

Значај Леонарда Фибоначија за успон модерне математике у Европи.

Ненад Швракић

ХЕК - XX.III.MMXVIII



...a??) – 1250(Pisa?)



*Leonardo Pisano, Fibonacci
with Hermit Crab
1175 - 1220*



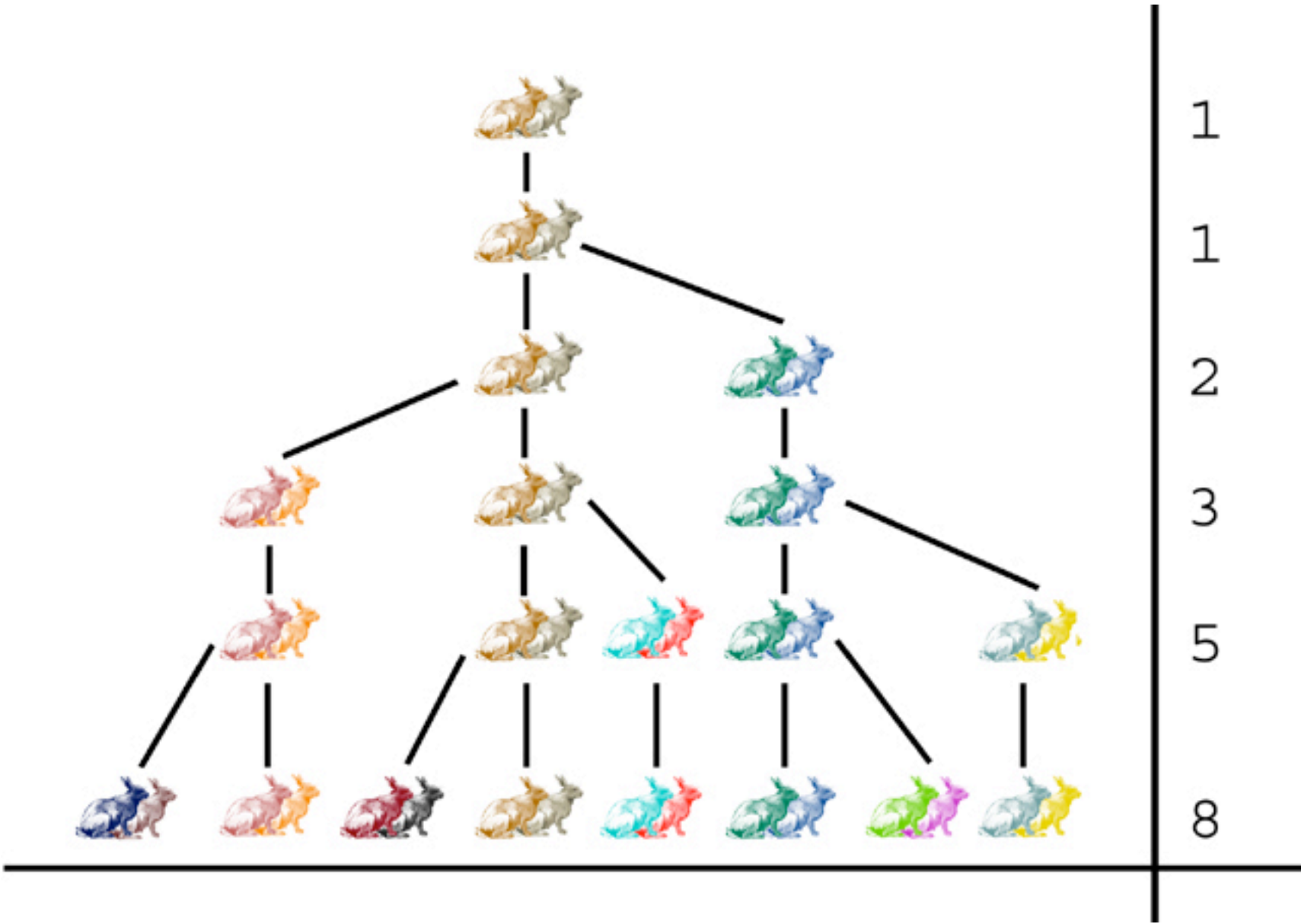
Italy

Rome

Tyrrhenian Sea



Ako par zečeva svakog meseca rodi novi par koji od drugog meseca već može da se razmnožava, koliko će zečeva biti posle 12 meseci? (377)



$$1+1=2$$

$$1+2=3$$

$$2+3=5$$

$$3+5=8$$

$$5+8=13$$

$$8+13=21$$

$$13+21=34$$

$$21+34=55$$

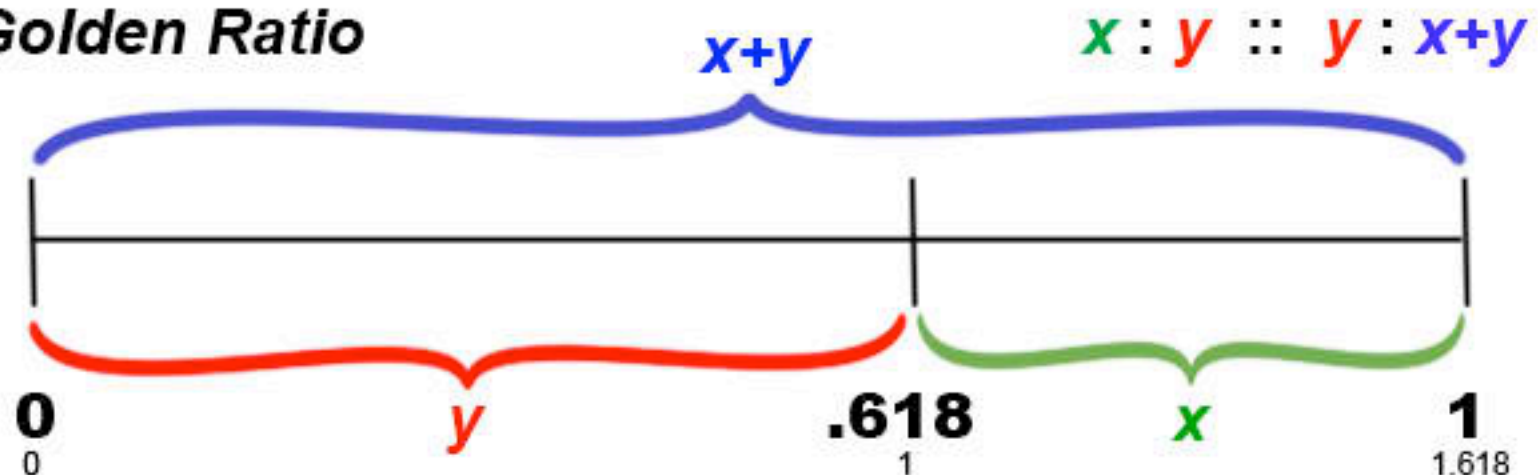
...

