate



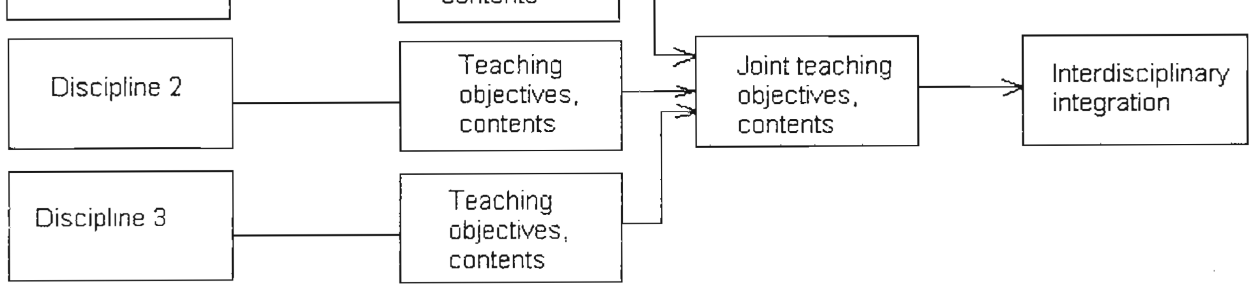
List natural and cultural growth which is significant for that type of climate.

Figure 4 *Action research report (an example of a task from test 2: Lenček, L., Zorn, B., Hozjan, F. (2007))*

Jure is an amateur runner. He decided on the number of km he would run each day of the school year. The table below shows, how many km Jure ran every month. Draw a diagram and calculate the average number of km which Jure ran in the whole year. Insert the average value into the diagram.

Month	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	Maj	Jun.
km	8	6	3	2	4	6	9	11	12	15

Diagram 2 *An example of the interdisciplinary planning (Hodnik Čadež, Filipčič (2005))*



An Approach To Textbook Writing

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Abstract. This article consists of authors thoughts on writing mathematical textbooks, poses some of the problems encountered during the writing process and provides some possible solutions for the problems in question. It mentions areas such as: illustrations and graphical presentation in general, language and style, problem with technical terms that do not have an adequate translation in Serbian, choice of adequate difficulty level, exercises that are used for reviewing lessons, pre-solved examples and accompanying exercises, as well as other complements to the textbook.

Keywords: Textbooks, textbook writing, mathematics.

1 Introductory Remarks

Educational process consists of many strongly interconnected components, so balance (a quality in teaching) can be achieved even if one of the elements is not entirely good. The goal is to contribute to the improvement of all components and make the whole process as better as possible. Here we shall discuss textbooks for Mathematics, more precisely – creating textbooks that should make teaching more effective and make students achieve better results.

Textbook writers encounter many problems, and their success in solving these problems is evaluated by their colleagues and their students. If an author wants his work to be accepted, he should, therefore, "please" both sides. A question imposes itself: what is more important – present or future? Here, the "present" represents authors colleagues, and the "future" represents all those who will use the textbook. I believe that future is much more important. So, a textbook should be good – usable, likable, interesting and stimulating for users (students).

In writing textbooks we also have cases we see in literature – cases when one's first book is a brilliant achievement, while the following ones lack the quality of the first one. There are also cases when younger authors, if they manage to cross certain bureaucratic barriers, publish very good textbooks, in which they convey their enthusiasm and knowledge, and so the book contains plenty material, and possibly even a non-standard structure, and is very successful as a whole, and therefore usable and used.

Among reviewers of textbook, there is one that is never mentioned inside the book's covers, but whose judgement is very objective – and that reviewer is: time. If there would be an objective criterion of textbook quality, than we would have no more than a handful of textbooks from specific areas of research; however, there is no such thing, and we are facing an abundance of textbooks, some of them continuing to exist only because they are written by teachers who are teaching students at a certain faculty and require the use of their own books, and some simply because there are no others.

Often are textbooks bought only because the teacher asked the students to do so in the beginning of a semester. Of course, it is easier to use the textbook of "your own" teacher, since the symbols used in class will be pretty much the same with those used in the textbook. However, any other textbook is welcome, and older pupils and especially university students should be encouraged to use other textbooks for the same subject. Because every subject requires a good textbook, no matter who its author is.

It is a fact that the distribution of previous knowledge and intelligence of readers is a normal one, and that alongside a majority of those "average" readers, we also have two minority groups – those who find average too much, and those who find average to be too little. But these categories should not be determined in advance. With teachers engagement, and with a good textbook, it is possible to improve success rate, reduce the number of those below average, and all of this without having to lower the criteria for students (a method that has, undeservedly, gained much popularity worldwide).

And how, then, to make the good ones even better? Is it possible? Yes, it is, through endeavour, personal improvement and critical analysis of one's own work when writing a textbook. Let us eliminate the egoism that only feeds the author's vanity or forcibly follows an actual, "modern" trend. It is quite possible that something that reaches us as very modern, already begins to be understood as a mistake in the place it originated from an is already being abandoned there. We should keep in mind that we are working with living beings, and that in vivo experiments are not recommendable.

Distribution for students' interest in a subject and distribution of persistence are probably both normal distributions. There too can a good textbook improve the average value, and reduce variance. Courses that consist of different mathematical subjects are in that sense more favourable, because they give more chances for "new beginnings". It is true for bigger differences (switching to geometry after arithmetic) or smaller ones (function in several variables, vector functions). However, in a one-subject course, listeners' interest can be maintained at a good level with good dosage and selection of material.

2 What To Avoid?

Author must be aware of the huge difference between him and the students. Author has been researching a certain area for twenty, thirty or more years, while the student is only getting acknowledged with these terms. Author cites certain situations, concepts and statements almost automatically, not asking the reader, and maybe not even himself, the question: why is this concept introduced, what will be its use, what is its connection to other similar concepts. You can often get the impression that the author is not teaching but simply repeating by heart lessons he learnt a long time ago. But it's not the author that should be doing that, although he too will be evaluated – by students who like or dislike his textbook and successfully or unsuccessfully learn from it.

No form of forcible likability, colourful illustrations, too monolithic or too broken text is in any way good. Everything should be carefully measured according to the age and previous knowledge of the students. Tendency a long time ago adopted for primary school textbooks – to integrate them with workbooks in which students write or draw themselves, with a goal to facilitate work – has an opposite effect: overlooking important parts of the subject, students often display less ability to memorise it, and even a certain amount of disparagement towards the subject.

It is a well-known fact that students in the beginning of high school or their studies at the university, as well as in the beginning of every semester, take their obligations with a certain amount of optimism, expecting some success and/or progress, which obviously is not happening, since we often face a lack of interest in students and witness their abandoning studies altogether. There are many different reasons to this, but one of them is certainly our current topic – textbooks. Had the textbooks been more adapted to their users, had they found in them answers to some of their questions, had these books contributed to their success, we wouldn't have so many books remain unopened, or be torn at the end of the school year (I don't know if this form of vandalism exists elsewhere...).

Language of lecture and that of textbook cannot be the same. It was said a long time ago: "Verba volant, scripta manent – Words fly away, writings remain." Neither can a textbook be written with colloquial phrases, with an excuse that they make it "closer to the children", nor can it be written only with dull and difficult technical vocabulary. Author knows where he is leading his student, and along this path he adds digressions and commentaries, which, however useful they may be, need to be adequately distributed and not to burden the textbook (as they would if they appear in inappropriate places). Few more words on language: A special problem in mathematics are those terms that are not in accordance with Serbian language, and especially the problem with computer-related terms that often get literally translated to Serbian, or kept in their original form. For instance, in statistics, "stem and leaf diagram" is, in Serbia, usually left untranslated.

A textbook is, because of his purpose, a specific whole, which is different from a research project, an encyclopaedia and a monograph. However, a textbook should contain details from research projects, encyclopaedias and extraordinary old books like "Fichtengoltz".

Mathematics cannot be divided into theoretical and practical. Theory should contain practical, numerical examples, while applied mathematics should also be followed by a theoretical basis for the selected method, conditions for its application, etc.

A textbook is not about recounting facts but about explaining – for its purpose is to teach. Learning must not be reduced to a memory training, it is important that student understands what he has been taught. It is also of some importance to provide an explanation as to why we learn a certain subject.

3 What To Do?

New textbooks have a modern graphical design, with full colour drawings, and often accompanied with a CD. However, the old textbooks are sometimes much easier to learn from, and they offer a wider range of information that can be learned. Both the students and the textbook writers can learn a lot from old textbooks. So, what makes old textbooks so good? They are good because of their gradual and detailed elaboration of the topic in question, font size that is not tiresome for eyes, and, for example, the fact that they always had an explanation of how to read a specific mathematical symbol (after "n!" it said "read: n-factorial"). This is necessary and useful because many of the students do not know how to read the formulae, a fact not hard to verify. Signs and marks in old textbooks were visually acceptable and easily recognisable (with an exception of gothic script, which is not so well applicable nowadays). Old textbooks contained plenty examples, even the simplest ones, and that is something we often neglect thinking that such examples are trivial or well-known. But, if the author knows them, it doesn't necessarily imply that the student will also be familiar with them.

A textbook should contain details that will make it "come to life". I think here on comments, additional historical information, anecdotes, images and illustrations, special symbols for pointing out the more important elements in a lesson or the difficulty of the exercise. For instance, a French probability and statistics textbook has a crossed fork and knife symbol for the difficulty level, and even a text accompanying one of difficult exercises (marked with four such "crosses"): "If you solve this, you must contact us!"

Author must find a way to commend his students where needed, at least like it is said in that old joke, that in the middle of a very thick textbook it was written: "You got this far – well done!"

Illustrations are very important in many areas of mathematics, starting with Venn's diagrams in set theory up to the complex drawings in descriptive geometry. I suggest that in a textbook there should be at least some hand-drawn sketches, because they are like those drawn by the student, while solving an exercise or analysing a proof. Drawings with many details, like in descriptive geometry, should be drawn gradually, in the process of their creation, because a whole drawing can confuse the reader, as he cannot understand from where to start looking at it, and simultaneous reading of the notes usually written along such drawing can become very tedious. The best thing is to gradually introduce details on a drawing, following the introduction and explanation of a construction or proof.

Mathematics is a universal language with its own symbols, reading rules and grammar. Textbook writer must take that into consideration. Let us compare learning mathematics with learning foreign languages – what are the tendencies in foreign language learning? A thorough learning of a foreign language is comprehensive. We can see it from book titles like: "Cours de langue et de civilisation française" or methods like: AVGS (audio-visual-global-structural) method. Something of the kind should exist in textbooks for mathematics. Among other things, explanations should be accompanied by information about what preceded the area of research, and what follows it. There should also be textbooks for teachers, with additional explanations, exercise suggestions, and so forth. And, as what is said for languages, that they can be learned the best way in the proper language environment, is also true for mathematics. How can you learn to speak "mathematical", if your language environment is that of everyday life? But we can enter the magical world of mathematics once we have the entrance ticket – a good textbook.

How to teach mathematics is not a topic we discuss here, but it is closely related to textbooks. Namely, does the teacher only stand for the first reading out loud, or will he say something more or less than it is said in the book? How to make students listen to the teacher if he will only say the same thing that is already written in the textbook? What are the respective roles of the teacher and the textbook? Textbook should be a well-kept road to go along, teacher – a road sign, and the student – a traveller who is to walk down that road, look around the place with his/her own eyes, spot all the ascents and dangerous places, take a look at the surrounding scenery as well, become aware of those faraway lands and know that there is so much more to experience in the world around him. Students shouldn't be forced to face roadblocks.

In writing textbooks, the author must, aside from his expertise, to take into consideration his emotions, as well as emotions of the reader. It is already sufficient for him just to think how he himself had problems solving some exercises, not finding explanation or help in the textbook he used during his schooldays. Author should think about books he had found difficult to understand, and compare them with his own work. Nothing must be left untold or unfinished in the textbook, all the exercises should be solved, theorems proven, at least in an additional chapter at the end of the book. Author should also think about books he enjoyed reading, and compare them with his work as well. All in all, author should form a critical attitude towards his work, improve it and adapt it for students' use.

Textbook should also raise the general cultural level of the reader. In an era of an overall debasement of culture, a textbook is a minor weapon, but we mustn't look down on the strength of small things. In any case, a textbook must not be a part of these culture debasement trends – if we cannot win them, we needn't join them either. A textbook should maybe even start with a form of a Hippocratic Oath: "...We will teach conscientiously from the very beginning, we shall not damage anyone on purpose, or make him hate mathematics..."

A textbook should be written to guide the reader carefully through the subject. And, just like in a theatre, a character first comes on the stage and then is addressed or introduced by another character, when a mathematical subject comes up for the first time, its appearance and behaviour are observed, and only then is its mathematical "name" known to us – that is to say, a strict definition shouldn't come first.

Quantity of material is also important. It is hard to find the right measure in any job, including textbook writing. Some writers adopt the "fully done things are boring" principle, and leave many proofs and calculations or readers to do themselves. Solution, however, lies on the other side, because the textbook is written *for* the students and so detailed explanations of basic elements are necessary for understanding the subject itself, as well as for constructing a good foundation of overall mathematical culture. Sentences should be relatively short and easy to understand. After giving the basic explanations, one should offer more material, point out to other possible research subjects, give additional exercises that require linking different subjects. For instance, in analysis, write down the condition for the function to be convex through the adequate determinant.

A textbook is usually a result of a period of years of author's teaching work. Such experience is desirable, but it must also be followed by success analysis through exams. Students' results will be connected to the quality of the textbook as well, so this factor must be taken into consideration before publishing a second edition. Given the fact that certain courses in our faculties have a relatively small number of listeners, it happens that a single edition of a textbook is used by several generations of students. If some possibilities of improving a textbook are noted, it would be good to take into consideration printing a 20-30 page booklet as an addition the book, and hand it to new generations of students. If a textbook is commissioned through an open competition, then author of the chosen textbook should teach at least one generation of students using it, to see whether his ideas can be realised, and only then, with possible additional corrections, publish the textbook.

A textbook should be planned according to lessons, because it gives students a certain feeling of advancement. In a case like this, for example, a two-hour lesson needs 10-15 pages for the basic text (introduction, elaboration, conclusion), which are followed by additions, comments, exercises. To make comments stimulating, they should be put after the main body of the text and written in a clear language. The additional part ought to contain some more difficult questions, but the answers to all of them *must* be found in the lesson. And if a question cannot be answered directly from the lesson, then one should write some guidelines and advice for it. Some difficult exercises can also be written here, but they too require a written solution, or at least a (relatively) detailed explanation of how to do it.

How to motivate students with a textbook? Can a mathematical text make someone willing to go into further research? What are the sentences an author should use to encourage his readers? Maybe some instructions considering these questions should be given in a Teachers' manual that goes with a textbook. Because that is the place for methodical instructions and notes on improving the educational process.

Recognise, repeat, remember – these rules of studying are realised through the textbook as well. Topics should be presented in a way that their content and form is easy to remember, and additional questions and exercises are there to enable recognition and repetition (and, therefore, help reader fully adopt a certain subject). For instance, exercises that enable recognition are those in which reader is required only to recognize a certain structure, like a type of differential equation.

Textbooks for mathematics intended for students with better previous knowledge and for students studying mathematics, are certainly different from those written for non-mathematicians. However, no matter what will be the future specialisation of the reader, a maths textbook (on general or particular subjects) contains mathematical models. Meaning of a mathematical concept does not depend on the area it will be used in: meaning of Pythagoras' theorem is the same, no matter who is using it. Mathematical concepts must be explained as a general case, and special cases connected to the specific area of research must be stated as special cases: for example, concept of a derivative can be explained to the physicists through the concept of speed. If a concept is explained only in one specific situation, it may remain unrecognised in another specific situation. For non-mathematicians, definition of a concept is most adequate to be given when used (concepts infimum and supremum used when defining the limit, not as a specific characteristic of the set of real numbers).

