

УНИВЕРЗИТЕТ У БЕОГРАДУ
МАТЕМАТИЧКИ ФАКУЛТЕТ



Марија Катић

РЕШАВАЊЕ ПРОБЛЕМА p -ХАБ
МЕДИЈАНЕ НЕОГРАНИЧЕНИХ
КАПАЦИТЕТА СА ВИШЕСТРУКИМ
АЛОКАЦИЈАМА МЕТОДОМ
ПРОМЕНЉИВИХ ОКОЛИНА

мастер рад

Београд, 2023.

:

МИШКОВИЋ,

,

:

МАРИЋ,

,

РАДОЛИЧИЋ МАТИЋ,

,

:

: *p*-

(. hub)

(.)
(.),
NP-

p-

AP

p-

, UMAPHMP,

, RVNS,

1		1
2		5
2.1	6
3	e	11
4		15
4.1	20
5		22
5.1	22
5.2	25
6		28
		29

1

$$\min_{x \in D} f(x) \tag{1a}$$

$$g_i(x) \leq 0, \quad i=1, \dots, L \tag{1b}$$

D is the domain of f and g_i . D is a subset of \mathbb{R}^n .

$$\begin{aligned} & f \\ & : g(x) \leq 0 \quad g(x) \leq 0 \quad g(x) = 0 \end{aligned}$$

a

g_i

f

$$\min_{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n} c_1x_1 + c_2x_2 + \dots + c_nx_n \quad (2a)$$

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = 0, \quad i = 1, \dots, L, \quad (2b)$$

$$x_k \geq 0, \quad k = 1, \dots, n \quad (2c)$$

$$x_1, x_2, \dots, x_n$$

Programming).

NP-

Integer Linear Programming, MIP (MILP),

(Mixed

NP-

(Facility Location Problems)

(hub)

1.

p -

e
 $\alpha < 1.$

•

•

max

min-max
min-sum

min-sum

Min-

1.

•

p -

•

•

NP-

[1].

2

3

4

5

CPLEX

2

p-
(. Uncapacitated multiple allocation *p*-hub median problem,
UMApHMP) :

α_i

p-

NP-

, NP- [29].