The Fibonacci Sequence

Zlatni presek

A	B	B / A
2	3	1.5
3	5	1.6666666666...
5	8	1.6
8	13	1.625
...
144	233	1.618055556...
233	377	1.618025751...
...

Golden Ratio



Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610...

$$3 \times .618 \approx 2$$

$$13 \times .618 \approx 8$$

$$55 \times .618 \approx 34$$

$$5 \times .618 \approx 3$$

$$21 \times .618 \approx 13$$

$$89 \times .618 \approx 55$$

$$8 \times .618 \approx 5$$

$$34 \times .618 \approx 21$$

$$144 \times .618 \approx 89$$

$$X=1.61803398875... \quad 1/X=0.61803398875...$$

Tj. $X-1/X=1$ pomnožiti celu jednačinu sa X

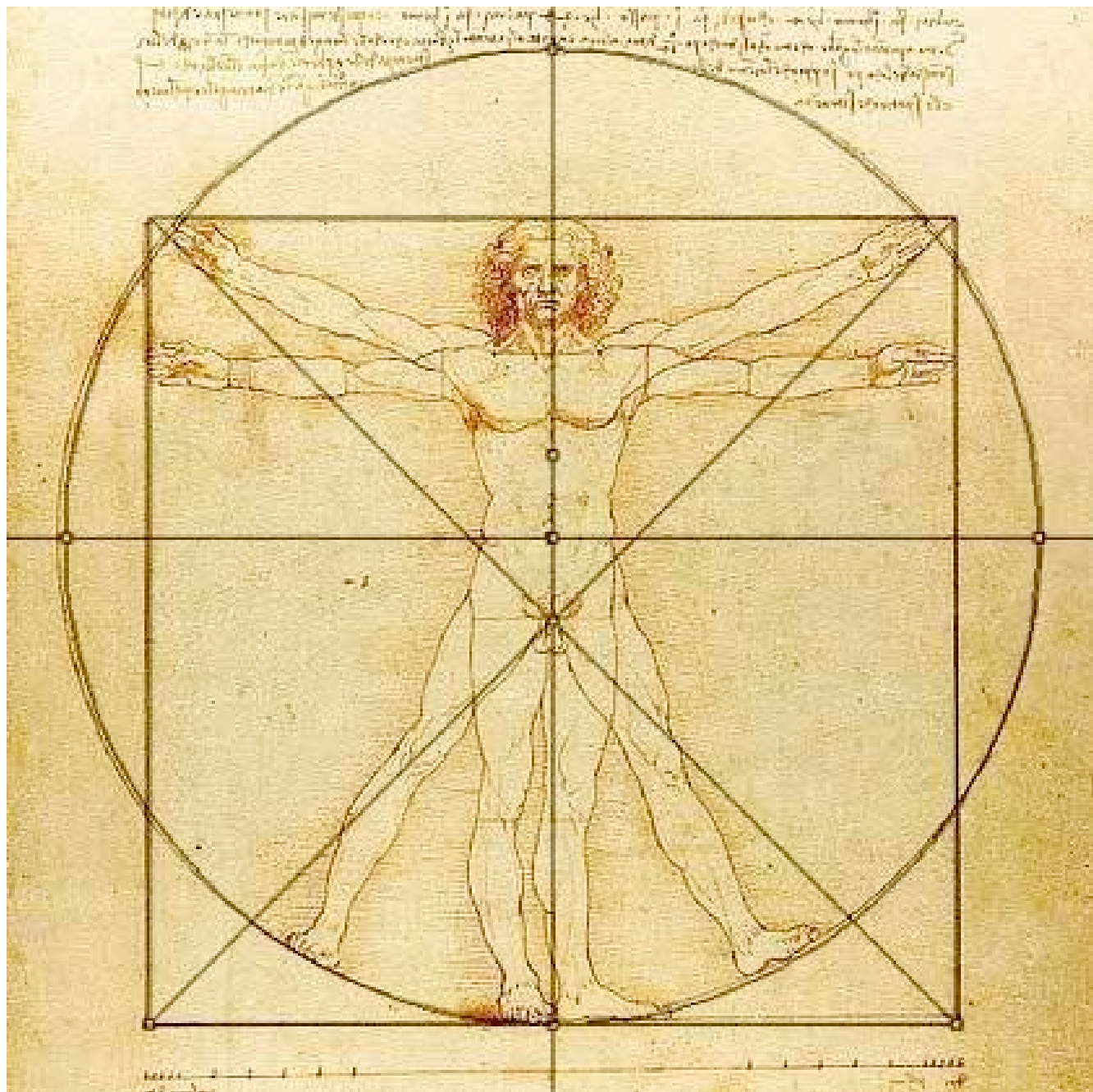
Dobije se

$$X*X-1=X, \text{ ili } X^2=X+1$$

Tj.