One thought about teachers' textbooks: such a book should have three parts for every topic. The first part would contain the lesson itself, as it is in the students' textbook. The second part should, then, be an analysis of the lesson, notes about connections between newly introduced topics and ones learned in previous lessons. The third part would contain things that a teacher with an academic degree should know about these topics (in geometry, Euclid's axioms as an example). Why should the lesson be in the teachers' textbooks in the same form it has in the students' textbooks? Because we teach future teachers what they need to do, and so they have to be provided with concrete models during their studies, and not sporadically but systematically.

Taking proofs into consideration, we should point out the difference between theorems like Roll's theorem (establishing the existence of a point) and theorems like Newton's method for solving equations (giving the algorithm for the series of approximate values). When dealing with statements of the form $A \Rightarrow B$, we should use examples to show that $B \Rightarrow A$ is not true in general case. Proofs for non-mathematicians should mostly be those deriving from definitions or already proven statements, while mathematicians should be acquainted with proofs that use special constructions. It would be best to explain these constructions, and, eventually, point out to possible modifications (proof that if in the sample space there is an infinity of elements, then at most a countable set of elements could have a non-zero probability).

4 What's Next?

Each of the problems posed in this article should be analysed in detail on the existing textbooks in Serbia and abroad, so as to create more comprehensive standards for improving mathematical textbooks.

Such an analysis of the problem of textbook writing can have only one conclusion, and that is the writing of a textbook according to the objective measures of quality, some of which are stated here, and to the fundamental sense of goodwill for the younger generations who can give much if they are given much.

[illegible] ose th[illegible]
every sequ[illegible]
rational. Th[illegible]
conjectures b[illegible]
The third part [illegible]
know about the [illegible], to mean [illegible]
the reason be[illegible] gives insight [illegible]
in people [illegible] takes [illegible]
to be provided [illegible] these models [illegible]
systematically.

Taking proofs [illegible]
theorems like Bol[illegible] establishin[illegible]
Newton's method for solving equatio[illegible]
approximate value [illegible]. When dealing with [illegible]
the examples to show [illegible] and $\beta = 1$ [illegible]
mathematicians would [illegible] these
guidance, while [illegible] these
computations. It is [illegible] better to expl[illegible]
out impossible [illegible] than
examples, [illegible]

4 Wh[illegible]

teachers [illegible]
technique[illegible]
theorems [illegible]
Such [illegible]
and that [illegible]
show [illegible]
young [illegible]

On Some Elements of Chinese Mathematics Pedagogy and Practise (1)

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Abstract. This paper is the first in a series of articles dealing with Chinese mathematics pedagogy and practice. It consists of a short review of contemporary mathematics education in China, with references to Chinese beliefs and tradition, whose influence to the educational system should be noted. We will see why teachers have high position in Chinese society, how they teach, and how does the teacher-training system work. Next papers will focus on the standard for mathematical textbooks and the standard for mathematical curriculum in China.

Keywords: Teaching, paedagogy, mathematics in China.

1 Introduction

In order to improve our understanding of the needs for curriculum changes and become aware of the methods for achieving them, I intend to summarise some of the points of the mathematics teaching practice and curriculum reform in the People's Republic of China.

System of education is very complex, as it has to be in accordance with the socio-economical conditions of each country. Still, when searching for the best solution for improving our own curricula, we must also study from other countries' examples, even those that are very different from our own. Chinese educational system is different than Serbian in many aspects: structure of the school system, daily schedule of both teachers and students, and curricula.

However, as it will be shown from this overview of the Chinese educational system and their thoughts on textbook writing, there are many general concepts that are, or should be incorporated in our educational system and current efforts in the field of textbooks and curricula reforms.

The outstanding performance of Chinese students in international competitions or tests has also attracted author's attention, and is a second reason for the study of Chinese educational system.

2 Brief Overview of the Primary and Secondary Education in the People's Republic of China

The history of Chinese educational system is very long because Chinese civilization has about 5000 years. The ancient Chinese culture has a great influence on China's modern life, including the contemporary educational system. At the same time education, and especially mathematics education as its important part, is one of the basic supports for the development of China. The development of economy facilitates educational development – modernisation of schools, introduction of IT.

The Ministry of Education has the responsibility for education in China. The education system provides free primary education for six years (some provinces may have 5 years for primary school but 4 years for middle school), starting at the age seven or six, followed by six years of secondary education for ages 12 to 18. At this level, there are three years of middle school and three years of high school. The Ministry of Education reported a 99 percent attendance rate for primary school and an 80 percent rate for both primary and middle schools. Since free higher education was abolished in 1985, applicants to colleges and universities competed for scholarships based on academic ability. Private schools have been allowed since the early 1980s.

The system of public education in the People's Republic of China includes primary schools, middle schools (lower and upper), and universities. Nine years of education is compulsory for all Chinese students.

China has a vast and varied school system to provide for the needs of different parts of population. There are preschools, kindergartens, schools for the deaf and blind, key schools (similar to college preparatory schools), primary schools, secondary schools (comprising junior and senior middle schools, secondary agricultural and vocational schools, regular secondary schools, secondary teachers' schools, secondary technical schools, and secondary professional schools), and various institutions of higher learning (consisting of regular colleges and universities, professional colleges, and short-term vocational universities).

The two-semester school year consists of 9.5 months, and begins on September 1st and March 1st, with a summer vacation in July and August and a winter vacation in January and February. Urban primary schools typically divide the school week into twenty-four to twenty-seven classes of forty-five minutes each, but in the rural areas, the schedule is more flexible.

The school has two daily sessions. The morning session of four classes starts at 8:00 a.m. and finishes at 11:30 a.m. After a two-and-one-half hour lunch break, the students return to school for two more classes. The classes end at 3:40 p.m., followed by an hour-long, after-school homework session. Students leave school around 4:40 p.m. There are usually 40 to 60 students in class. But problems with discipline do not exist, teachers are respected, and students' behaviour is excellent.

The primary-school curriculum consisted of Chinese, mathematics, physical education, music, drawing, and elementary instruction in nature, history, and geography, combined with practical work experiences around the school compound. A general knowledge of politics and moral training was another part of the curriculum. A foreign language, often English, is introduced in about the third grade. Chinese and mathematics accounted for about 60 percent of the scheduled class time.

Mathematics is regarded as one of three major courses in schools. Mathematics curricula are based on cultural inheritance but also use all the advanced experiences in mathematics education from other parts of the world.

In primary schools the students are very active in mathematical classrooms, while in high schools, the classroom teaching is more teacher-centred. The reason for this change can be found in the fact that the students have to learn for the entrance exams for higher education.

The ancient examination system for selecting government employees, with its more than one-thousand-year history, is also one of the reasons why hard learning is accepted by all students. Any person, regardless of the family background, could become government employee once he had passed the examination. This possibility of getting a better position in society as a result of one's own hard work and learning still influences students to do their best at school.

Chinese mathematical education is connected with the so-called "Paradox of Chinese Math Learner". In everyday school practice there is a lot of memorization, and students often have to learn rules and formulas by heart, but Chinese students won a lot of awards in international competitions or tests.

3 Teachers

In Chinese history teachers had a prestigious position since the times of China's great philosopher – great teacher Confucius, entitled "The Teacher of all Ages". Confucius' lectures were in form of conversation, and he used to say that "Teacher and student promote and enhance each other". Nowadays, all over the world, we rediscover this method, and understand that the best and probably the only good method of teaching is the one which "promotes and enhances both the teacher and the student".

Mathematical teachers in China have the role of a guide in mathematical education, and their mathematical belief is to encourage the students to study hard, and they take responsibility if the students do not study hard and work well. Many Chinese sayings are concerned with teaching and studying: "If you don't climb a mountain, you will never know how high the Heavens are", "Teachers open the door. You enter by yourself", "Unpolished jade never shines", etc.

Mathematics education in China is based on basic knowledge and skills, followed by the development of creative thinking. In practice, these parts are not separated. Students are encouraged to memorize, compute both in mental and written forms. Computation and calculation are basic in Chinese mathematics education in primary schools.

During the weak all teachers arrive at school 15 to 30 minutes before 8:00 a.m. and leave around 5:00 p.m. each day. A significant part of their non-teaching time, about four and one-half hours, is devoted to preparing lessons, correcting students' homework, and dealing with student discipline problems. The teachers have the responsibility of everything in the class – he/she deals with all the problems that could arise with students.

Following the traditionally high status of teachers, teachers in China are respected in the society and they have the leading position in classroom. In addition to the existing respect, September 10 was designated Teachers' Day in 1985, the first festival day for any profession and indicative of government efforts to raise the social status and living standards of teachers.

To improve the quality of teaching, the government has started the Nationwide Program of Network for Education of Teachers. Its aims are: to modernize teachers' education through educational information, provide support and services for lifelong learning through the teachers' education network, TV satellite network, and the Internet; to improve the teaching quality of elementary and high school faculty through training and continuous education of high quality and efficiency. In concrete teaching, Mathematics teachers should communicate not only with each other, but also with teachers of correlative subjects in a greater degree.

In-service training for primary-school teachers was designed to raise them to a level of approximately two years' postsecondary study, with the goal of qualifying most primary-school teachers by 1990. Secondary-school in-service teacher training was based on a unified model, tailored to meet local conditions, and offered on a spare-time basis.

In-service teacher training for Chinese teachers is district or school based. A half-day is reserved each week for teacher training. During that time, students do not attend classes. The trainers are all master teachers and use only one centralized standard curriculum that is followed daily. This curriculum serves as a guideline for teachers' daily lesson plans. A master teacher contacts the weekly in-service training administrator, who attends the training session as a teacher. Most of the administrators have backgrounds in teaching mathematics or science. Teachers attend the training not for extra credits, for pay raises, or for teacher-certification requirements, as is often the case in some other countries.

4 Mathematics Curriculum in China

To better respond to the socioeconomic changes in the 21st century, China has carried out educational reforms. These curriculum innovations draw great attention worldwide. Previous national 9-year compulsory curriculum designed in 1992 was found to be inadequate in a number of ways: "... the contents and the ways of learning/teaching were overloaded and too difficult for pupils to complete. A lot of knowledge or curriculum contents were backward ... The curriculum ignored to some extent individual development and students creativity. The design and implementation of curriculum was too centralized, and the development and ability of locally-based and school-based curriculum were very weak... " [3]. Those were the reasons for changes of the curricula. The new Standards were developed in 2001 after consulting 1200 experts from different fields, such as: educational, psychological and subject experts. Many aspects of foreign curricula have been incorporated. Developers of Mathematics curriculum worked together with the developers of other curriculum standards and jointly conducted a lot of research. The curriculum reform preparation is also based on the experiments in some regions of China. The results were the *Guidelines for the Curriculum Reform of Basic Education*.

The Ministry of Education (MOE) developed national curriculum standards for all subjects. In the first 12 years the subjects are: Chinese, Math, Foreign Languages (*English, as well as Russian, Japanese, French*), and Moral Education for grades 1-

12, Primary Science (grades 3-6), Integral Science (grades 7-9), Physics (8-12), Chemistry (9-12), Biology (9-12), History (7-11), Geography (7-11).

In the past, the curriculum was centralized, i.e. subjects were the same all over the China. The idea of decentralization means that now the National curriculum takes no less than 80%, Provincial curriculum no more than 15% and School curriculum no more than 5%.

"Mathematic thinking", "sensibility and attitude", "knowledge and skill" and "problem solution" are listed as four key objectives, which materialize the priority on sensibility and thinking ability development of students and reflect the spirit of the time.

Following the new *Standard* students shall obtain necessary knowledge and skills, at the appropriate difficulty level with the goal to develop students' minds. But mainly, difficulty level was reduced. Also, the curriculum of mathematics is now more thought-oriented. The new math Standard puts emphasis upon the application of Mathematics, connection between Mathematics and living practice, and synergies between Mathematics and other subjects.

The new *Standard* pays respect to student's personality development, but it also tries to form student's responsibility to learn. Some interesting Math material can reinforce students' interest to learn Mathematics, as well as to develop the students' capability to analyze and solve mathematic problems. From this angle, the opinion of "learning Math happily" has positive meaning. But one must recognize that, for most math contents, students need to make efforts and some hard work as well.

Mathematics Curriculum Standard of High Schools aims at helping high school students develop calculation skills, math consciousness, space concept etc. The curriculum scheme encourages quick learners to learn more and prevent slow learners from being frustrated. Thus, high school math *Standard* proposes required math 1-5 and elective math 1-5. All students have to learn required math, but, if they are interested, they can choose any elective math to learn.

The objective of new math standards and textbooks proposes step-by-step development and takes into account the age and cognitive abilities of students. Context, essence and application have priority. The new math standard tends to promote integration of math and science, in order to foster student's scientific spirit and improve their hand-on competence. The new math standard improves learning activities of students, and fosters their awareness of originality and pays attention to an application of IT.

It should be noticed that the new Standard, according to the Chinese authority, should be revised because: "The new math *Standard* seems not good enough in the following aspects: the thorough understanding of international teaching practice as well as its integration with China's present situation; the *Standard* needs to reflect the spirits of the mathematics for the masses and with more elasticity. On the cultivation of students, too much emphasis is put on the respect over student's feelings but less on the sense of responsibility. In order to get rid of the above weaknesses, MOE has organized revision of the mathematics curriculum *Standard* and will publicize a new version in 2007."

5 Textbooks

With permission from MOE, one can edit textbooks according to national curricula. Textbooks have to be sent to MOE and get approved by the National School Textbook Examination Commission.

In the light of the *Standard*, some sets of math textbooks were compiled with fundamental nature, diversity, and selectivity, meeting the social requirement of the contemporary era, reflecting scientific advancement, attaching importance to the developing process of knowledge, and fitting in with learning of high school students. These textbooks cover required courses from Math 1 to Math 5, all together five volumes, and the elective course 3-4, "Symmetry and Group" one volume.

The set of math textbooks published by the People's Education Press has placed stress on embodying the fundamental nature, diversity, and selectivity of new course of high school math, comprehensively fulfilling the essential ideas related to math, Math course, Math learning, Math teaching, evaluation, and employing information technology.

The development of the teaching resources includes teacher's books synchronized with the compilation of textbook so as to incarnate the concept of new mathematics courses for high school.

Information technology is a powerful tool and has its place in textbooks. In textbooks there is a special column "Information Technology Application" which introduces the application of information technology. There is also one part starting with: "Calculator or computer can be used as well…" In consideration that China's vast territory has different economic levels the textbook presents specific requirements for the use of calculator and marginal notes for the contents to be completed by computer only. The teacher's and student's books will include complementary introductions on the use of information technology.

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Competence Approach in Mathematics Teachers Education[1]

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Abstract. The competence approach is generally recognised as the main component of the higher education during the last years mainly because it answers the demand of the labor market. Building key competences in mathematics meets great difficulties in preparation future math teachers because of the complexity of the basics that need a contemporary math teacher in his work. The experience of the author in dealing post-graduate courses is shared in the article. The interaction of key-competences is under consideration and the impact of the application of professional program packages in teaching process is in the focus of the discussion. The concept of *synthetic competence approach* is revisited in perspective of education of mathematics teachers. Two example of good practice are given, one of them is how to organize context-likely education in short-time courses.

Keywords: competence approach, context model, key competences, synthetic competence.

1. Introduction

The profile of the Institute of Mathematics an Informatics (IMI) of the Bulgarian Academy of Sciences includes post-graduate training and PhD programs for teachers in mathematics and informatics. This allows seeing from one hand the output of the university education of mathematic teachers and from the other hand the needs of the modern teaching practice

[1] This paper is based on the talk given at the first conference of TEMIT in Nis (May 2008).

in secondary school mathematics. Trying to manage the learning process for acting teachers one should keep in mind the present problem areas in mathematics education in secondary school (Lazarov, 2007). The complexity of secondary school mathematics education calls the need of some guidelines in organizing the postgraduate learning process of teachers. We adopted the competence approach as a conceptual base for this purpose.

2. The competence approach – general view

The competence approach stresses the educational process on the result at the end of the education. The main point is that not the sum of the knowledge acquired by the person but the ability to act adequately in different kind of problem situations is crucial for the successful social realization of this person. Ivanov (Ivanov at all., 2003) points four streams of competence approach to be performed.