$$\min_{u, w} \sum_{uw} (c_{uw}) = \sum_{uw} \min(c_{uw}) \quad (3)$$

 c_{uw} u, w c_{uw}

algorithm) [18] [43],

 n , p . $O(pn^2)$,

(Floyd-Warshall

4,

 $\alpha = 1$,3. $\alpha < 1$,

4

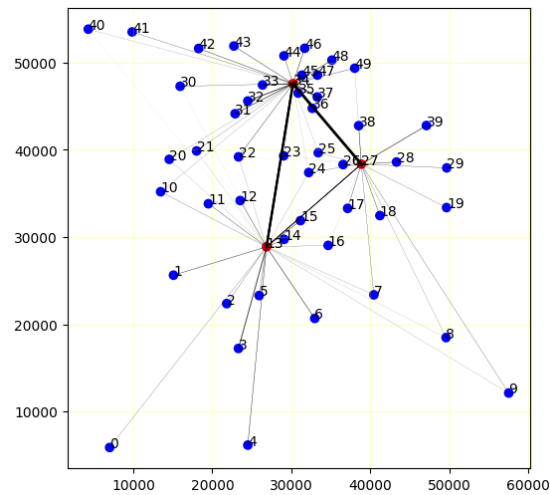
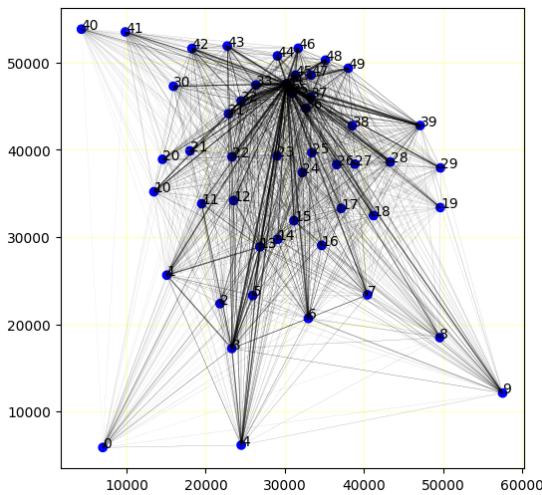
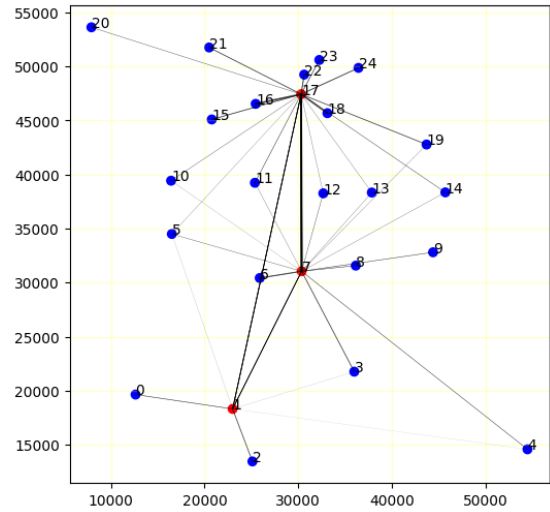
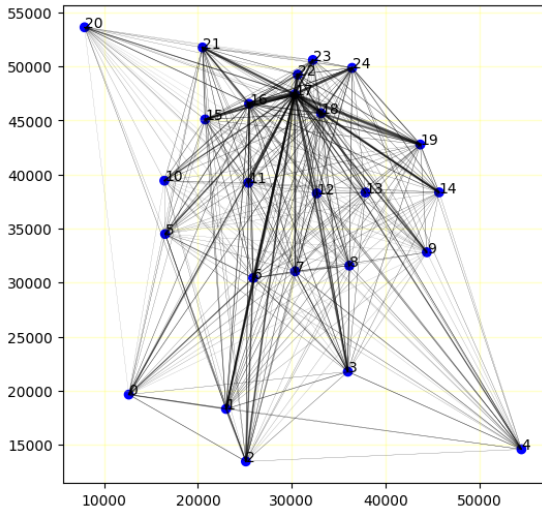
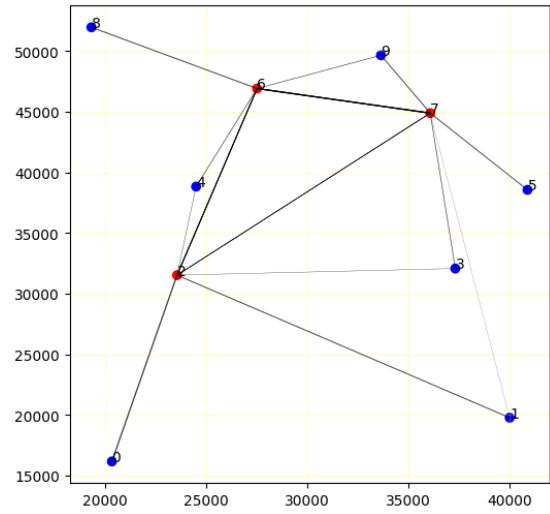
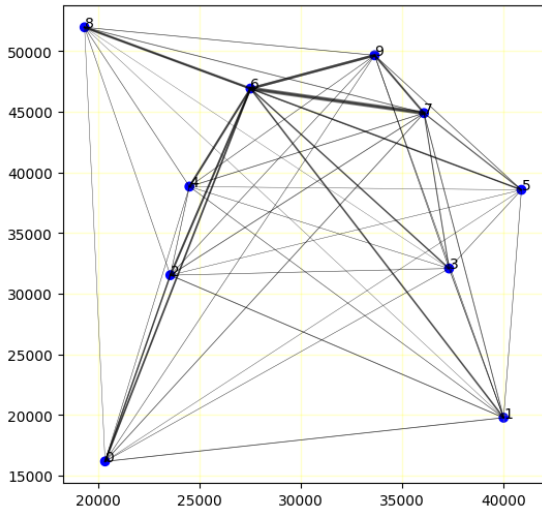
1

 p *AP* (Australia Post)

2.1

 p

[1], 1992.