$$1.61803398875*1.61803398875=2.61803398875$$

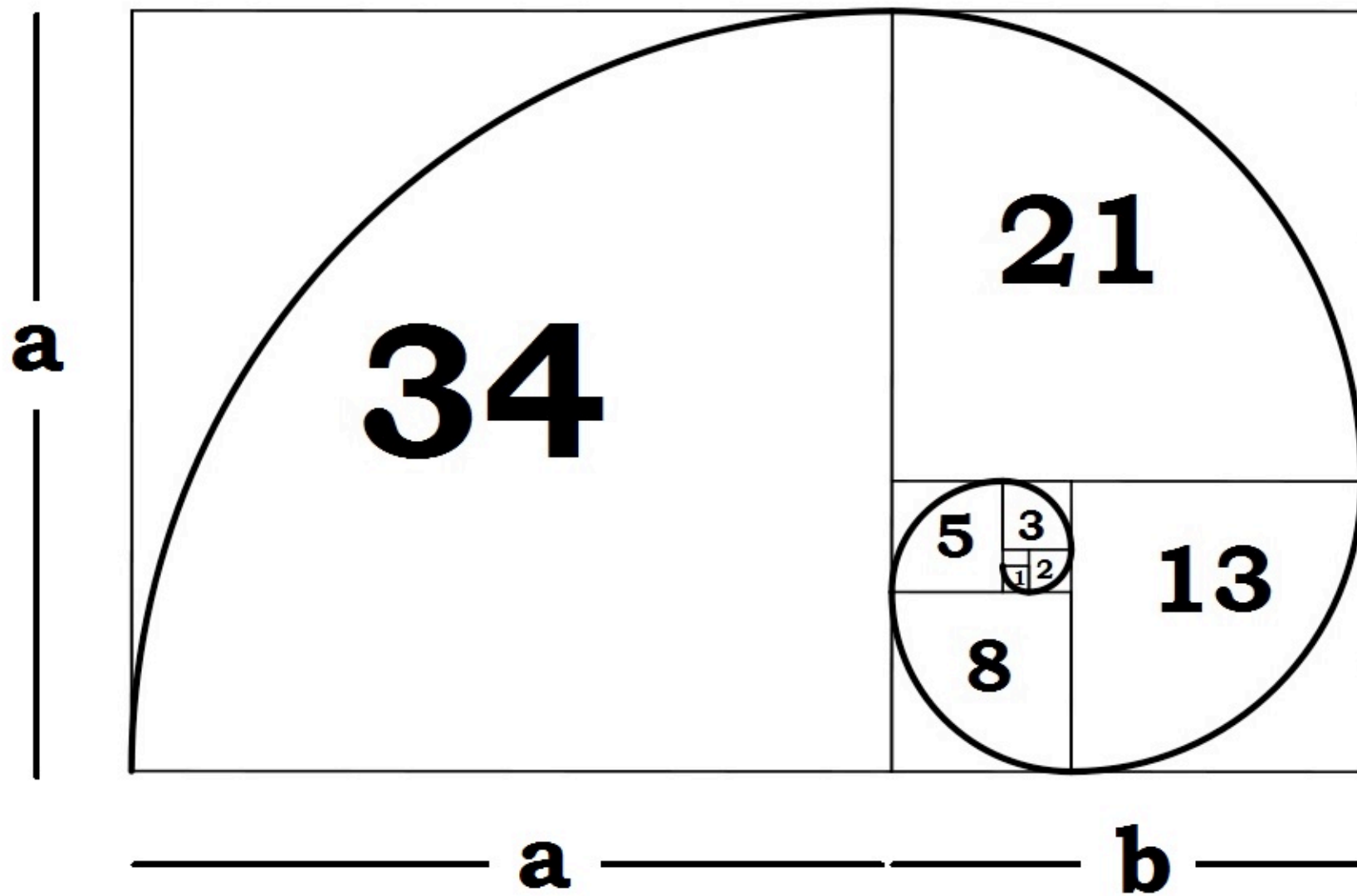
$$X=(1/2)*(\text{sqrt}(5)+1); \quad 1/X=(1/2)*(\text{sqrt}(5)-1)$$