1. Forming key-competences on meta-subject level as understanding text, dealing with information of different sort, working in group.
2. Forming general subject skills, in particular mathematical modelling.
3. Highlighting the applied aspects of the education including mathematical abilities on functional level.
4. Renewing the content of the education in everyday live direction.

It is easy to see that such point of view is good for the education of students of non-mathematical subjects but does not serves the aims of preparing math teachers. To clarify the previous sentence we step on the Program report (European Commission, 2003). There are eight key-competences (given below as Appendix) which are supposed to be formed at the end of the compulsory education. Since the math teacher is supposed to be a builder of some key competences she/he is expected to act on a higher level than the required output for the learners. It is pointed in the Program report that

Many of the competences overlap and interlock: aspects essential to one domain will support competence in another. Competence in the *fundamental basic skills of language, literacy, numeracy and in information and communication technologies (ICT) is an essential foundation for learning, and learning to learn supports all learning activities. There are a number of themes that are applied throughout the Reference Framework: critical thinking, creativity, initiative, problem-solving, risk assessment, decision-taking, and constructive management of feelings play a role in all eight key competences.*

Having in mind the listed above themes and the teacher's mission to build up them in her/his students, we advocate that the successful mathematics education of secondary school students could be performed by a teacher who has a kind of synthetic competence.

3. The synthetic competence

The concept of *synthetic competence* was introduced recently in the context of extracurricular activities of secondary school students and distance education for university students (Lazarov, Vasileva 2007; Lazarov, Uluchev, 2007). The synthetic competence in such a perspective was defined as

The ability of use mathematical knowledge (concepts and methods) combined with a level of skills to use computer technologies for solving problems; the ability of analyze the initial data of a problem and to transform them in appropriate form to apply computer technology; the ability to evaluate the outcomes and to manage their application.

But the concept of synthetic competence applied to the mathematics teacher education needs a complement of the above definition. The listed characteristics are necessary but for a mathematics teacher it should be taken into account a large scale of competences needed to deal with data sources, some "artistic" and presentation skills, etc.[2] A key knowledge is also the contemporary range of the program packages appropriate for a particular material in mathematics education. And here the point is to consider all of these knowledge, skills, and abilities in their interaction.

For having the whole picture it is important to clarify the place and the role of competences in didactics of mathematics. Since the main

[2] A comprehensive guide book in this direction is *I*Teach Methodology Handbook* (Sendova at all. 2007).

didactical principles were stated in the past and have relatively long evolution in non-computer era a lot of them need revision and renewing. Under revision should be also the main cell of the educational process – the lesson. But since we have not enough own research-based conclusions in this area we leave some space for future consideration of these problems and the inclusion of corresponding components of the concept of synthetic competence for mathematics teachers.

4. A sort of good practice

The teacher in mathematics A.G. is also postgraduate student at IMI. He teaches advanced students and has remarkable success – in 2007 one of his students gains the first price in an international contest for the projects about implementation of ICT in mathematics. As a supervisor of this particular student's project A.G. needs competences in some branches of mathematics, related to the extended mathematics curriculum but going far beyond it. He also needs competences in a software product by which a lot of illustrations have been done and even some conjectures were stated. Since the student's work on the project was during the whole term a special didactical technology was needed to keep student's activeness steady on high level for a relatively long period. The final product made by the student shows pretty high abilities in design of the applications that could be a kind of talent but more likely is a mastered skill in this area. After having a look at the references and the citation list of the project one can see a carefully selected collection of books and papers that points to a responsible attitude to the sources.

It is clear that A.G. is an experienced teacher who has competence to cover such a large area of necessary conditions needed to be supervisor of advanced (and gifted) students – in fact a kind of synthetic competence. A.G. proved this evaluation in the 2008 with acting as a supervisor of another two quite successful students' projects. Thus we can talk about a successful didactical technology for ICT based mathematics education in secondary school extracurricular activities. Unfortunately the didactical technologies applied in preparing projects as an extracurricular activity meets serious difficulties in implementation in everyday classroom practice.

But neither the deep knowledge of the extracurricular secondary school mathematics, nor other subjects directly related to the listed above abilities were included in the university curriculum when A.G. was university student. Fortunately, the situation in Sofia University now changes.[3] A lot of compulsory and optional courses in subjects that are in between the traditional and modern topics allow the preparation of the future teachers to face the challenges of the ICT organized mathematics education. In fact the competence approach is accepted in education of mathematics teachers in Sofia University.

5. The competence approach and the context model

The implementation of the competence approach takes more clear shape in post-graduate and doctoral courses for mathematics teachers at IMI. We share the ideology that the main goal of mathematics education with respect to ICT in upper secondary school should be to provide opportunities for the learner to build up:

- a terminology basis directly related to mathematical methods;
- a basis in mathematical methods that focus directly on the problem solving;
- competencies with respect to the scope of application of certain mathematical method;
- competencies with respect to the realization of certain mathematical method through software packages;
- competencies with respect to the quality interpretation of the results obtained after the application of certain mathematical method.

In fact, these five points describe the essence of the *competence approach* as a base for organizing the secondary school mathematics education with a significant ICT component. The practice for organizing further education of acting mathematics teachers, adopted by us, is directed to ensure a kind of synthetic competence, necessary to perform educational process in secondary school mathematics, which output guaranties the five points listed above.

[3] We refer to the talk of Ivan Tonov given at The first conference of TEMIT in Nis (May 2008).

Recently we knew about a model developed in Russia that is most appropriate for the realization of the competence approach. Sergeeva (2008) described so called *context approach* in the frame of the *competence model*. Since the paper contains only a brief list of the ideas we are not ready to discuss here what the competence model means. This is why we prefer to adapt the main results to our terminology and we are going to talk about the *context model* which is a way to put in practice the competence approach.

The elements of the context model we describe below are modifications of the ones, given by Sergeeva. For the purpose of mathematics teacher education we propose three general forms of activities.

- Attending basics courses given at lectures and seminars on the fundamental mathematical topics as set theory, mathematical logic, introduction in calculus, analytic geometry, psychology etc.
- Participating applied courses as didactics, didactical models, didactical technologies, software packages etc. performed in an interactive mode, perhaps a kind of laboratory activity.
- Doing practice and preparation of diploma thesis where the most of teacher's real work is modified in problem situations.

The above three form correspond to Sergeeva's three components of studying modules. The difference comes from the specifics of target groups. Sergeeva deals with the students in economics and our learners are future teachers in mathematics. Further we point some details about the three forms of activity.

5.1 Basic courses in the context model

The fundamental knowledge in mathematics for the future math teachers should be adapted for their professional needs. We advocate the implementation of program packages wherever it is allowed by the theory. For instance the largest part of the symbolic calculations in finding derivative or indefinite integral could be successfully performed by packages. This could set free some time for discussions about the conceptual base of the methods and the interpretation of the results. Another advantage of the implementation of program packages is the opportunity to visualize a large amount of examples which is impossible by paper-and-pencil technique, e.g. the behaviour of some implicit functions could be relatively easy shown by an animated graphic.

Another way of the context orientation of the fundamental courses is to remember Felix Klein's ideas about interpreting the *elementary mathematics from an advanced standpoint*. Our experience shows that highlighting the connections between the rigorous theory and the mathematics presented in secondary school is very fruitful in teachers' education.

This section of the context model supposes a directed education where the students are relatively passive side. However, the seminars allow students to participate more or less active in forming their basic knowledge in (higher) mathematics.

5.2 Participating applied courses

The ideology of the context model expects applied courses for mathematics teachers to be organized in an interactive form. The theory could be presented in a problem-solving mode where ICT are a significant part of the education. Learners are supposed to be active in taking decisions during the whole course but they still act reproductively.

Our experience in teaching applied courses shows that the motivated learners build up competences in the matter faster and more efficiently when they have a bit of free space for experiments. Since our learners are acting teachers their perspective is always to the direct application of the theory. On the other hand they often give ideas related to their own experience. It is another case with the university students but nevertheless we think that giving the learners the opportunity to express their opinion and to discuss over it is a key point on this stage of education.

This part of education should guarantee learners all components of the synthetic competence needed for the future mathematics teacher to be formed. The learners are expected to have whole concept base build up.

5.3 Practice and diploma thesis

The most significant form in the context model is related to the practice work. During the practice students are challenged to prove themselves in real work tasks. Putting a student in a problem situation he/she is urged to search general solution which calls his/her synthetic competence on test. This form allows to fill up most problematic gaps in general education and to complete the synthetic competence of the learner where it is necessarily.

A successful practice of IMI is postgraduate students to be involved in the projects of the Department of education. They are encourage to participate also in commissions, editorial boards etc. Such an inclusion allows learners to have a general picture on the educational process in secondary school, to build up criteria for external evaluation as well as self-estimation. As a collateral product learners get contacts with the experts from IMI for advises and help in the future (the competence to know who is the competent person in a particular area).[4]

During this final stage of education all components of the synthetic competence interact and the future mathematics teacher is expected to start a successful career. Further professional development of the teacher should not be affected by lack of competence.

We advocate the defend diploma thesis instead a heavy exam as the closure procedure of the education of future teachers to be accepted. Our experience shows that successful passing an exam does not guarantee competence to start teaching at school. Moreover, the final exam often includes topics from the whole course of education and the examinees are not allowed to use books or other sources of information. In fact they are tested both one-sided and restrictively which contradicts the ideology of the synthetic competence approach. All this disadvantages are skipped when a diploma thesis is defended after a serious preparation. As a bonus the university equips itself with a collection of didactical items some of which are appropriate for implementation in school practice.

6. Another example of (partially) good practice

The context model as it was described above requires the whole university course. However, some parts of the model were tested in 2008 during a short-time course with acting teachers. The main feature of the context model is the *problem situation* as a basic educational unit (Sergeeva, 2008). We examined how it works during a one-week course for teachers that are involved in extracurricular activities with advanced students.

[4] Similar approach is accepted in the University of Sofia as it was reported at the General meeting in Borovez, 2008.

Four pairs of lower secondary school teachers were asked to prepare educational module on different topics from a list of themes related to mathematical competitions – one to any pair. The teachers were given a book, containing papers about the topics under consideration (Makrides 2006). Any module is expected to be suitable for extracurricular activity appropriate for a particular target group – namely the teachers' own students. The structure of a module should contain a brief theoretical and historical introduction to the topic (not given in the book), collection of problems (most of them from the book) and a kind of check test (designed by the teachers). The pairs were asked to present their works in a well equipped cabinet simulating real class activity. During the presentation the role of the school students were taken by the other participants they could either propose solutions, or ask questions, or require additional explanations. Meanwhile some comments and recommendations were given by the lecturer (here he acts as a tutor – since the learners were experienced teachers a position of preacher is never comfortable for any lecturer). A final discussion was accepted both as summary and assessment.

The structure of the module was chosen by the following reasons. First teachers were given a quite well structured unit as a base – the themes from the book. There were also recommendations how-to-use the book's units. But the adaptation for a particular target group requires some changes to be made – some problems were removed and some others were included. The historical part of the introduction supposes a web search. The final test calls the use of electronic tables. The presentation could be performed by both paper-and-pencil and multimedia including online work.

The context-model-likely form of organizing the course was accepted with enthusiasm. The learners have already their own results in teaching and they got the opportunity to share both success and problem areas they had met. So the learners' activeness was high. High was also the willingness to apply ICT in the presentations but the result was far from satisfactory. The problems were specific for any pair of teachers and only the lack of routine was common. But the most slippery place was the lack of criticism in using the web-sites. Here the unreliable sources together with some gaps in fundamental knowledge lead to dangerous deviation in preparing materials for advanced students.

Nevertheless the results are optimistic – teachers demonstrate erudition but they show themselves also as open minded to the new trends. And most important – they recognize that the synthetic competence is crucial for effective, attractive and as result successful education.

7. Conclusions

The competence approach in higher education is the answer of the demand of the labour market – no doubt. But finding appropriate models for realization of the competence approach is in progress. We have no witness for the effectiveness of the context-model yet. But it is an alternative to the classical model of forming fundamental knowledge and skills by lectures and seminars. Our observations say that the output of the classical model applied to preparation of future mathematics teachers is not satisfactory. Just-finishing-university teachers are not ready to start teaching ICT based mathematics at secondary school. More over, they have no guidelines to improve their competence. More ambitious try to perfect their skills in ICT which, in our opinion, is dead-end road – students are and always will be a step ahead.

Our view-point is that mathematics teachers do not need skills but competences – they act most as supervisors then as instructors. Teacher's role seems like the conductor of the orchestra – he does not play the most of the instruments but he knows exactly when and how to include any of them to obtain a harmony. The skills required in a particular key-competence are not obligatory for having synthetic competence. The teacher is not supposed to master computer skills on the level that is usual for the new generation. But she/he needs a deep understanding of the mathematical concepts and a general view on the mathematical methods. Also she/he is expected to know where and how some program packages could be applied – all this from the perspective of a didactical technology.[5] Teacher needs competence to present a mathematics topic in attractive mode but not loosing the effectiveness. She/he should have a reliable data-base of sources and the competence to evaluate the reliability of the new ones.

We believe that the competence approach could be accepted as a basis in designing the new curriculum for mathematics teachers and the building of synthetic competence as a goal could be declared. Taking into account the long tradition of the University of Belgrade and the University of Nis in producing high qualified mathematics teachers we advocate an evolutionary turn to the ICT organized education and secondary school teaching practice in mathematics. The main ideas of the context model seem to be appropriate for realisation of the competence approach.

[5] A lot of examples for MATHEMATICA-based applications were presented by colleagues from Novi Sad in the November-2008 meeting in Belgrade. So it stand the question how this good practice could be turn to a regular practice.

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Appendix

From the EC Report
(http://ec.europa.eu/dgs/education_culture/publ/pdf/ll-learning/keycomp_en.pdf):
the Reference Framework sets out eight key competences:

1) Communication in the mother tongue;
2) Communication in foreign languages;
3) Mathematical competence and basic competences in science and technology;
4) Digital competence;
5) Learning to learn;
6) Social and civic competences;
7) Sense of initiative and entrepreneurship;
8) Cultural awareness and expression.

The key competences are all considered equally important, because each of them can contribute to a successful life in a knowledge society.

Reflecting Mathematics Teaching Practice

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Abstract. Teaching practice is generally recognised as an important activity during the preparation of prospective teachers. Effective teaching practice develops the student's ability to learn from his/her experience by reflecting on it. A research at the Faculty of Education in Ljubljana has shown that participants that are involved in prospective teachers' teaching practice (university teachers, mentors in institutions, prospective teachers, and others) put different emphasis on reflection and show different understanding of reflection of teaching experience. It was also found that prospective mathematics teachers' ability to reflect teaching practice does not improve during their study. In the article we present the importance of an appropriate theoretical conceptualisation of the teaching practice and in particular of the reflection on the teaching practice. Developing the ability to reflect experience should be a constituent part of the formation of prospective teachers and should be considered in planning students' activities in teaching practice.

Introduction

As mathematicians would say, knowing mathematics is a necessary, but not sufficient condition to be a good teacher. And knowing about how to teach mathematics is a necessary, but not sufficient condition to be able to teach mathematics. In fact, teaching mathematics is related to a kind of "*knowing in action*", specifically in the context of the learning process in the classroom environment. In order to develop such "*knowing in action*" all educational program for prospective mathematics teachers include, in one way or another, some forms of teaching practice, in which prospective teachers develop competencies related to "*knowing in action*".

Students' teaching practice is usually implemented as various kinds of activities in real (and sometimes just realistic) classroom environment. Taking as criterion the level of involvement of prospective teachers in the teaching activity we can speak of three basic forms of teaching practice:

- attending (observing) someone else's teaching,
- preparing and carrying on a lesson in a real classroom under close supervision of the faculty staff, and
- working for a period of time in a school or in some other institution under the guidance of a mentor.

The first two forms of teaching practice usually occur in parallel with student's regular study at the faculty, in both cases the students work together with faculty teachers and with mentors at visiting institutions. In the third form of teaching practice the students move for a certain period (2 – 6 or more weeks) to an institution (school). During this period they work in the institution under the guidance of a mentor. There are indeed many ways in which the three mentioned forms take place (e.g. the working practice may take place in some consecutive weeks or just on one day in a week during a whole semester). Besides the three mentioned forms, there are several other practice-oriented activities, e.g. simulation at the faculty of school-teaching situations, various types of visits to educational institutions.