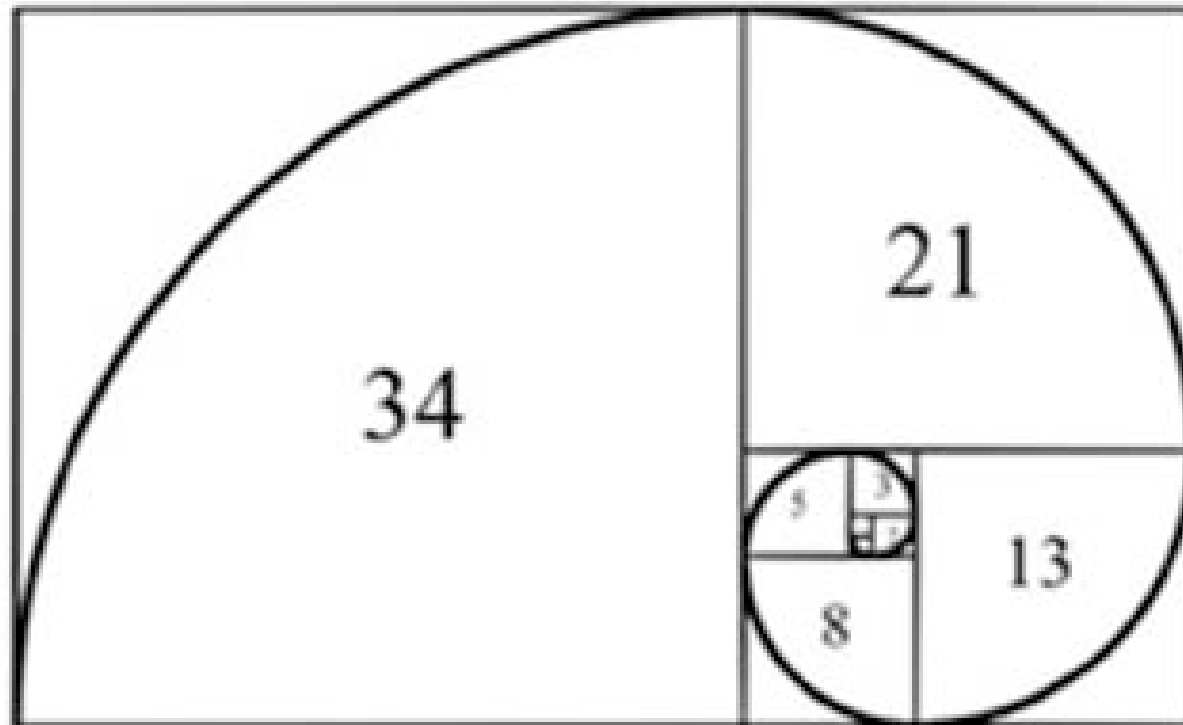


$$B/A = \varphi = 1.61803398\dots$$



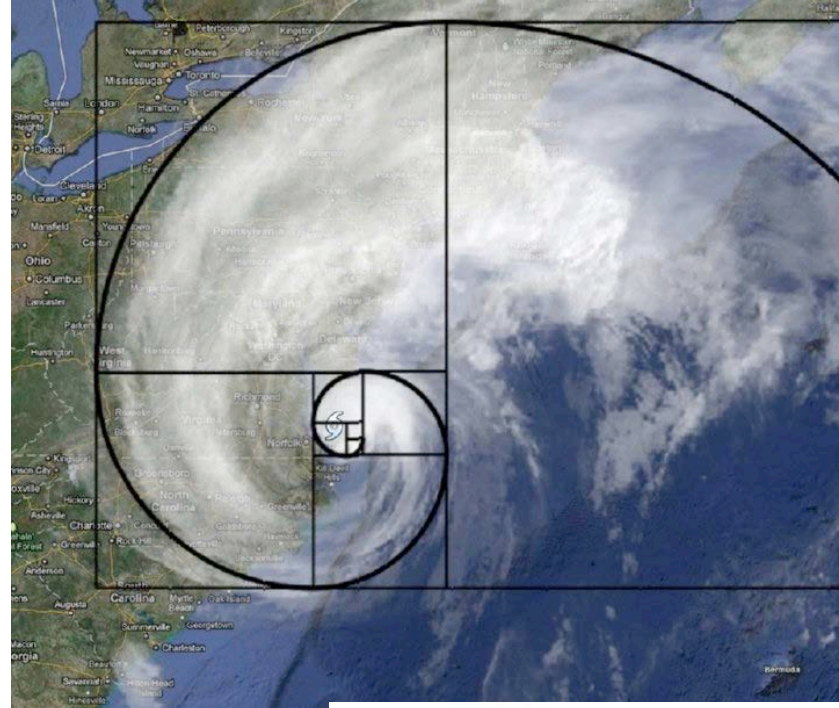
Fibonačijeva spirala



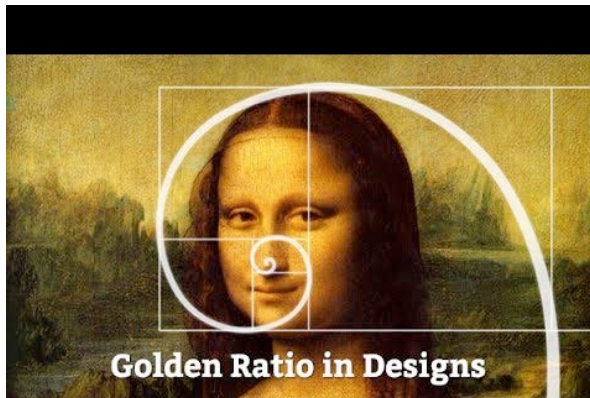


$$\begin{aligned}
 0 + 1 &= 1 \\
 1 + 1 &= 2 \\
 2 + 1 &= 3 \\
 3 + 2 &= 5 \\
 5 + 3 &= 8 \\
 8 + 5 &= 13 \\
 13 + 8 &= 21 \\
 21 + 13 &= 34 \\
 34 + 21 &= 55 \\
 55 + 34 &= 89 \\
 89 + 55 &= 144
 \end{aligned}$$

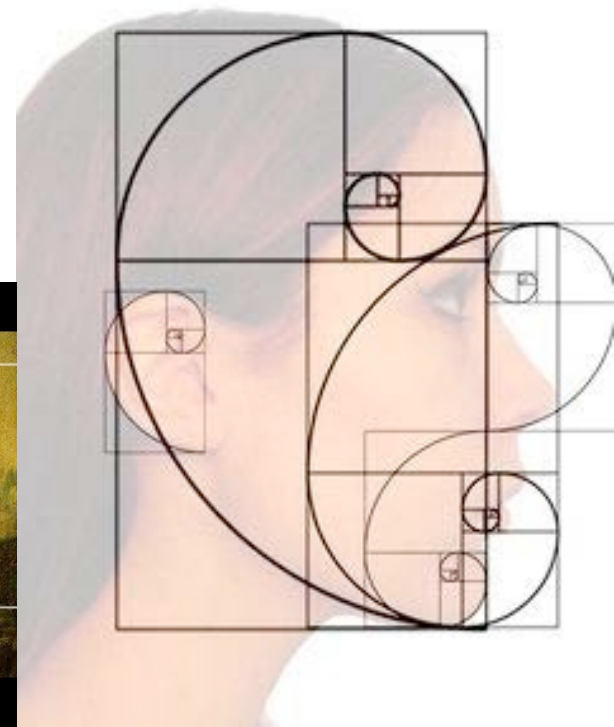
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...



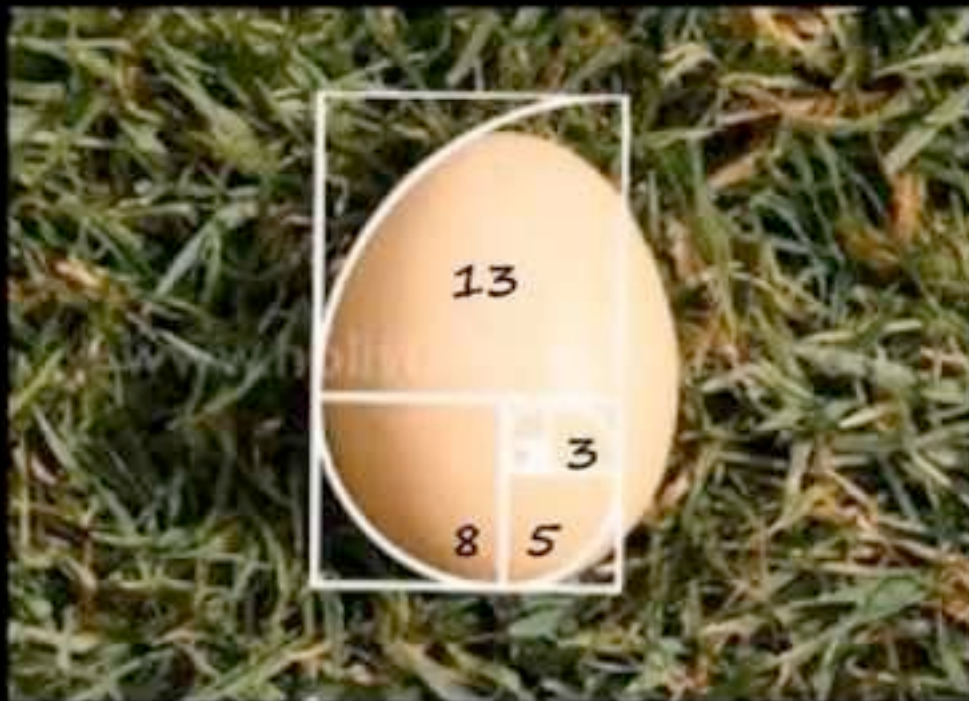
Age of Aquarius spirale



Golden Ratio in Designs

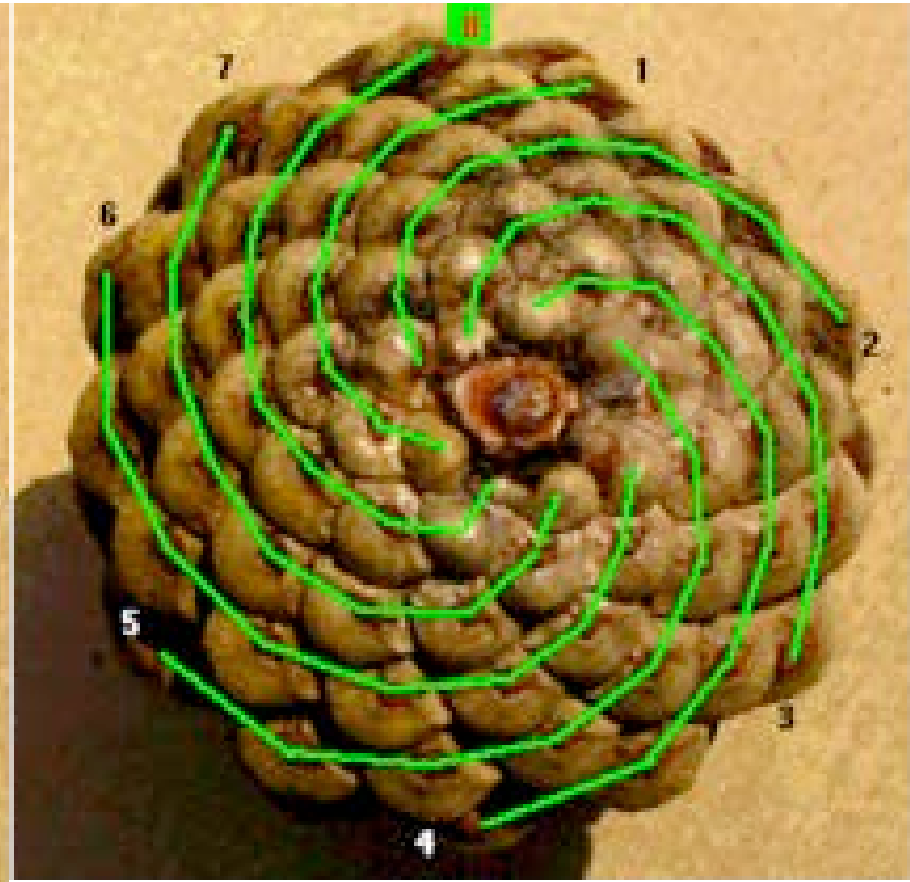
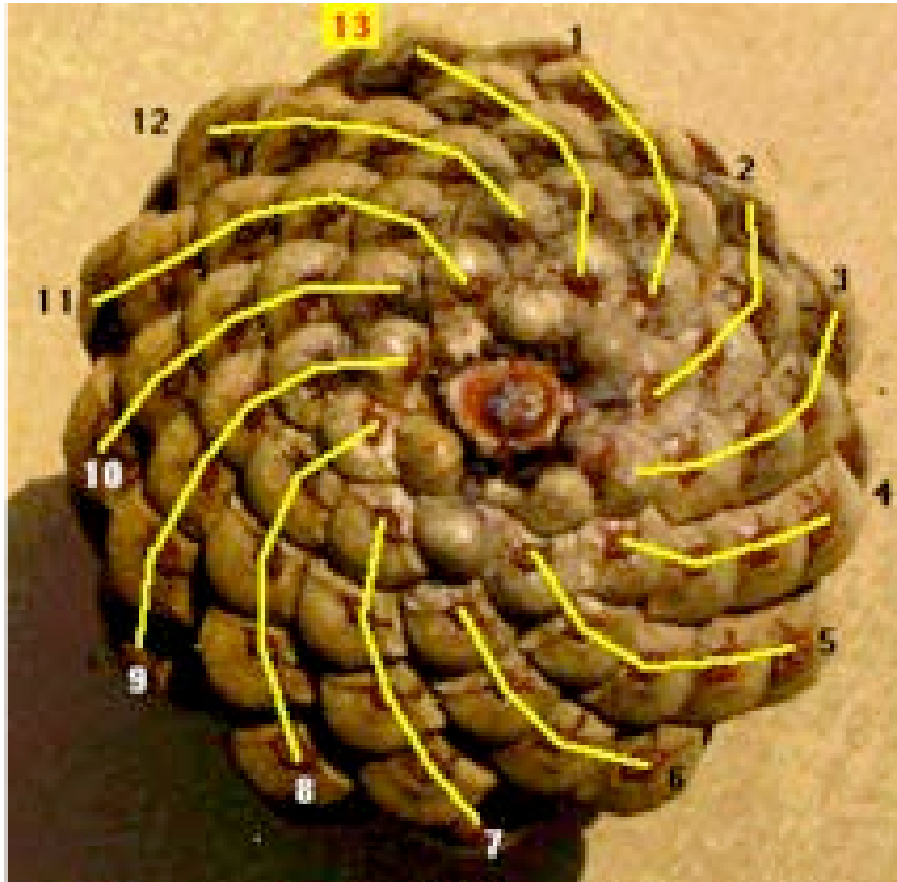


Ovo je ipak previše!



Ipak, nije sve lažna vest....



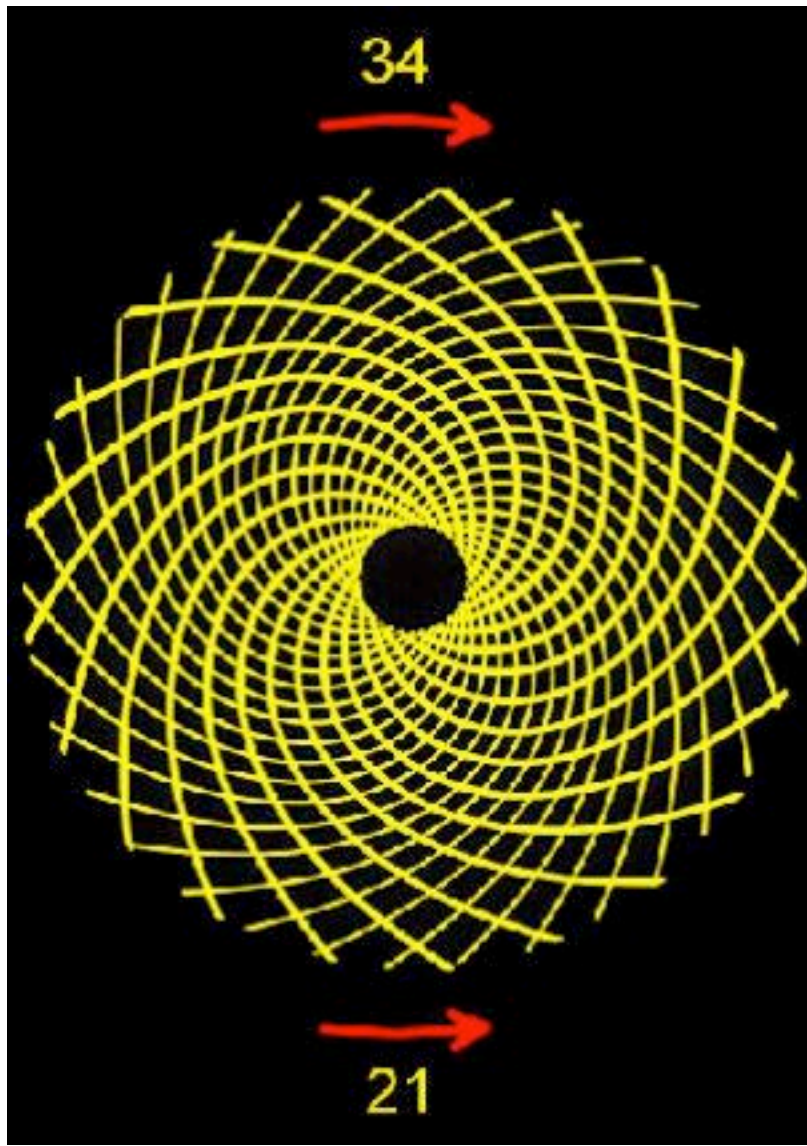






34

21
































































MDCCCLXXXIV

БЕОГРАД

БЕОГРАД



Vavilonska notacija

 1	 11	 21	 31	 41	 51
 2	 12	 22	 32	 42	 52
 3	 13	 23	 33	 43	 53
 4	 14	 24	 34	 44	 54
 5	 15	 25	 35	 45	 55
 6	 16	 26	 36	 46	 56
 7	 17	 27	 37	 47	 57
 8	 18	 28	 38	 48	 58
 9	 19	 29	 39	 49	 59
 10	 20	 30	 40	 50	

Fibonači je mnogo poznatiji i značajniji po drugoj vrsti doprinosa.
On je prvi uveo dekadnu notaciju u matematici.