The aim of teaching practice is not just seeing something or gaining some experience in classrooms. Prospective teachers are also supposed to learn from their experience by analysing in and reflecting on it. In this article we present the importance of an appropriate theoretical conceptualisation of reflection on teaching experience.

Some theoretic considerations

Though it seems obvious that teaching practice is essential in the formation of prospective teachers, it is not easy to set a theoretical basis for student's learning during teaching practice and we do not even attempt to give a generic answer to this issue. Common ideas about teaching practice, e.g. that teaching practice »empowers students' knowledge« or that mediates between the previously learnt theory and working practice, are, to be honest, more justifications of established programs for teacher formation than explanations of the learning process that occurs in teaching practice. Rather we shall focus on a specific question: *What is the role of reflection in student's learning during teaching practice.*

We start from the well known dichotomy concerning the explicit (prepositional) knowledge and implicit (tacit) knowledge. The distinction between these forms of knowledge can be found in (Ernest, 1998; Magajna, 1998) for school mathematics context, and in (Berliner, 1994; Korthagen, 2001) for the teaching context. In teaching practice prospective teachers learn implicitly since the object of prospective teacher's learning is not in the focus of his attention. In fact, during the work in a mathematics class the prospective teacher's attention is focused on the students' learning of mathematics class and not on his/her learning of teaching. Though implicit learning over a long period of time may result in good performance, it is important that students make their implicit knowledge explicit and learn effectively from their experience. Reflecting is a kind of reasoning, however in this article we shall restrict ourselves to the act of reflecting experience as means to make implicitly learnt knowledge explicit.

We thus consider reflection (here) as reasoning about experience, and this clearly involves implicit (tacit) and/or explicit (prepositional) knowledge. Our approach to reflection will be based on activity theory (Wertsch, 1985). According to activity theory, actions are the considered the constituent units of mind. Actions are always goal oriented, i.e. a subject does an action in order to achieve a goal – note that goals usually emerge without being noticed as one is involved in an activity. Actions are only meaningful in the context of an activity (e.g. school learning). It is plausible that reflection that happens *in action* occurs at a rather implicit level. During a work in an activity (e.g. teaching in a class), goals constantly emerge and one has to select actions to achieve goals and modes of carrying on the actions. Thus, in general, reflecting in action is reflecting about modes of achieving goals. On the other hand, reflection that does not happen within action, i.e. reflection *on action* may involve a broader view about goals or even about activity in which actions occur.

Reasoning on how to execute an action (i.e. about the means to achieve a goal) is referred to as *technical reflection*. Reasoning about goals, their appropriateness, their relevance, causes and consequences is called *practical reflection*. Finally, interpreting the goals in the context of activity (i.e. questioning about adopted conventions, tools, artefacts) is referred to as *critical reflection.*

A related hierarchical categorization of the level of explanation of experience was developed by O'Houlon. Starting from the most simple, the levels of explanations are: *description* of modalities of executing actions (facts, feelings), *interpretation* of actions in terms of goals (why an action was carried on), *reflection* on the goals (deduction, evaluation, hypothetical situations), *integration* of experience and theory (viewing experience holistically in the light of different theories and critically considering the contextual activity).

The way in which students and mentors reflect on teaching experience depends on how they understand the mentor's role. In this respect Maynard and Furlong (1994) distinguish three basic models that describe the relationship of mentor's and prospective teacher's role in teaching practice.

- The *apprenticeship model*, in which mentors acts as models, is based on transferring established practices in an activity (we ignore here other dimensions of apprenticeship). Basically, the role of the mentor is to act as a model for the prospective teacher and to direct him what to do and how to act in various situations.
- The *competence model*, aims at competent decision-making and professional functioning of prospective teacher in teaching practice. Mentors provide appropriate learning situations and give suitable feedback to the prospective teachers.
- Finally, in the *reflective model* mentors act as colleagues who help prospective teachers to make sense of their experience and to find ways of professional improvement.

There is, obviously, a great deal of simplification in the above description, nevertheless one is tempted to associate the apprenticeship model to technical reflection, and critical reflection to the reflective model.

Using the theoretic model to research reflection in teaching practice

We illustrate the use of the described theoretic model with two excerpts from a research project *Partnership of Schools and Institutions* carried on by a group of researchers at the Faculty of Education in Ljubljana and several related educational institutions in Slovenia. One of the researched topics was the participants' perception of various elements of teaching practice of prospective teachers. For this purpose 327 mentors in institutions and 32 faculty teachers responded to a questionnaire about existing teaching practice of the prospective teachers. The questionnaire consisted of a number of closed questions, evaluations on Likert type scales, and open type questions. In many respects the views of the two groups of respondents on teaching practice were very similar. But there was a relevant difference that was detected on several questionnaire items that were directly or indirectly related to reflection of prospective teachers' experience. In such questions the answers of the mentors consistently pointed to a pattern, different from the pattern of the faculty teachers. In terms of the presented theoretic model the difference can be formulated as follows: mentors (at schools and institutions) put more emphasis practical reflection while faculty teachers emphasize technical and, possibly, critical reflection.

In fact, in one of the questionnaire items the respondents were asked to choose two aims they found most relevant among several listed aims of the teaching practice. The results of the categorised answers of the respondents (Table 1) indicate that mentors, in contrast to faculty teachers, consider teaching practice from the viewpoint of their daily work in the institution. They find practical skills to be of great importance, almost in the centre of the teaching practice. Faculty teachers, on the other hand, emphasize students' cognitive development, and, compared to mentors, put more emphasize on reflection of teaching experience and on the motivational aspects of teaching practice.

Table 1. Perception of the aims of teaching practice by faculty teachers and mentors at institutions. Each respondent chose at most two aims.

Categories of aims of teaching practice	*Faculty teachers (N=327)*	*Mentors (N=32)*
Relating theory to practice	50%	62%
Learning practical skills	**38%**	**62%**
Becoming acquainted with the cultural milieu of the institution and with the specifics of the workplace	31%	35%
Learning by reflecting one's own or someone's else experience, experimenting	**31%**	**14%**
Increasing motivation for the study (experience functions as a motivation for further study)	31%	5%
Assessing the level of achieved teaching competence	19%	0%
Other aims (learning about technology; personal development)	13%	4%

In another series of questionnaire items we investigated expectancies and satisfaction of the faculty staff about various aspects of the work of mentors in institutions. In most aspects the satisfaction of the faculty staff was reasonably correlated to the expectancies. However, again, in two areas the situation was different. The first area is mentors' work in helping prospective teachers to prepare a lesson and to execute the lesson. Both, mentors and faculty teachers find this area important, and mentors fulfil the expectations of the faculty teachers. The second area is about analysing lessons, reflecting experience, and evaluating the prospective teacher's work. Here the expectations of faculty teachers are very high, but the mentors' contribution is, according to faculty teachers, modest (faculty teachers infer about the level of reflection of prospective teacher from various documents, diaries, etc.). Again, we can observe that mentors view teaching practice, and in particular the reflection of experience, on a practical level, while faculty teachers emphasize also the technical and critical level of reflection. Similar observations emerged in some other questionnaire items. Note that, although faculty teachers find reflection to be very important, they provide the prospective teachers and mentors with little or even no guidance for analysing and reflecting teaching practice experience. They simply assume that mentors and prospective teachers are able to properly reflect teaching experience.

Using the theoretic model to evaluate prospective teachers' progress

The described model of reflecting teaching practice was also used for evaluative purposes. In year 2006 the teaching practice for prospective mathematics teachers took place only in years 3 and 4 of their study at the faculty, we wanted to know whether the students in year 4 reflect their teaching experience at a higher lever than their peers in year 3. For this purpose prospective maths teachers in years 3 and 4 were systematically asked, *as they attended maths lessons of their mentors*, to formulate and addressed to the mentor just one question (per lesson). In this way 557 questions, which served as units of analysis, were collected from about 100 students in their 3rd and 4th year of study. The questions were categorised according to the level of requested explanation and to the content of the question.

Table 2. Requested levels of explanation in questions (N=557) that prospective teacher asked their mentors about the observed lessons

Level of requested explanation	*Year 3*	*Year 4*
Description		
Interpretation		
Reflection		
Integration		

As expected, prospective teachers formulated no questions that related observed lessons to theoretical issues (integration of theories and experience). At least in part this is due to the fact that the addressees of the questions were mentors and not faculty teachers. The distributions in Table 2 for years 3 and 4 are significantly different (χ^2 is significant at level 0.018), but note that the questions in year 3 ask for a higher level of explanation than questions in year 4. The probable reason is that currently there was no special emphasis on developing reflection in prospective teachers and that the main course in didactics of mathematics took place in year 3.

Here are some illustrative questions of the various levels. Questions at the level of description ask for modes of operation i.e. what to do in given situation and how to do. Examples:

I've noticed that you stricty use mathematical terminology. Do you allways speak in this way?

How do you react if a pupil does not do his homework?

If pupils have troubles with some homework problem, do you discuss about it with pupils or you show the solution.

Questions at the level of interpretation ask for explanation of the goal of an observed action, i.e. what is the purpose of a given action, why mentor did something. Examples:

Why did you split the class in three groups of different sizes?

Why were the pupils asked to estimate the circumference of that object and not to measure it?

What did you intend to acchieve by asking pupils how to find out the volume of that irregular body?

Why have you ignored the pupil who complained that his triangle is not as it should be.

Questions at the level of reflecting goals of actions ask about hypothetical goals, compare possible actions to achieve a goal, question the appropriateness of a goal. Examples:

Would the pupils be able to develop the formulae for the 6-edged pyramid without your help?

During the class you found that the OHP is broken. How did you feel and what possible solutions you had in mind?

You started the class by asking pupils about the properties of circles. Do you think this is appropriate also for more able and for less able pupils?

You explained % in this way (...), while I did this that way (...). Which one do, according to you, the pupils understand better?

Finally, questions at the level of integration ask about explanations of observed phenomena in terms of a theory, compare various theories, question the conventions, social roles, used tools, ways in which mathematics is learnt in schools. As mentioned the students did not ask such questions to their mentors.

Table 3 shows the distribution of questions with regard to content categories, more precisely with regard to the object in the centre of the question. Note that in the analysis more categories were used, but since the questions related to pupils' understanding, their actions and behaviours, legal, and organisation questions were sporadic, all such questions were joined to a single category. A qualitative analysis showed the prospective teachers were, in general, asking relevant didactic questions about methods of schoolwork, pedagogical and psychological issues. On the other hand, most questions are teacher-oriented and from year 3 to 4 there is no significant shift of attention from teacher's actions to pupils' understanding, which came as an unpleasant surprise.

Table 3. Content of the questions (N=557) related to lessons observed by prospective teachers

Teacher actions related to methodological issues	51.9%	47.3%
Teacher actions related to psychological and pedagogical issues	24.9%	27.3%
Pupils' actions, pupils' understanding, legal and organisational issues	23.2%	25.4%
Total	100.0%	100.0%

Conclusion

Teaching practice is generally recognised as an important activity during the preparation of prospective teachers, and reflection is generally recognised as an important element in teaching practice. However in the implementation of the teaching practice the participants are too often concerned with organisational and technical aspects of the endeavour. A research at the Faculty of Education in Ljubljana about teaching practice of prospective mathematics teachers has shown the importance of an appropriate theoretical conceptualisation of the teaching practice and in particular of the reflection of the teaching practice. In the article we present some theoretical considerations on reflection and ground them on the theory of activity. A good theory of reflection is important for several reasons. First, it allows finding out by appropriate research methods facts related to reflection. Second, it allows evaluating the quality of reflection (on teaching experience) of prospective teachers and in this respect to follow their development during the study. Third, it allows conceiving models for the preparation of prospective teachers and their mentors that would develop the quality of reflection of prospective teachers and improve their experiential learning.

A study at the Faculty of Education in Ljubljana has shown that teaching practice of prospective teachers is carefully planned and, in general, the realisation of the teaching practice is good, due to considerable efforts of the university staff, mentors and prospective teachers. But the research brought to light some problematic areas, and two of them concerned participant's reflection on teaching experience. First, mentors consider practice and, in particular, reflection of teaching experience, in a different way than faculty teachers, which causes a sort of tension between participants. Second, during their study the prospective teachers (at least in some programs) show no qualitative progress of the level of reflection of teaching experience. Thus a model of teaching practice that would elicit different levels of reflection and would also consider the cognitive and professional development of the prospective teachers in this respect was needed. Currently we are developing models of teaching practice that would take care of both needs.

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Towards an Integrated Course in Didactics of Mathematics

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Abstract. The present text contains an extended version of the lecture delivered at the Tempus meeting in Jena, June 4 – 8, 2008.

Mathematics is often seen as something very fixed and fossilized. From such a stand point it follows that school teachers have to learn only one thing more – how to transfer it to their pupils. This is particularly true for traditional books on didactics of mathematics which, besides of being rather preaching, they do not enter into subject analysis either. On the other hand, a far reaching reform of school mathematics, known as "New math", had for its primary aim reorganization of school mathematical content based on set theory and abstract structures. Lacking a clear didactical ground, this trend was doomed to failure. Nevertheless, a good organization of the subject matter and the idea of structuring are enduring contributions of this trend to didactics of mathematics.

The II ICME, Exeter, 1972, was a turning point, when and where formalism was exposed to a sharp criticism and intuition stressed as an essential constituent of knowledge. In the time that followed, a variety of diverging trends have appeared:

overemphasis on "life value" of mathematics,
consideration of mathematics as a "tool subject",
promotion of general modes of thinking (piles of intellectual games with uncertain educational effects),
cutbacks in syllabi and inclusion of new more pragmatic contents, etc.

All such tendencies lead astray, ignoring the developmental role of mathematics as a central school subject.

For our further considerations, let us notice that all contents of school mathematics have one of the following aspects:

historical – when they are seen as once they were, *in statu nascendi*,

scientific – when they are seen exposed in a logically compact way,

didactical – when they are transposed to be suitable for learning

Related to the widely accepted principle that ontogenetic development follows phylogenetic one, some acquaintance of teachers with main facts from history of mathematics (and education) is indisputably important. A proper selection of these facts should always be dependent on the specific subject matter.

A good acquaintance with the scientific aspect serves a specialist in didactics of mathematics to be able to undertake a logical analysis of the relevant contents. To say it in other words, such knowledge is the ground upon which the meaning of the fundamental mathematical concepts is sharply established, which helps the teacher avoid formation of quasi-concepts and possible shifts in meaning (so often present in primary school textbooks). To some degree, such knowledge should be considered as useful even for primary school teachers.

If history of mathematics and mathematics as science are two pillars which support didactical analysis, the third is the science of learning and experience of teaching. Investigations of the ways we perceive and conceive were traditionally a preoccupation of philosophy. Nowadays, exposed to experimental verification, it is predominantly the subject of psychology. Figuratively speaking, the psychologists now provide us with the landmarks which outline terrain of our interest, but they cannot tell us how to pave it. And the way how this paving is done is learnt through didactical analysis of the whole subject matter.

Without any doubt, the following didactical tasks should be respected:

Mathematics contents have to be elaborated and didactically transformed with coherence and order at all levels of teaching and learning,

Teachers have to know the school contents of mathematics in a deepened way, what is an essential prerequisite for successful transferring of that knowledge.

To deepen understanding, teachers have to get acquainted with creation of concepts and their systems throughout the whole course of their development, starting with the intuitive ground and going to higher levels of abstraction. Following Freudenthal ([1], [2]), we understand **didactical analysis as examination of teaching themes trough their decomposition into constituent parts and then, their reorganization according to the aimed didactical tasks**. Such an analysis is a complex piece of work which combines historical and scientific aspect of treating topics of interest as well as the study of those processes related to the intellectual development of the learner. Integrated into one course (and taught by one person), these areas of knowledge taken up to the measure of necessity, are the basis for didactical analysis of the subject matter. Without any doubt, this analysis should be the core of each course in mathematical didactics and, as such, it should enable teachers to form a masterly knowledge of the subject matter and make him/her be able to follow didactical transpositions of the teaching themes with a full understanding. In what follows, we exemplify didactical analysis in the case of some key themes of school mathematics: Set Theory, (Logic, Development of Number Systems, Variables), etc.