I=1, II=2, III=3, IV=4, V=5, VI=6, VII=7, VIII=8, IX=9, X=10, L=50, C=100,
D=500, M=1000

Da je danas 2000 godina, pisala bi se MM,
današnja godina se piše MMXVIII – pet znakova više

4000=MMMM, aha!

5000=V=ICD

Crta iznad broja znači taj broj x1000

Izuzetaka ima kao što ih ima u nekim jezicima, uključujući Srpski.

Veliki brojevi su “nepravilni” !

Pravilo je da veće brojeve konstruisemo sabirajući manje sve dok ne dodjemo do 5, 10, 50, 100, itd., a onda proces nastavljamo dodajući im prethodne manje. Recimo $11=XI$, tj., $10+1$, 12 je XII tj. $10+2$, 15 je XV, 16 je XVI, 103 je CIII, tj. $100+3$, itd.

Pravilo sabiranja: Kad se manji broj nadje iza petice ili desetke, ili 50, itd., onda se on sabira, tj. 6 je VI, 23 je XXIII, 300 je CCC, itd. Hm, jedino taj broj IV što je nesiguran. Naime, 4 se ne piše kao IIII već IV. Novo pravilo!

Pravilo oduzimanja: Kad se manji broj nadje ispred petice (desetke, stotine, itd.), onda se on oduzima, tj. 4 je IV, 40 je XL, 39 je XXXIX, itd. Medjutim, 8 se ne piše kao IIX (dva manje od 10), već VIII, 5 plus 3. Aha, poboljšanje pravila oduzimanja: samo se jednom oduzima, a sabira se tri puta. Dakle, 3 je III, ali 4 je IV, 300 je CCC ali 400 je CD. Godina 1389 bi se, dakle, pisala kao MCCCLXXXIX.

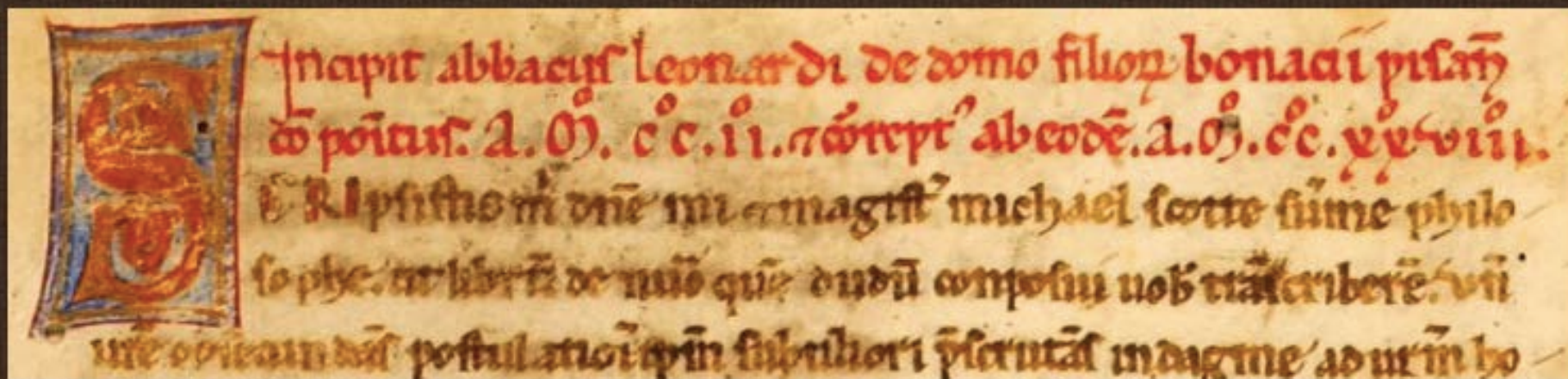
Dobro, sada umemo da pišemo brojeve, ali kakva su pravila sabiranja i oduzimanja takvih brojeva? Recimo $XII+XV=XXVII$ ($12+15=27$), dakle prepíšemo sve znakove i poredjamo ih po veličini, ali ovo ne važi za zbir $CLXXIV+XXVIII=CCII$, ili razliku $CLXXIV-XXVIII=CXLVI$. Ukratko, osnovna aritmetika u ovoj notaciji je veoma nepraktična, o množenju i deljenju da ne govorimo.

The problem with the French is that they don't have a word for entrepreneur.



Ove godine je 52. super bowl

Ovde počinje *Liber Abaci* – knjiga o računanju (1202)



“Here begins the Book of Calculation composed by Leonardo Pisano, Family Bonaci, in the year 1202. [...]”

De manu

unus	duo	tres	quatuor
			
			
			
			
			
			
			
			
			

LIBER ABBACI

LEONARDO PISANO

PUBBLICATO

SECONDO LA LEZIONE DEL CODICE MAGLIABECHIANO
C. I, 2016, *Badia Fiorentina*, n.° 73.

DA

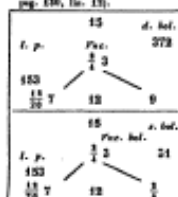
BALDASSARRE BONCOMPAGNI

SOCIO ORDINARIO DELL'ACCADEMIA FLORENTINA DE' LUINI, E SOCIO
CORRESPONDENTE DELL'ACCADEMIA DELLE SCIENZE DI TORINO,
DELLA REGIA ACCADEMIA DELLE SCIENZE DI NAPOLI,
E DELLA SOCIETA' ACCADEMICA DELLE SCIENZE
DELL'UNIVERSITA' DI BERGAMO.

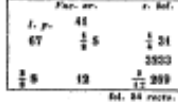
ROMA

TIPOGRAFIA DELLE SCIENZE MATEMATICHE E FISICHE
VIA SABA NUM.° 211
MDCCLVII.

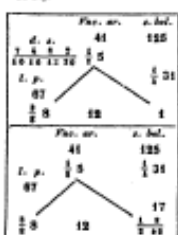
* 278 et pone + (fol. 52 verso,
lin. 23-25; per. 129, lin. 30 —
pag. 126, lin. 17).