1. Set Theory

Elements of set theory, once widely present on school curricula, now as a result of counter reactions on the New Math trend, they have been either completely banished from elementary school contents of mathematics or they have found their right place and didactical shaping. (By the way, this possibility of right didactical transformation is often overlooked by those professional mathematicians who identify school mathematics with the foundations of their own discipline). We find that Serbian syllabi indicate the place and suggest the corresponding didactical shaping of set theoretical contents in a right way, but still let us ask what is the number of school teachers who do understand it properly?

As for didactics of mathematics, **the general language of set theory is an indispensable means of unifying descriptive situations in the underlying phenomenology**, helping so a teacher see the general in the mass of diverse particular cases.

Facing the problem of logical foundation of the real number system, R. Dedekind was led to create set theory to be a language more general than the traditional languages of algebra and geometry had been and with the **concept of set being more general than all other concepts of classical mathematics**. Examples related to this concept exist at all levels of abstraction and its full meaning develops gradually with the broadening of the class of these examples. We sketch here three stages of this meaning that are found in Serbian school practice..

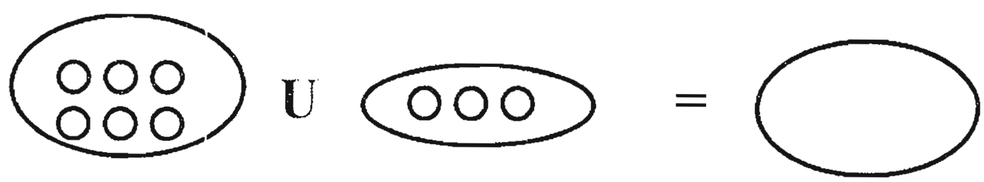
1. 1. Sets at sensory level

The first steps in teaching arithmetic have always been related to sets being the groups of real objects (beads, counters, etc.), no matter if the word "set" has or has not been explicitly used. Many words in natural languages, for example in English, the words "flock", "flight", "shoal", "bundle", etc. mean exactly the same as the word "set" does, except that they specify the nature of grouped elements, for example, "goats", "birds", "fish", "banknotes", etc. And in no natural language the universal meaning of the words "set" and "element" has been developed. Nowadays, teaching of mathematics in school creates that universal meaning, whereupon it enters into spontaneous use.

A set is at sensory level when all its elements are visible things (including also their pictorial representation). Drawings representing arrangements of dots, circlets, etc. are also examples of sets at sensory level.

Two physical objects occupying different places in the outer space are naturally considered to be different. Similarly, in the case of pictorial representation, **two iconic signs occupying different places should be considered to be different**. When this requirement is not respected, various confusions may arise, producing blunders which are often found in primary school books. Besides being "barbaric" (a mixture of iconic and syntactic signs), the examples that cause quite a lot of confusion are, for instance, the following two:

$$\{ \triangle, \bigcirc, \square \} = \{ \bigcirc, \square, \triangle \}$$

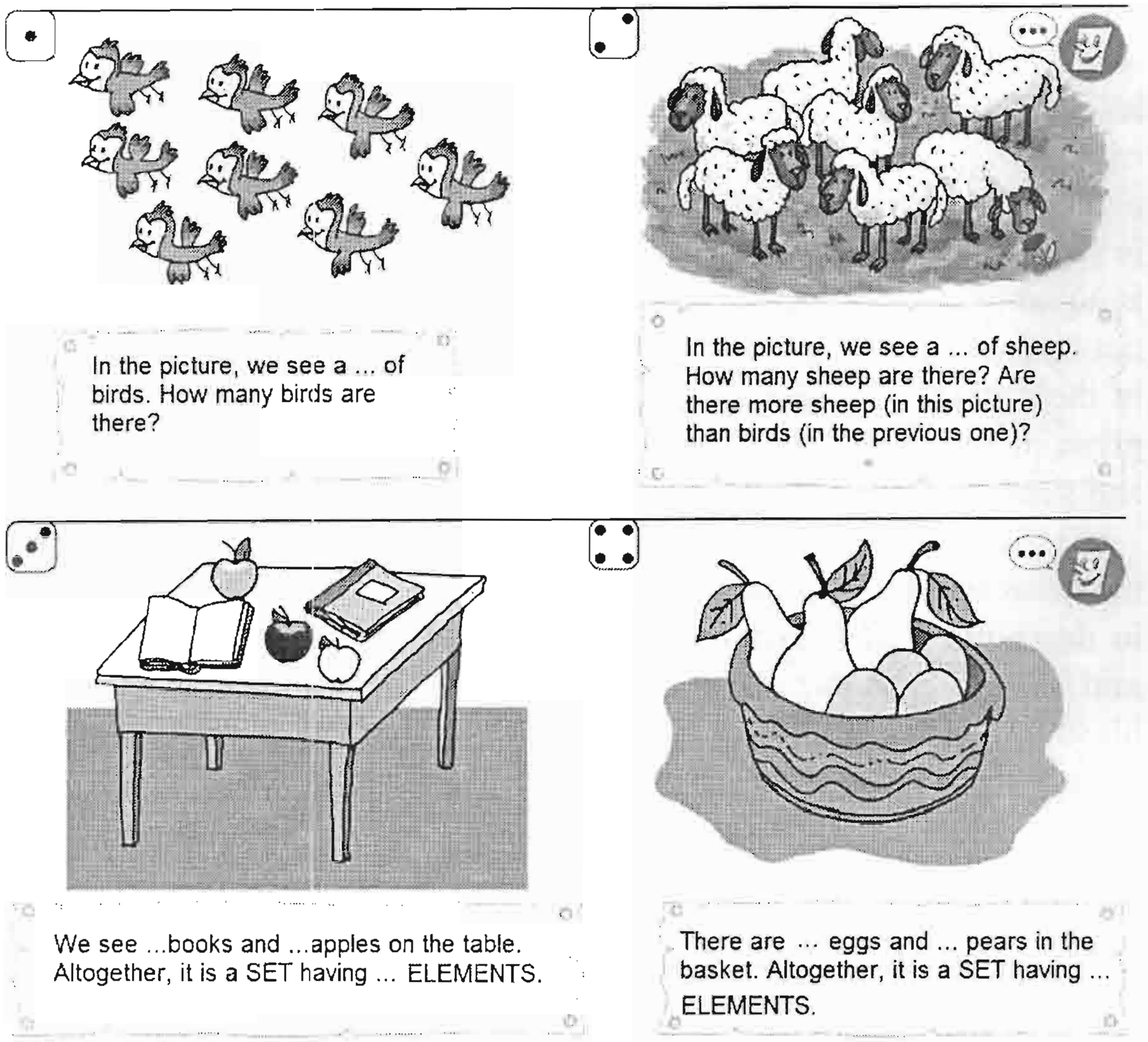


(In the first of these examples, one could not see the ground upon which two shapes are taken to be equal. Their alignment excludes the idea of congruence; their understanding to be symbols for concepts of triangle, square and circle is an unusual labeling, etc. In the case of second example, transportation of all given circlets into the blank, enclosed space is evidently the expected solution of this "equation". Then, each circlet is equal to its transported copy (why?), while pairs of other ones should not be (why?). Those who compose exercises of this kind evidently have in their mind the activities of rearrangements of plastic chips, what the given pictures cannot suggest. Arising cases of confusion caused by this and similar violations of rules of iconic representation have no excuse.)

At this stage (first two classes of primary school) no set theoretical notations should be used. Simply, the **words "set" and "element" are used in description of scenes belonging to the underlying phenomenology and children are let assimilate their meaning spontaneously**. Let us illustrate it by a lesson from the very beginning of arithmetic teaching, discussing first the teacher's preparation for this lesson.

The teacher should understand that

- this lesson belongs to the theme "Counting",
- children know to recite number names up to 10,
- the process of counting already starts to develop the idea of number,
- this process begins with observation of collections of concrete objects (sets at sensory level) and then it is proceeded by **neglecting of the nature of these objects** and **of any kind of their organization** (Cantor principle of invariance of number),
- didactical aim of this theme is to let **children assimilate the meaning of expressions "how many", "altogether", "to remain"**, etc. and their **answers will be given by word of mouth** (orally), without writing (or reading) numbers,
- in this way **children perform set theoretical operations as mental operations** expressed in words of natural language,
- this **theme is introductory for the next one "Block of numbers up to 10"**, when notations for numbers are introduced, meaning of operations "addition" and "subtraction" is established and when children train their unsteady hand writing first arithmetic codes (digits and operation signs).



In set theory, product of cardinal numbers is defined via the direct product of sets. In arithmetic, **product of natural numbers relates to multiplicative schemes** representing *m* sets, each of cardinality *n*. A very convenient example of such schemes is rectangular arrangement of uniform iconic signs as the following one is

O O ... O
O O ... O
...
O O ... O

Such **iconic representations are conveyors of meaning and, at the early level of learning mathematics, they play the role analogous to definitions at the higher levels**.

1. 2. Elements being conventional symbols

The second stage begins with the **first use of set theoretical symbols**: brackets, symbol for "belongs", "does not belong", **and when the elements of sets are conventional signs** – symbols for numbers and letters. It is important to say here that the **shape of conventional signs serves only for their recognition and that such two signs are equal whenever they mean the same** (represent the same number, letter, etc.). A reasonable motivation for the use of these sets is the solving of simple inequalities: $a + x > b$, $a + x < b$, $a- x > b$, $a - x < b$, etc. where the sets of solutions have for their elements natural numbers represented in decimal notation (3rd and 4th class of primary school).

At this stage sets are also denoted by single capital letters, the operations "union" and "intersection" as well as the relations "equality" and "inclusion" are introduced for the first time (5th and 6th class of elementary school),

A particularly subtle technical task is the **use of notations when the elements of sets are written in the form of a sequence and when ellipsis is used to suggest ordering according to the law of general term. Ellipsis is also used to suggest an unbounded ordering of elements** and the first idea of infinite set is conceived in the children's mind with their acquaintance with the set N of all natural numbers. The subtlety of the **use of ellipses is understood by teachers as soon as they see their logical function of being an implicit form of expression of the principle of total induction.**

An often encountered **misunderstanding of Venn's diagram is the situation when they are prematurely used in the form of regions in the plane**. Namely, all geometric objects are continuous entities and they cannot be taken as sets of points before the system of real numbers is built and the correspondence with the points of number line established and, in the case of plane, before the coordinate system is introduced.

1. 3. Sets given by predication

The next stage in development of the concept of set begins with **examples of sets given by predication of characteristic properties** of their elements. Let us denote the predicating property by $p(x)$, then

$$S = \{x \mid p(x)\}$$

denotes the set of all x (belonging to a universal set) which have the property expressed by $p(x)$. The first examples of such representation of sets are usually very simple, for instance,

$$S = \{\, x \quad N \mid x > 5\},\quad S = \{\, x\ Q \mid\ x < ¾ \,\},\ \text{etc.}$$

By matching sets and set theoretical operations and relations with the corresponding predications, the sharp logical meaning of the conjunctions "or", "and", "if ... then ... " is established:

A -------------------------------- $p(x)$

$A \cup B$ -------------------------------- $p(x)$ *or* $q(x)$

$A \cap B$ -------------------------------- $p(x)$ *and* $q(x)$

$A = B$ -------------------------------- $p(x)$ *is equivalent to* $q(x)$

$A \quad B$ -------------------------------- $p(x)$ *implies* $q(x)$.

In logic, $p(x)$ is called a predicate or a propositional function and in the case when A and $p(x)$ match, A is called the truth set of $p(x)$. In the real teaching situations, $p(x)$ is taken to be a **condition** and A the set of **all** ***x*** **which satisfy that condition**. Practically, in the teaching situations, $p(x)$ is an equality or an inequality or else a sentence expressing a property as, for example, the following one: *x is a whole number divisible by 5*, etc.

The above correspondence between sets and related conditions should be **elaborated procedurally**, i. e. in the form of concrete exercises. If it starts with the sixth class syllabus, it **should be continued throughout all subsequent syllabi and should permeate the whole content of mathematics**.

These considerations lead us evidently to the discussion of basics of logic, no matter if they exist or do not exist on some school curricula. But to end this teaching theme we include a case from our personal experience. Teaching a digested course of logic for students at teacher training faculty, I assigned them with a list of exercises also including the following one:

Prove that $x^2 + x - 2 < 0$ *implies* $|x| < 2$.

A student brought a solution on two pages, with a long sequence of implications, also admitting that she had been helped in doing this exercise by a mathematics teacher. Students of mathematics often learn logic very formally, what this solution evidently proves.

(The expected solution runs as follows: The set A of solutions of $x^2 + x - 2 < 0$ is the interval (-2, 1) and of $|x| < 2$ is the set B, being the interval (-2, 2). Since A is contained in B, the above implication is proved to be true.)

References

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... what ...

(1) The expected solution runs ...

$x - 2 = 0$ in the interval (2, 1) and ...

(-2, 2). Since it is continuous in ...

... one ...

Ref ...

[1] Freudenthal H., Mathematics as ...
Publishing Company, 1973.

[2] ... Waerden and ...
19[illegible]

Competences for Computer Science Teachers

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Abstract. *In the process of modernisation of study programs at the Faculty of Education a set of nineteen competences was defined for future teachers of computer science in primary and secondary schools. Competences comprehend expert and didactic knowledge and skills from the subject field and pedagogical and psychological knowledge for teaching. We carried out an inquiry about the achieved level of these competencies in the current program and about the desired level of the competencies in the modernised, "Bologna" programs among graduates of our current study program - "teachers", their headmasters - "employers", and teaching staff in the study program - "academicians". By means of the analyses of results of the poll we found out some highly desired and relatively poor achieved competences. This was important information for the modernisation process.*

Introduction

In the process of modernization of study programs at the Faculty of Education, University of Ljubljana, after careful study of the results of Tuning project (Gonzales & Wagenar, 2005) and a wide discussion in Slovene academic sphere, we selected a set of nineteen competences, which are crucial for teachers of computer science in primary and secondary schools in Slovenia. We carried out opinion poll among interested professional groups (i.e. teachers, headmasters, and people from research organizations and academia) to find out to what extent these competencies are achieved in current programs and what are the expectations for the modernized study programs.

Competences generally comprehend expert and didactic knowledge and skills from the subject field and pedagogical and psychological knowledge for teaching. They are defined (Gonzales & Wagenar, 2005) as dynamic combination of knowledge, understanding, skills and abilities. Competences are formed in various course units and assessed at different stages. Competences are related to five different areas:

(1)personal,
(2)development,
(3)professional,
(4)social,
(5)action.

Personal comptences are related to determination, self-confidence, intelligence, initiative, responsibility, sincerity, confidence, communicativeness and other values of the teacher.

Development part enables evolution of teaching and professional abilities of the teacher, leads the realization of the teaching process, enables the creative use of new findings related to the profession and allows recognition of the personal needs of the pupils.

Professional competences comprehend the pedagogical, psychological, philosophical and other knowledge needed for work in the classroom, and incorporate the creative use of time for realization of the subject-specific goals.

Social part of the competences is related to the techniques of communication, interaction, involves knowledge about problems solving, motivation and organizing team work.

Action part is related to the practical activities of the teacher in and out of the school

The set of nineteen competences (**Table 1.**) was divided into generic competences (competencies 1. - 9.) and subject-specific competences (10. - 19.) for the field of computer science. Generic competences were common for all teacher students at the Faculty of Education. The remaining ten competences are specific for the computer science teachers. Specific competences for the field of computer science were selected according to the ACM/IEEE Computing Curricula (ACM/IEEE Joint Task Force on Computing Curricula (2001). IEEE Computer Society

Association for Computing Machinery, ACM, Computing Curricula 2001, Computer Science, — Final Report —(December 15, 2001). URL: http://www.computer.org/portal/cms_docs_ieeecs/ieeecs/education/cc 2001/cc2001.pdf (retrived July 18, 2008)., 2001). The authors of this document are professors from prominent universities and members of prestigious international computer science organizations, such as IEEE Computer Society and Association of Computing Machinery (ACM). Computing Curricula document defines »the body of knowledge«, which should be mastered by the graduate students of computer science, describes the core of the graduate computer science programs, defines the teaching goals, suggests models of syllabus and describes the content of courses.

The poll was carried out by means of web questionnaires, which were completed by all three target groups, i.e. "teachers", "employers", and "academicians". They assessed, on a scale from 1 to 4, achieved competences of our graduates and suggested, on a same scale, the level of desired competences for the graduates in our modernised study programs.

1) Understanding and application of general and didactical knowledge in the field of teaching.
2) Interdisciplinary knowledge linking.
3) Application of special pedagogical knowledge to teaching pupils with special needs.
4) Pedagogical classroom/group guidance.
5) Organising active and independent learning, qualifying learners for effective learning.
6) Qualification for the assessment of learners' achievements and for preparing feedback information.
7) Communicating with experts from educational fields and ability to establish and maintain partnership relations among them.
8) Collaboration with parents.
9) Formation of integral assessment of individual or group needs, about their strong and weak fields.
10) Knowledge and understanding of essential facts, concepts and theories from computer science.
11) Identification and analyzes of the problems and planning strategies for their solving.
12) Selection and application of adequate theory and tool for specification, planning and realization of the system.
13) Considering the principles of human-computer interaction in the system planning and evaluation.
14) Critical analyses and evaluation of the systems' or their parts in accordance with the specifications.
15) Knowledge and application of social, professional and ethical aspects related to computer technology.
16) Knowledge and understanding of didactic particularity of computer science and informatics.
17) Theoretical and practical knowledge for effective integration of information and communication technology (ICT) into the field of education.
18) Ability for information management.
19) Usage of professional terminology and appropriate language through professional and pedagogical work.

Table 1. List of competences for graduates - future primary and secondary school teachers of computer science.

We compared the results of analyses on the domain "Computer science" with the results of competence analyses on the faculty level, which included the results of inquiry on different study programs.

Methods

By analyzing the results of inquiry we wanted to overview the estimations of achieved and desired competences. We used descriptive statistical methods (Gonzales, J., Wagenaar, R. ed. (2005): Tuning Educational Structures in Europe II (Universities' contribution to Bologna Process, Final Report – Phase 2). Bilbao: University of Deusto, University of Groningen.nzales, J., Wagenaar, R. ed. (2005): Tuning Educational Structures in Europe II (Universities' contribution to Bologna Process, Final Report – Phase 2). Bilbao: University of Deusto, University of Groningen.

Miller and Miller, 1999). We drew graphs of sorted estimations for each group of participants and graphs of sorted differences between desired and achieved level.

Sample of respondents for the Two-subject Computer Science Program

There were 129 participants (respondents) involved in the web inquiry. The number of participants was relatively small, although the inquiry was completed by almost all the graduates on the computer science program and almost all the teaching staff, professors and teaching assistants who teach in the computer science program. As the number of employers who took part in the poll was relatively small, we merged them with the group of academicians. Consequently, the respondents can be divided into two main groups:

- 83 teachers, i.e. graduates of the study program Mathematics and computer science, (64%),
- 46 academicians, i.e. headmasters, professors and teaching assistants, (36%).

Web based inquiry

The web opinion poll contained a set of 19 competences, listed in **Table 1.** related to the future computer science teachers. Subject specific competences, related to field of computer science, were defined according to the ACM/IEEE Computing Curricula. General education and didactics competences were defined regarding to the results of the project Tuning and considering the results of discussions on competences at the Faculty of educationl (Tancig & Devjak, 2006).

Respondents estimated the achieved competences level in the current program and the desired competence level in the modernized Bologna programs among graduates of our current study program. They used marks from 1 to 4 to estimate the level of mastery or amount of knowledge, where marks were assigned the following meanings: 1 – nothing or almost nothing; 2 - low level; 3 – pretty high level; 4 – very high level.

Statistical methods

Competences had been assesses by the estimates average (arithmetic mean) of all respondents and of each single group (teachers and academicians). The results are presented in the following graphs:

- Graph of estimations of the achieved and desired level of competences with confidence intervals, (95% confidence level);
- Graphs of sorted (descending) estimations of the achieved/ desired level for each respondent group;
- Graphs showing sorted (descending) differences between the desired and the achieved level;

Results

We started the competence analyzes with drawing graph of estimates of achieved and desired competences with confidence intervals on 95% confidence level (Fig. 1.).

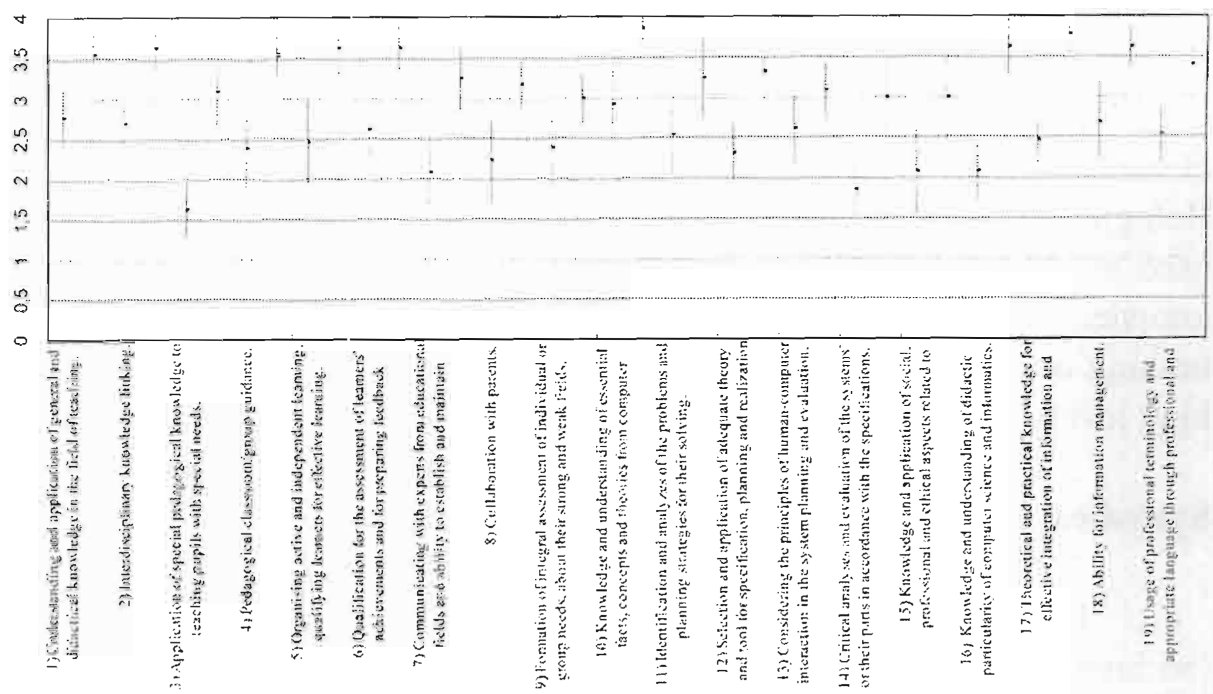

Fig. 1. Assessed competences with confidence intervals, on the 95% confidence level (for each competence there are given estimations for achieved and desired level).

We compared the estimates of achieved competences for the both groups of respondents, teachers and academicians. We found out that the group of teachers (Fig. 2) estimated that the following competences were achieved best: 1) understanding and application of general and didactical knowledge in the field of teaching" and 2) interdisciplinary knowledge linking. The same group gave the worst marks for the competences 7) communicating with experts from educational fields and ability to establish and maintain partnership relations among them, 14) critical analyses and evaluation of the systems' or their parts in accordance with the specifications, 15) knowledge and application of social, professional and ethical aspects related to computer technology and 18) ability for information management.

Academicians assessed the best achievement of the competences 16) knowledge and understanding of didactic particularity of computer science and informatics and 18) ability for information management. The same group gave the worst marks to the competence 3) application of special pedagogical knowledge to teaching pupils with special needs.

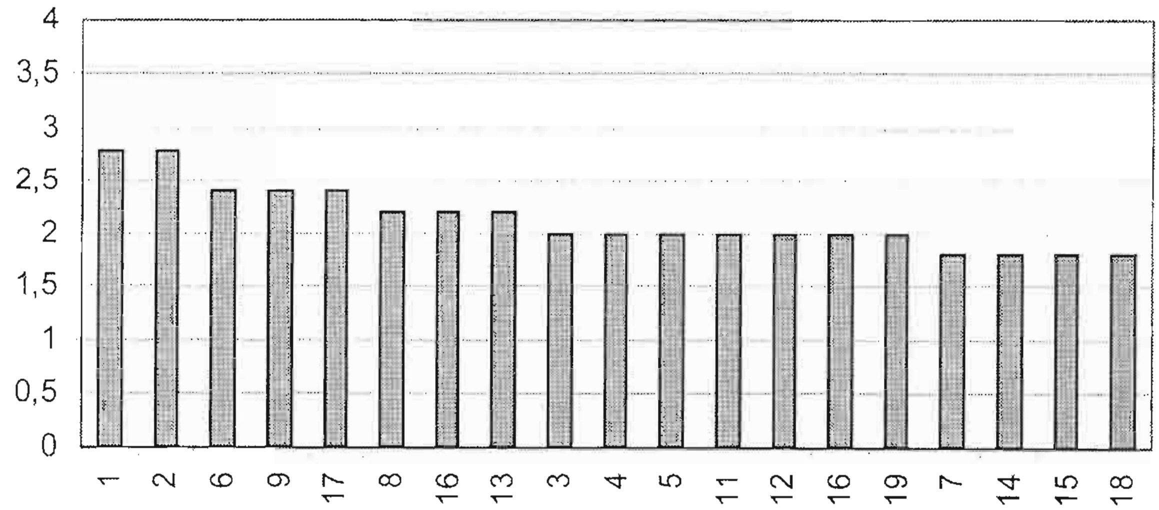

Fig. 2. Estimations of achieved competences assessed by teachers

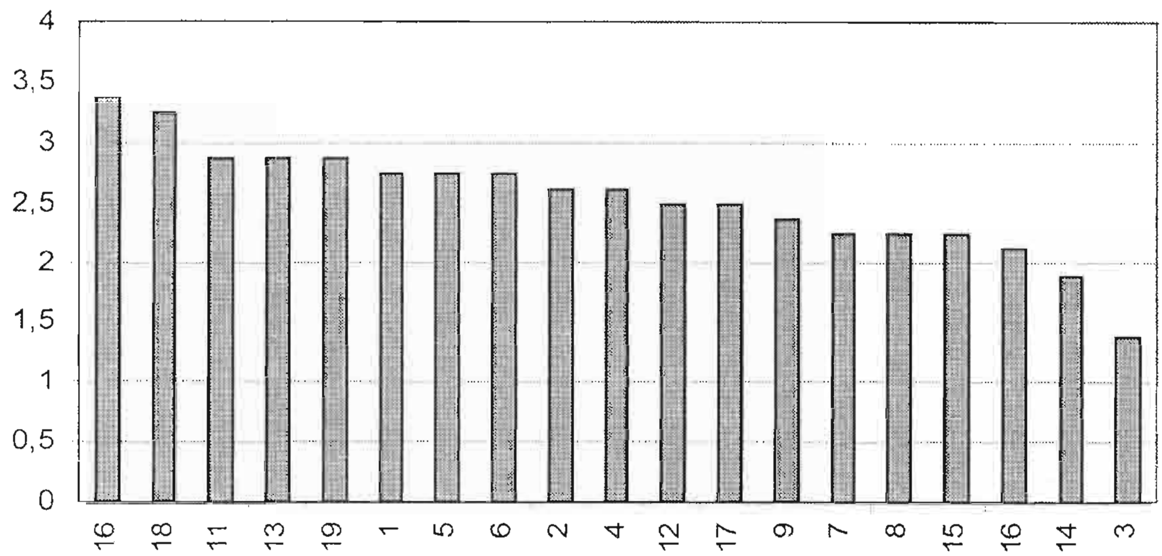

Fig. 3. Estimations of achieved competences assessed by academicians

We compared the estimates of desired level of competence attainment for the both groups of respondents . We found out the group of teachers (Fig. 4.) stressed the importance of *2) interdisciplinary knowledge linking, 16) knowledge and understanding of didactic particularity of computer science and informatics* and *17) theoretical and practical knowledge for effective integration of information and communication technology (ICT) into the field of education.* Academicians (Fig. 5.) stressed the meaning of *16) knowledge and understanding of didactic particularity of computer science and informatics, 5) organising active and independent learning, qualifying learners for effective learning* and *17) theoretical and practical knowledge for effective integration of information and communication* *technology (ICT) into the field of education.* They gave the minor importance to *9) formation of integral assessment of individual or group needs, about their strong and weak fields.*

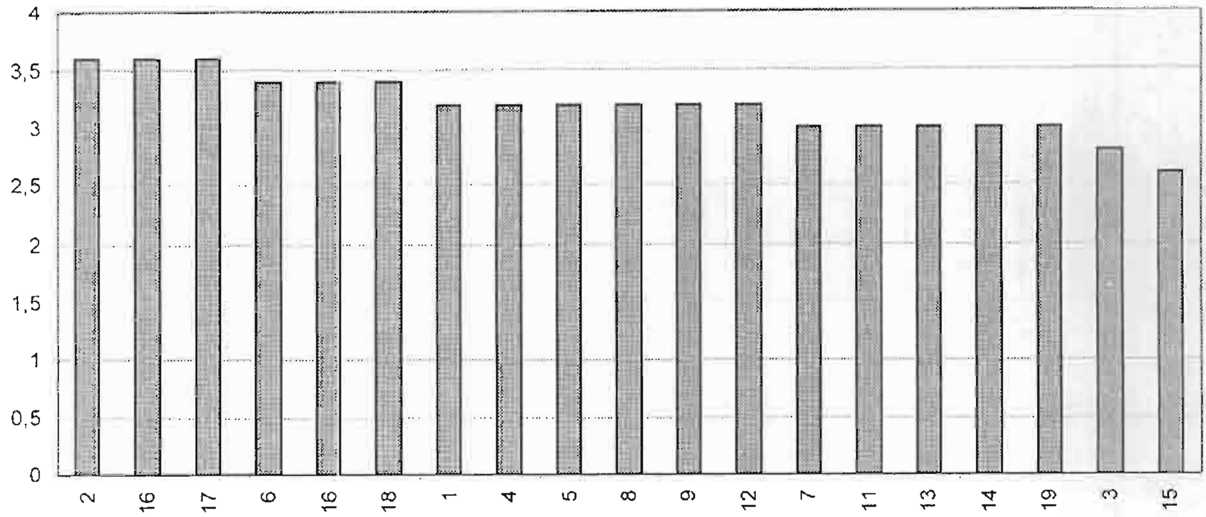

Fig. 4. Estimations of desired competences assessed by teachers

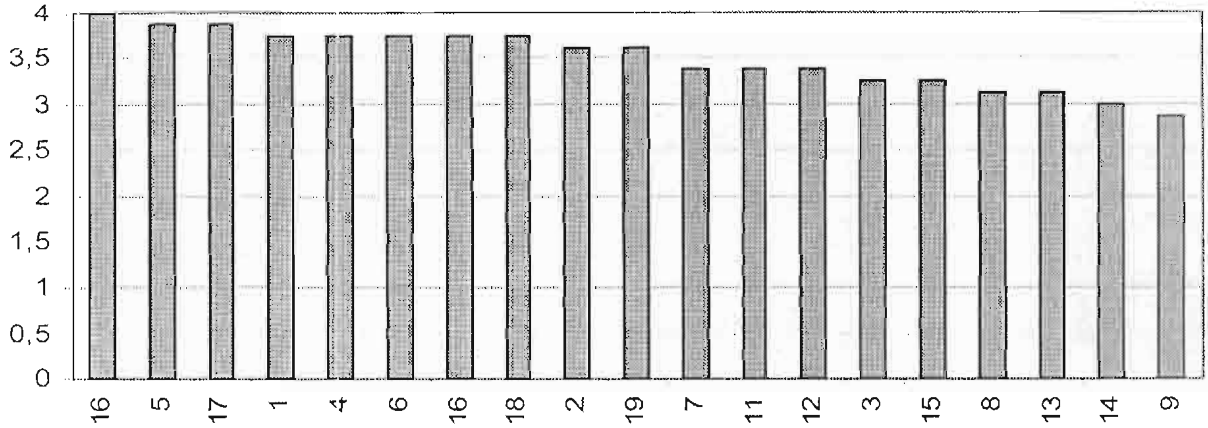

Fig. 5. Estimations of desired competences assessed by academicians

We also compared the differences between the desired and achieved level estimates for the both respondent groups. We found that for teachers (Fig. 6.) the difference was the biggest for *18) ability for information management* and the smallest for *1) understanding and application of general and didactical knowledge in the field of teaching.* For academicians (Fig. 7.) the difference was the biggest for *3) application of special pedagogical knowledge to teaching pupils with special needs* and the smallest for *13) considering the principles of human-computer interaction in the system planning and evaluation.*