* ponam euitabis + (fol. 52 verso,
lin. 36-38; pag. 126, lin. 43-47).



* 21 probabis pone ipse + (fol. 54 verso,
lin. 1-10; pag. 120, lin. 37-34).



que est $\frac{1}{2}$ et $\frac{1}{3}$, et per 12, et per ruptos, hoc est per $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$; euitabis inde $\frac{1}{2}$ de 15, hoc est quod multiplicabis 9 per tertiam partem de 15, scilicet per 5; que per 12, erunt 45; que divides per $\frac{1}{2}$ et $\frac{1}{3}$, exhibit $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$. In alia vero descriptione multiplicabis 3, que sunt super 4 per 15; que per 12, erunt similiter 45; que divides per 21, et per 12, et per omnes ruptos, hoc est per $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$, exhibit $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$, hoc est denarii $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$.

De eodem.

Irem sint libre $\frac{1}{12}$ et 12 cuiusdam bolsonalie, que sit ad uncias $\frac{1}{2}$ s argenti; et in libra ipsis sint soldi 21, et denarii 2, hoc est soldi $\frac{1}{2}$ 21; et libra argenti valeat libras 8, et soldos 7, et denarios 6, hoc est libras $\frac{1}{2}$ s: fac soldos de libris $\frac{1}{12}$ et 12, erunt soldi $\frac{1}{2}$ 209, quos ponas sub soldis $\frac{1}{2}$ 21, ut hic ostenditur; et multiplicabis 209 per suam uirgulam, erunt 2322; que pone super $\frac{1}{2}$ 216, et super ipsa pone pensam ipsorum, que est 21, per 11: similiter facies de $\frac{1}{2}$ s, et habebis 41 super ipsa, quorum pensa est s: hoc idem facies de $\frac{1}{2}$ s, et habebis 67; et pro pensa 1, et super $\frac{1}{2}$ 21 habebis 125: deinde multiplicabis 222 per 41, et per 67, et per 4, que sunt super uirgula de 21; et divides summam per regulam de 125, que est $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$, et per 12, et per ruptos tertii reliquorum numerorum, scilicet per 12, et per 8, et per 8, et aptabis ruptos, et euitabis, et probabis, semper exhibunt libre $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$ et 12 pro pretio dictarum librarum 12, et soldi 9, et denarii s.

De eodem.

Et si pretium unius soldi eiusdem bolsonalie reperire uolueris, describes t sub $\frac{1}{2}$ 21, et multiplicabis ipsum 1 per 41, erunt 41; que multiplicabis per 67, erunt 2747; que relinques multiplicare per 4, que sunt sub uirgula post 21, et non divides per 4, que sunt in regula de 8, que sunt sub 41: ergo divides 2747 per 125, et per 12, et per 2, que remanent de 8, que sunt sub uirgula sub 41, et per 8, que sunt sub uirgula sub 67; et aptabis ruptos, exhibunt $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$, ut in hac descriptione ostenditur; hoc est perun minus de denariis $\frac{1}{2}$ 27, uidelicet $\frac{1}{2}$ unius denarii, minus per unamquamque libram; et hoc cognoscitur ita: quod pretium soldi est denariorum $\frac{1}{12}$ et 27, hoc est denarii $\frac{1}{12}$ 27; a quibus usque in denariis $\frac{1}{2}$ 27 desunt $\frac{1}{12}$ unius denarii: ergo si unicuique soldi defuerint $\frac{1}{12}$ unius denarii et libre, scilicet soldi 28, decerunt $\frac{16}{125}$, hoc est $\frac{1}{2}$ unius denarii, ut prediximus.

De eodem.

Irem si queratur pretium de denariis $\frac{1}{2}$ s eiusdem bolsonalie; aut de soldis $\frac{1}{2}$ 21 facies denarios, qui sunt 273; et ponas sub eis dictos denarios $\frac{1}{2}$ s; uel de ipsis denariis $\frac{1}{2}$ s facies partes unius soldi, scilicet $\frac{1}{2}$ 21; et ponas ipsas sub soldis $\frac{1}{2}$ 21, ut sint soldi sub soldis, ut in hac descriptione ostenditur; et multiplicabis denarios 8 per suam uirgulam, erunt 17; que multiplicabis per 41; que per 67; que per 4, que sunt sub uirgula post 21; et divides per 125, et per 12, et per ruptos, scilicet per $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$, et per 8, et per 8; et euitabis, et coaptabis, et exhibunt $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$, hoc est denarii $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{12}$.

Irem quidam habet soldos 11, et denarios 7, hoc est soldos $\frac{1}{12}$ 11 cuiusdam bolsonalie, que est ad uncias $\frac{1}{2}$ s; et intrant in libra ipsis bolsonalie soldi 25, et denarii $\frac{1}{2}$ s, hoc est soldi $\frac{1}{12}$ 25; et libra argenti ualeat libras $\frac{1}{2}$ s; describes questionem, ut hic ostenditur, et multiplica 25 per suam uirgulam, erunt 425: similiter multiplica omnes numeros per suas uirgulas; et habebis 12 super $\frac{1}{2}$ s, et 167 super $\frac{1}{2}$ s, et 128 super $\frac{1}{2}$ s.



DECLARATION (1865): The Rulers and People of Pisa in the year 1865 after ignoring old crushing falsehoods and where the will of the Elders was to study what was better known and proven about Leonard Fibonacci ordered the city archives to file a copy of the decree by the same Most Eminent Republic of Pisa that one monument equal to so great a man survive.

DCXXIV godina je prošlo od:

DECREE (1241): In consideration of the honor brought to the city and its citizens and their betterment by the teaching and zealous cooperation of that discreet and wise man, Master Leonardo Bigolli, as well as by his regular patriotic efforts in civic and patriotic affairs, the Pisan Commune and its Officials in certain right and conscious of our prerogative to make recompense for work that he performed in heeding and consolidating the efforts and affairs already mentioned confer upon this same Leonardo so meritorious of our love and appreciation an annual salary or reward from the Commune of 20 free denarii and the usual accompaniments. This we affirm with the present statement.

DCXXI godina je prošlo





About 500 meters along the Arno from the *Ponte di Mezzo*, at 30 Lung'arno Mediceo, is an imposing, three-story building with attractively decorated grey stone walls. This is the *Palazzo Toscanelli*. It originally belonged to the Lanfranchi family, but then passed on to Alessandro della Gherardesca, who made a number of architectural modifications. In 1821-22, it was the home of Lord Byron and his circle of friends, during the period in which Pisa was called the "Paradise of Exiles". From here Lord Byron departed for Greece, where he met his death. Today the palace hosts the *Archivio di Stato* (State Archives) of Pisa.