Fig. 8. shows the differences between desired and achieved competence levels for all respondents. The difference is the biggest for

competence *16) knowledge and understanding of didactic particularity of computer science and informatics* and it is smallest for *13) considering the principles of human-computer interaction in the system planning and evaluation.*

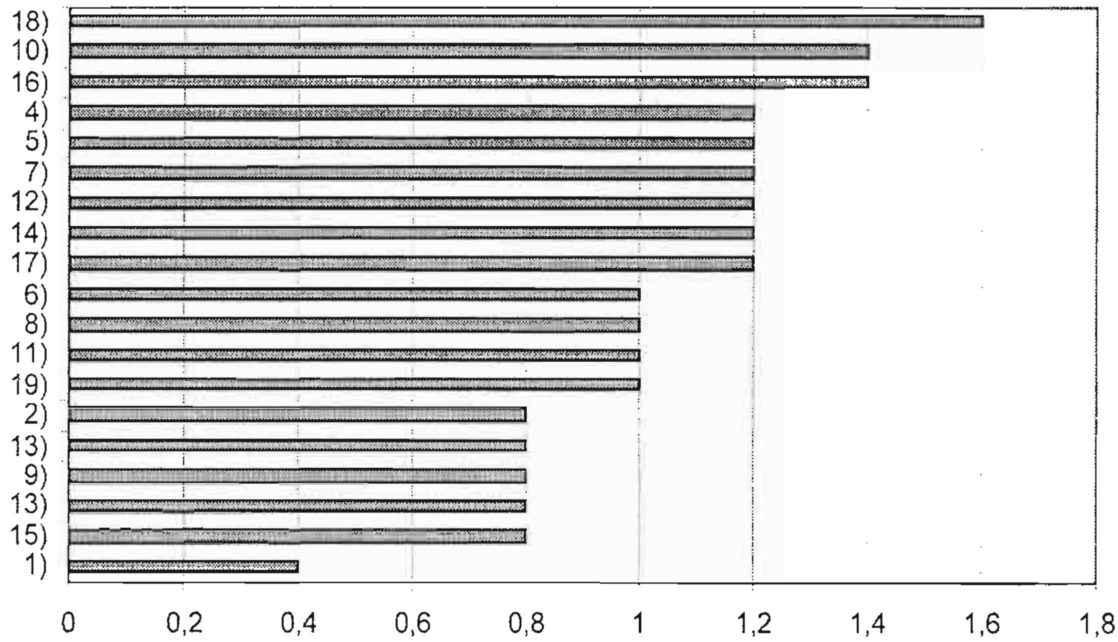

Fig. 6. Differences between desired and achieved competences assessed by teachers

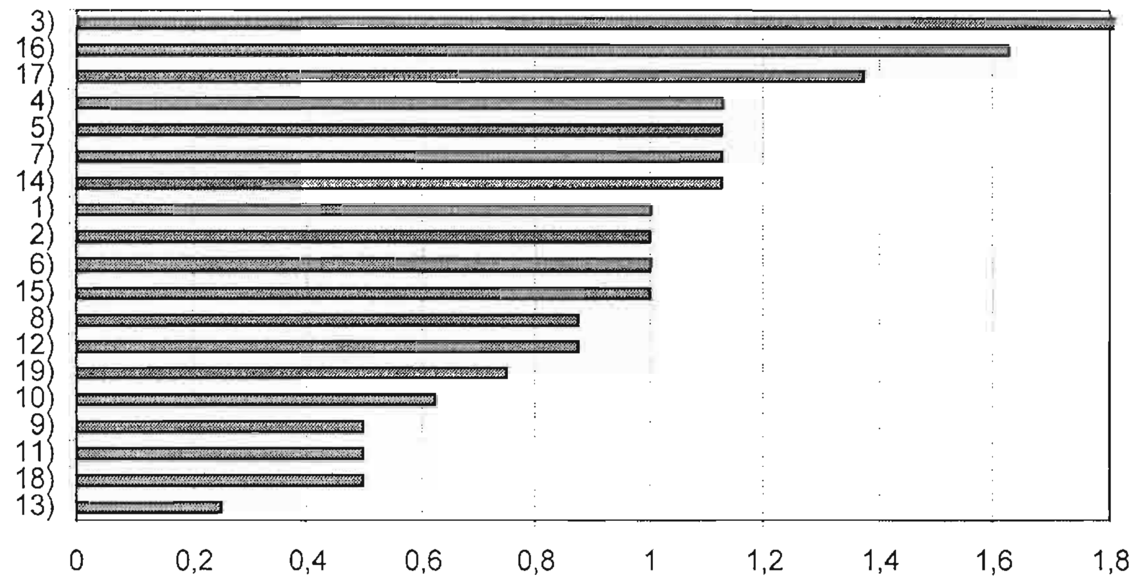

Fig. 7. Differences between desired and achieved competences assessed by academicians

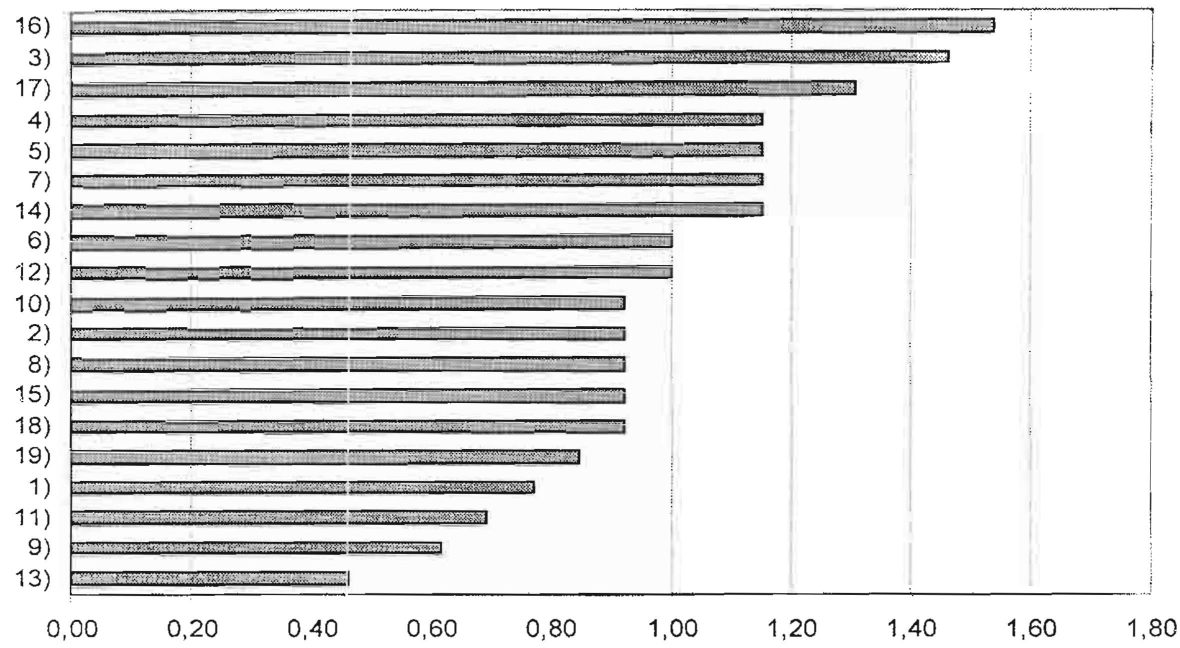

Fig. 8. Differences between desired and achieved competences (all)

Discussion

Partnership between faculty and partners in practice - teachers and headmasters - played an important role in the competence analysis. We found out that the number of participants in the poll was relatively small, although we actually invited all the graduates to participate in the poll and they responded to our invitation. Computer Science study program is running only for a few years now and the number of graduates is small. But information gained from the partners was quite useful for our work.
Figure 3. shows that the estimation of achieved competences for all three participating groups varies substantially. Teachers think general teaching competences are achieved better in comparison with specific competences, but academicians thing that specific computer science competences are achieved good enough.

Graph in the Figure 1. confirms that for all respondents the following competences are important: »knowledge and understanding of didactic particularity of computer science and informatics« and »theoretical and practical knowledge for effective integration of information and communication technology (ICT) into the field of education«.

We focused our attention on the competences, where achieved level was relatively low the desired level was relatively high. Figure 8. shows seven competences with the biggest difference between the desired and achieved level. Among them we expose the following competences: *16) knowledge and understanding of didactic particularity of computer science and informatics* (highly desired), *3) application of special pedagogical knowledge in teaching pupils with special needs* (lowly achieved) and *17) theoretical and practical knowledge for effective integration of ICT into the field of education* (highly desired).

Abovementioned estimations of the achieved competences confirm that there was not enough emphasis on the didactic topics for teaching of computer science in the current study program, students were not adequately taught about the particularity of teaching pupils with special needs and they did not obtain enough applicative knowledge for integration of ICT in the education. We found out the competence *3) application of special pedagogical knowledge to teaching pupils with special needs* was relatively lowly estimated by all respondents on the level of the Faculty of Education (Tancig & Devjak, 2006). In general, the results of the competence analysis on the level of the faculty, where 14 study programmes were analysed, are to great extent in accordance with the results concerning study program Computer science.

We took into account the results of competence analysis in the design of new curricula in the framework of the Bologna processes. Abovementioned "critical" competences are appropriately "covered" and developed through different subject in the renewed computer science program.

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Teaching for lifelong learning, and lifelong learning for teaching

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Abstract. When re-thinking the design of the curriculum for pre-service teachers (in our case in Informatics and Information technologies) we should start with the main question – what these teachers-to-be should prepare their students for? The authors' provide arguments in support of the belief that the school should be oriented to building competences identified as crucial for the citizens in a knowledge/creativity-based society. The paper presents authors' experience in applying some good European practices focusing on lifelong learning in the context of the pre-service and in-service education of ICT-teachers in Bulgaria.

Keywords: Teacher education, competences, lifelong learning (LLL)

1. Introduction

One of the most important features of the citizens of the knowledge/ creativity based society is expected to be the lifelong learning ability. In order to cultivate such ability in their students, it is important for teachers and university professors alike to demonstrate that they themselves are ready to learn new things all the time (including from their students). In order to achieve this we should introduce a learner-centered methodology at all levels – from primary school to the University. "Teaching the way we preach" should be applied with special care when working with in-service teachers. Another important step is to build communities of lifelong learning teachers who are ready to integrate the innovative ideas of the younger teacher-to-be generation with the experience and expertise of the in-service teachers.

Before starting to educate the future teachers we should know what their students are expected to achieve according to the most modern standards.

2. Teaching Lifelong learners

2.1. Competencies for the knowledge society

According to the recommendations of the European Union Parliament and the Council, every member of the knowledge society is expected to achieve wide range of competencies the emphasis being on the following eight:

- Communication in the mother tongue;
- Communication in foreign languages;
- Mathematical competence and basic competences in science and technology;
- Digital competence;
- Learning to learn;
- Social and civic competences;
- Sense of initiative and entrepreneurship;
- Cultural awareness and expression.

From those we have identified "Learning to learn" to be the key competence for our setting, especially in such dynamic domains as IT and informatics. Although this competence has often been associated with the modern society it is quite in harmony with the ancient Chinese proverb: *If you give a man a fish he will eat for a day; if you teach a man to fish he will eat for a lifetime; if you* ***teach a child to fish*** *he may feed the world.*

Bringing this educational concept in the context of teacher education has been one of the main goals behind a methodology which was designed and implemented in the frames of the *Innovative teacher* European project (*I*Teach*) [1].

2.2. Methodology to Grow Citizens of Knowledge Society

The *I*Teach* methodology was developed in response of significant number of studies within the EC program *Education & Training 2010* [2]. According to them, a broad range of new competencies are expected from the teachers in the knowledge-based society, e.g. to design, develop, conduct, and facilitate teaching/learning process so as to enhance the so called *soft skills* by means of information and communication technologies (ICT). Thus, the focus of the *I*Teach* project has been on both developing a practical methodology and supporting tools for building *ICT-enhanced skills* – a concept coined to denote the synergy between soft skills and ICT skills. Through the collaborative effort of partners from seven European countries (Bulgaria, Germany, Italy, Lithuania, the Netherlands, Poland, and Romania) the skills identified as the *ICT-enhanced skills* needed the most in the countries involved were the following [3]:

- searching and selecting information
- presenting information
- working on a project
- working in a team

The *I*Teach* methodology is laid on the Project- and Problem-based learning methods. The main idea of the methodology is to build ICT-enhanced skills through *continuous, repeatable* and *gradually accumulated experiences and expanded activities* leading to concrete *goals* by performing specific *tasks* in different *context*.

This methodology is oriented to finding the balance between the full freedom of the learners to choose their learning path, and the necessity to follow strictly the fixed learning path offered by the teacher.

One of the main concepts of the methodology is that of *didactic scenario*. Each *I*Teach* scenario presents an educational goal to the learner, usually formulated as a *challenge,* gathered by brainstorming in the context of "real life" situations and taking into account the interests of the learners. To solve the challenge, learners need to pass through a set of *milestones*, each one associated with attaining specific results. Learners choose their own learning path while moving from one milestone to another. The trainers stay as invisible as possible, monitoring the workflow and helping/intervening only when there is a real need [4].

Thus, an *I*Teach* scenario represents a composition of tasks, to be executed in the context of an active learning environment. The metaphor behind such a scenario is a *path* (the process) traced by *landmarks* (the milestones) leading to the *peak* (the goal) as shown in Fig.1. Following the *milestones* to the final goal, learners gradually develop ICT-enhanced skills.

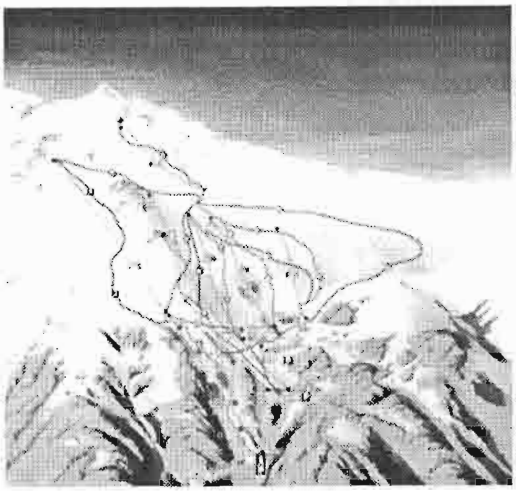

Fig. 1. I*Teach metaphor

The comprehensive description of the *I*Teach* methodology can be found in the *I*Teach* methodology handbook [5]. It contains explanation of all the active learning methods applied, and specific approaches for the development of *ICT-enhanced skills*. Last but not least, this handbook contains a rich set of exemplary didactic scenarios, designed and developed in support of the *I*Teach* methodology.

2.3. Implementing the *I*Teach* methodology

In order to support teachers to teach lifelong learners in the school settings, a set of textbooks (applying I*Teach methodology) for ICT in 5th, 6th and 7th grade [6] were developed.

The idea has been to meet the specific requirements defined by the national standards together with the more general ones envisioned for the needs of a knowledge/creativity based society. As Chickering and Gamson suggest [7], in order to be actively involved students should not only listen but also read, write, discuss, or be engaged in solving problems. Most important, they should be engaged in such higher-order thinking tasks as analysis, synthesis, and evaluation. Using active learning techniques in the classroom is found vital because of their powerful impact upon students' learning. Our approach is based on numerous studies having shown that strategies promoting active learning are superior to lectures in promoting the development of students' skills in thinking and writing.

These ideas are realized through the *I*Teach* methodology incorporated in our set of ICT textbooks by means of didactic scenarios containing:

- A short introduction in which the **main objectives, both technical and soft skills** to be acquired, are specified;
- **A challenge** - a creative task motivating the introduction of new knowledge and skills, stimulating students to find and describe different ways to the final goal and encouraging the spirit of cooperativeness (Fig. 2);

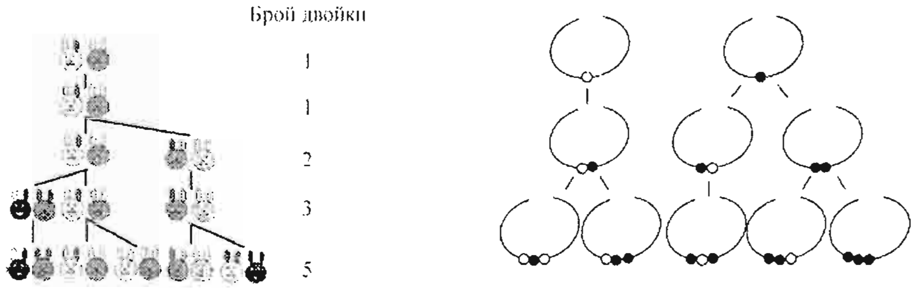

Fig. 2. The challenge: to calculate Fibonacci numbers in the context of grandpa's rabbits and grandma's necklaces

- **Creative tasks** spread in several subsections, e.g. *Roll up your sleeves*, *Give free rein to your imagination*, etc.
- **Variety of paths to solving the challenge**. To prepare the students for citizens of the knowledge/creativity based society we try to encourage them to discover various solutions to the problem and compare them according to specific criteria (rather than to apply recipes in the *click and drag* style). For instance, after the two columns with different descriptions of opening a specific file an empty column is added to be filled by the students with alternative ways they have discovered (Fig. 3).

Мога да го направя така:	Или пък така:	Открих и друг начин!
Стартирам програмата **Paint**. От менюто **Файл** избирам **Отво-рям...** Отивам на диска, отварям папка **Graphics** и отварям файл **2_kubcheta**.	Стартирам програмата Paint. Отивам на диска, отварям папка Graphics. Щраквам върху файла 2_kubcheta и го тегля до програмата Paint.	1. ………………… ………………… 2. ………………… …………………

Fig. 3. Describing various ways of opening a graphic file

- A **friendly and collaborative spirit** of addressing the readers

Typical for the structure of textbooks is that there is a common *thread* linking

- the tasks in a lesson (5th grade [8]),
- the lessons in a common ICT theme (6th grade [9]),
- the ICT themes in the whole textbook (7th grade [10])

in the didactic scenarios.

The implementation of these ideas is best achieved in the textbook for 7th grade, the *red thread* being the theme of coding. The *grand finale* is a project requiring the students to put together all the subject knowledge and skills acquired during the school year and to work creatively in teams for the achievement of a common goal. During the project the students are faced with problems from real life and are expected to understand in a natural way *when*, *how* and *which* ICT tools to apply so as to solve the challenge of restoring archeological artifacts (Fig. 4).

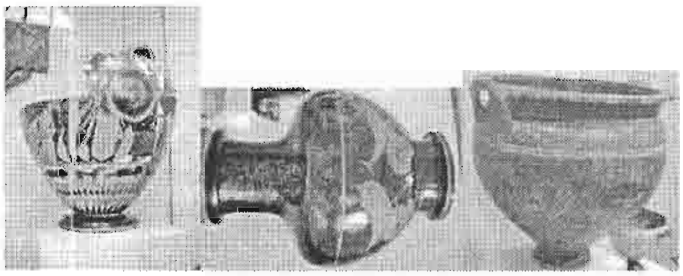

Fig. 4. The challenge of restoring ancient vessels

For the purpose students are encouraged to explore computer models (Fig. 5) and to decode a message (Fig. 6) with hieroglyphs so as to help a local museum to restore ancient Greek vessels and guess their function.

Fig. 5. The computer model developed on the basis of an *Elica* [11] application

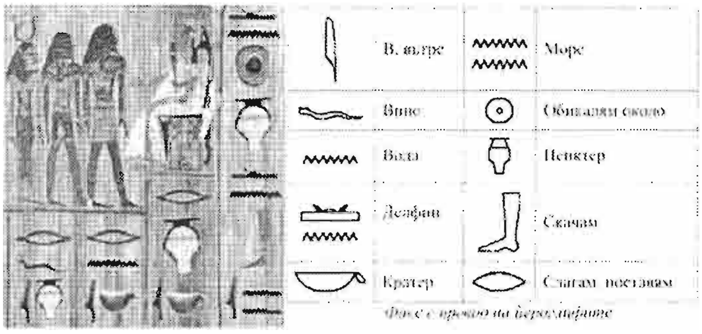

Fig. 6. The coded message

The project is a stand-alone entity in which students are expected to demonstrate their problem solving skills by means of ICT.

The project is designed as a prototype of a real large-scale project. The tasks in the project are connected at various levels of abstraction. Some tasks can be done in parallel, others require sequential execution. All these details are something which the students must accomplish as a first step of the project. They are also expected to split in teams and distribute work forces according to their skills and the amount of work.

The textbooks are not designed to serve as recipes for a good teaching. They offer examples for work on projects and could be treated as generators of ideas. We expect teachers together with their students to find their own challenges according to their interests and to follow the suggested approach.

3. Lifelong Learning to Teach

3.1. Educating Teachers of Lifelong Learners

It is easy to say "You have good textbooks. Follow the methodology we suggest!", but we found that the teachers needed special type of teaching and encouragement in order their imagination to be unlocked. As discussed by Syslo and Kwiatkowska in [12], changes in education may be expected according to the model for ICT development when the first stage (*Discovering ICT tools*) and the second stage (*Learning How to Use ICT Tools)* are passed, and the third stage (*Understanding How and When to Use ICT Tools*) is reached. Thanks to numerous training courses in ICT for mathematics teachers in Bulgaria the first two stages have been passed successfully. Now the most difficult and important one has come– understanding *How* and *When* to use ICT tools so as to achieve particular educational goals.

The *I*Teach* methodology suggests *How and When to use ICT.* Based on previous experience we were convinced that it was not possible to use the old manner of teachers' training if we wanted teachers to use innovative ways of teaching. That is why we would like to focus on the most interesting question here: "*How did we train teachers?*" The answer is: *recursively* - we, the team of trainers, applied the methodology itself to educate the teachers how to apply it. Usually the training sessions started with a short introduction of each participant answering 3 questions *In which area do you feel an expert?*, *Why do you think you are an expert?*, *How did you become such an expert?*. Through their answers we succeeded to demonstrate that the driving force to learn is the learner's interest and motivation. In addition, answering the third question, they realized that to become an expert you should apply your knowledge many times. But we would make these comments later. In the beginning, it looked just as a non-standard introduction of the participants.

After that, we would conduct a brainstorming session around the topic, which we had previously identified as important and interesting for the participants. As a result, the participants were grouped according to the specific interests they expressed during the brainstorming. Next, they had to solve their first challenge – to plan how they would work as a team on the project on the theme chosen by them. The teachers worked on their project exactly the way we expected their students to work - learning new things in the area they had chosen. Finally, they had to present their results and to share their achievements with the other participants. Finally, we reflected on their presentation, and asked them to analyze what, and how had been achieved during the training. In such a way the teachers discovered the main ideas of the *I*Teach* methodology by themselves. We guided them to derive the main conclusions about the training process in a group discussion. Thus they felt like co-authors of these new ideas. When you have the feeling that an idea is your own, then you are ready to apply it more easily and with a greater enthusiasm. And as a result, we observed high motivation and engagement in performing all the tasks, which unfortunately is missing in their traditional school practice. In addition, during the training they were encouraged to generate new ideas and challenges, and to prove to themselves that they could be innovative. These arguments were sufficient to motivate them to implement the methodology in their teaching.

It should be noted that the participants were active in the process of learning, playing a central role in it, and completing the training with specific final products developed by them. In addition they felt co-authors of the methodology, having in fact re-discovered its basic ideas during the course. When interviewed at the end of the course, the teachers expressed their high appreciation of the methodology for training teachers (Fig. 7), although in some cases were skeptical about its applicability in a real class setting.

Fig. 7. A teacher proud of her creative integration of various software tools. More than 500 pre-service and in-service teachers in the last two years were trained to apply the methodology.

First the methodology was probed with pre-service teachers from two Master of Science programs at Faculty of Mathematics and Informatics in Sofia University: E-learning and Technology of Mathematics & Informatics Education.

Next, it was applied during the workshop with in-service ICT-teachers being re-qualified from other subjects, e.g. mathematics, physics, arts, etc. The teachers trained proved the applicability of the methodology in their classes [4].

Later on, the methodology was introduced to pre-service teachers in Mathematics and Informatics in Bachelor of Science Course. Let's note that we would choose specific challenge for each group of students, based on their interests and background.

The workshop, organized as a satellite event of the *Annual Spring Conference of the Union of the Bulgarian Mathematicians*, Varna, 2007 with math researchers and teachers of highly achieving students in mathematics and informatics, was the next place where the I*Teach methodology was demonstrated in action [13].

During the trainings the teachers pointed out additional conditions for spreading the methodology on a larger scale:

- To continue the training within the newly built communities;
- To harness the power of the collective intelligence [14]
- To suggest to other teacher trainers and University's professors to apply similar methods during pre-service teacher education since – *One teaches the way one was taught.*

These three problems inspired our further activities.

3.2. Teachers and University Professors as Lifelong Learners

To provide a solution for some of the above mentioned problems, we decided to apply our "recursion approach" on the next level – to put some university professors in the role of learners [15]. In order to prepare them to teach teachers in implementing the *I*Teach* methodology, we used again the methodology itself. But we decided to combine the training of the university professors with the piloting of the Personal Competence Manager (PCM) - a tool developed in the frames of the FP6 TENCompetence project [16]. The idea was to use the PCM for both – continuing the training and providing an environment for community building.

Our observation from the *I*Teach* training workshops has shown that the knowledge and competencies gained do not finish with the end of the training [4]. Most of the teachers would face new challenges during their work in the class. Thus they would feel the need of on-going exchange of good practices within the professional community formed during the course. We identified a strong need of the trainees to continue their further competence development preserving all the information channels built during the initial training. The *I*Teach* trainers found the PCM as the most appropriate tool to provide teachers with a relevant support and ensure their lifelong learning. They considered PCM to be a tool for converting an established professional community into a virtual one, rather than just a tool for communication. In addition, we found it very easy to put all available *I*Teach* learning resources and information into the PCM system. But most of all, our expectation was to use successfully the PCM for teachers' competence development and to give them a chance to continue work on

e-learning materials in collaboration with other colleagues and students.
Which characteristics of the PCM have been identified as the most important ones for the *I*Teach* trainings? PCM is a system designed especially to support people's personal and Life Long Competence Development. In contrast with existing learning systems, designed around concepts like lecture, course, training program, the main concepts in the PCM are learning network, competence profile, and competence development program [17]. PCM gathers competence related information drawn from sources at multiple levels, and presents this information in a context, structure and format, which are determined by the user.
The PCM system is available as a service-oriented architecture with a Java Eclipse client, with new Web-based client interface planned to be available at the next stage of the TENCompetence software infrastructure development.

The PCM functionalities include forming/joining virtual *communities* (learning networks) with common professional and/or personal interests. Each community can develop different *competence profiles*. For each competence profile different *competence development plans* can be designed, leading to improving or achieving a set of specific *competences*. Each plan may contain various learning paths, comprised by different learning activities and supported by specific knowledge resources (Fig. 8).

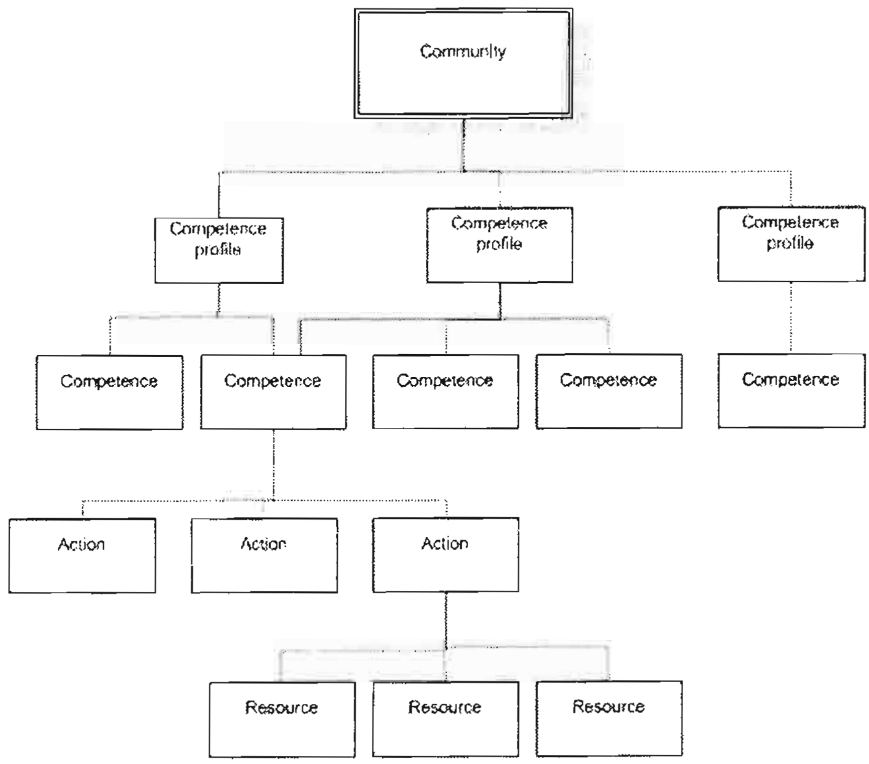

Fig. 8. Structure of the PCM community

When working with the Personal Competence Manager the users can choose their own competence development plans, follow them and thus built the desired competences. They can rate any existing object (plan, activity, resource) in relation to achieving specific competence profile. They can easily share their plans, ratings, resources and ideas using the embedded communication tools. The self-assessment instrument and *best way* map help learners to find the most efficient (for them) learning path through any competence development plan.

All these features make PCM the perfect tool for putting in action the idea of *collective intelligence*, followed by the I*Teach trainings so far. Having the power of the PCM tool, we chose the same *I*Teach* scenario, we had already used with the teachers, with one small change: asking the university professors not only in what area they have competence, but also to name three areas in which they would like to become competent (to achieve new competences). We turned the university professors in learners the same way we did with the teachers. The methodology succeeded once again. We managed to train professors to train lifelong learning teachers of lifelong learning pupils.

In addition, through the PCM we stimulated the professors to use the tool for sharing their knowledge and experience among them and the teachers, training later on teachers how to use tools like PCM to build their own communities or to join already existing communities in their area of interests.

4. Conclusions

In a nut shell, the most important competence the future teachers should acquire is the readiness to learn lifelong, to think creatively, to be ready to make choices and to defend their decisions. Not only is it challenging and motivating, but is applicable for the pupils they would teach, as well as for the professors by whom they have been taught. This holds especially for such a dynamic area as the information technologies.

Learning concrete facts, even gaining a deeper knowledge, available at a given moment, is far from being enough. After graduating, the teachers (IT teachers specifically) should continue to learn about the novelties in their subject, the methodology of teaching and would be ready to share the joy of learning with their students.

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