On a wall in the entrance hall is a stone tablet commemorating Leonardo. It was dedicated on 16th June, 1867. Following an introductory declaration written in 1865, it reproduces the text of a document of 1241, by which the Comune of Pisa (the town government — at that time Pisa was an independent state or republic) decreed that an amount of money should be given annually to Leonardo for his merits.

This proclamation is the only indication we have that Leonardo was still alive in 1241.

ORDO · POPVLVSQ · PISANVS
QVO · VETERI · CONTEMPTVS · CALVMNIA · OBTRITA
MAIORVM · VOLVNTAS · STVDIVMQVE
IN · LEONARDVM · FIBONACCIVM
NOTIORA · TESTATIORAQ · ESSENT
AMPLISSIMI · PISANAE · REIPVB · DE · EODEM · DECRETI · EXEMPLAR
QVOD · VNVM · SVPEREST · TANTO · VIRO · AEQVALE · MONVMENTVM
IN · PATRIO · TABVLARIO · AFFIGI · IVSSIT · A · MDCCLXV ·

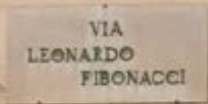
CONSIDERANTES · NOSTRE · CIVITATIS · ET · CIVIVM · QVEM · SVBSTINET · IN · AVDIENDIS · ET · CONSOLIDANDIS
HONOREM · ATQVE · PROFECTVM · QVI · EIS · TAM · PER · ESTIMATIONIBVS · ET · RATIONIBVS · SVPRADICTIS · A
DOCTRINAM · QVAM · PER · SEDVLA · OBSEQVIA · DISCRETI · COMVNI · ET · CAMERARIIS · PVBLICIS · DE · COMVNI
ET · SAPIENTIS · VIRI · MAGISTRI · LEONARDI · BIGOLLI · ET · PRO · COMVNI · MERCEDE · SIVE · SALARIO · SVO
IN · ABBACANDIS · ESTIMATIONIBVS · ET · RATIONIBVS · ANNIS · SINGVLIS · LIBRE · XX · DENARIORVM · ET
CIVITATIS · EIVSQVE · OFFICIALIVM · ET · ALIIS · QVOTIES · AMISERIA · CONSVETA · DARI · DEBEANT · IPSEQVE
EXPEDIT · CONFERVNTVR · VT · EIDEM · LEONARDO · PISANO · COMVNI · ET · EIVS · OFFICIALIBVS · IN
MERITO · DILECTIONIS · ET · GRATIE · ATQ · SCIENTIE · SVE · ABBACATIONE · DE · CETERO · MORE · SOLITO
PREROGATIVA · IN · RECOMPENSATIONE · LABORIS · SVI · SERVAT · PRESENTI · CONSTITVTIONE · FIRMAMVS



The Giardino Scotto is behind the trees. The marble street sign is on the brick wall on the far side of the bridge to the right.

Via Leonardo Fibonacci in Florence

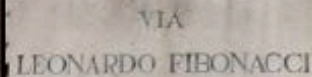
This single short street boasts no fewer than eleven "Via Fibonacci" street signs. Here are seven.



VIA
LEONARDO
FIBONACCI



Wine Bar
La Piazza



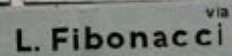
VIA
LEONARDO FIBONACCI



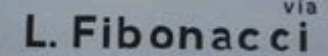
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LEONARDO FIBONACCI



VIA
LEONARDO FIBONACCI



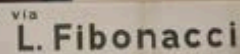
VIA
L. Fibonacci



VIA
L. Fibonacci



SENSO UNICO



VIA
L. Fibonacci

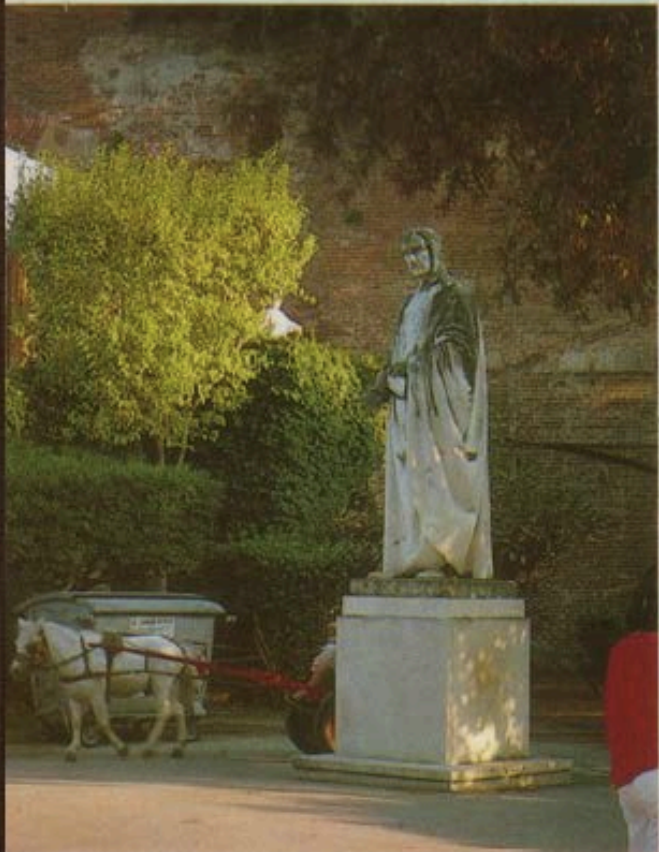
Statua je javno prikazana 1876 i postavljena je u čuvenom Camposanto.
Fašističke vlasti su je stavile ovde 1926. Statua je čudom preživela savezničko bombardovanje krajem rata i samo je neznatno oštećena

From 1926 to 1946 the statue was here



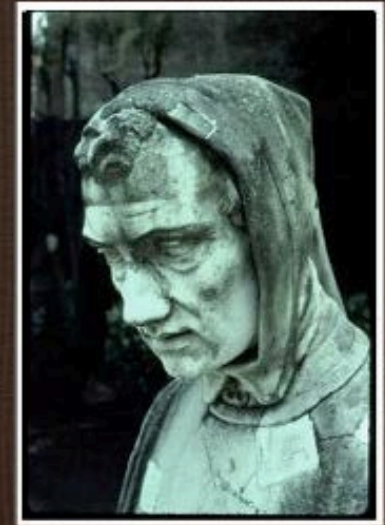
In 1966, the statue was moved here

PISA
Giardino Scotto



A postcard showing the Leonardo statue in a small public garden along the Lung'arno Leonardo Fibonacci

... where it got very dirty



Two photographs taken by Laurence and Judith Sigler in the 1980s, and one by Frank Johnson that can be found on many websites







Campo Santo



Paying Homage



.. the serious and learned Master Leonardo Bigollo