1:

p -
AP

$p = 3$ $n = f(10, 20, 50)g$

p -
Campbell, [10],

$$C_{ij} = C_{ji},$$

$$C_{ij} = \sum_{k=0}^{\min(i,j)} W_{ij} X_{ijk} C_{ik} + \alpha C_{km}$$

- $X_{ijk} = 1$ if $i - k - m = j$, 0 otherwise.
- $H_k = 0$ if $k = 0$, $H_k = 1$ if $k > 0$.
- $C_{ijk} = C_{ik} + C_{mj} + \alpha C_{km}$

$$\min \sum_i \sum_j \sum_k \sum_m W_{ij} X_{ijk} C_{ijk} \quad (4a)$$

$$\sum_k \sum_m X_{ijk} = 1, \quad \forall i, j, \quad (4b)$$

$$\sum_k H_k = p, \quad (4c)$$

$$X_{ijk} \leq H_k, \quad \forall i, j, k, m, \quad (4d)$$

$$X_{ijk} \leq H_m, \quad \forall i, j, k, m, \quad (4e)$$

$$X_{ijk} \leq 0, \quad \forall i, j, k, m, \quad (4f)$$

$$H_k \in \{0, 1\}, \quad \forall k \quad (4g)$$

(4a)

$$\sum_{k=0}^{\min(i,j)} W_{ij} X_{ijk} C_{ijk}$$

$$X_{ijkm} = \begin{cases} 4, & k = m \\ 3, & k \neq m \end{cases} \quad (4b) \quad (4f)$$

$$X_{ijkm} = \begin{cases} 4, & i = j \\ 3, & i \neq j \end{cases} \quad (4c) \quad (4e)$$

$$X_{ijkm} \in \{0, 1\}, \quad \delta i, j, k, m, \quad (4d)$$

$$X_{ijkm} \in \{0, 1\}, \quad \delta i, j, k, m, \quad (4d)$$

$$O(n^4) \quad O(n^3)$$

1998. e

[16]. $O(n^3) \quad O(n^2)$

$$n, p, \alpha, C_{ij}, W_{ij}, H_k$$

- $Z_{ik} = \sum_j W_{ij} H_k, \quad \delta i, k,$
- $Y_{kl}^i = \sum_j X_{lj}^i H_k, \quad \delta i, k - l,$
- $X_{lj}^i = \sum_k Y_{kl}^i H_k, \quad \delta i, l - j,$

$$\min \sum_i (\chi \sum_k C_{ik} Z_{ik} + \alpha \sum_k \sum_l C_{kl} Y_{kl}^i + \delta \sum_l \sum_j C_{lj} X_{lj}^i) \quad (5a)$$

$$\sum_k Z_{ik} = \sum_j W_{ij}, \quad \delta i, \quad (5b)$$

$$\sum_l X_{lj}^i = W_{ij}, \quad \delta i, j, \quad (5c)$$

$$\sum_l Y_{kl}^i + \sum_j X_{kj}^i - \sum_l Y_{lk}^i - Z_{ik} = 0, \quad \delta i, k, \quad (5d)$$

$$\sum_k H_k = p, \quad (5e)$$

$$Z_{ik} = \sum_j W_{ij} H_k, \quad \delta i, k, \quad (5f)$$

$$\sum_i X_{lj}^i = \sum_i W_{ij} H_k, \quad \delta l, j, \quad (5g)$$

$$Y_{kl}^i, X_{lj}^i, Z_{ik} \in \{0, 1\}, \quad \delta i, j, k, l, \quad (5h)$$

$$H_k \in \{0, 1\}, \quad \delta k \quad (5i)$$

$$\begin{aligned}
 & (5b) - (5d) \\
 Z_{ik}, Y_{kl}^i & X_{lj}^i \quad (5b), Z_{ik} \\
 & i. \quad (5c), X_{lj}^i \\
 & i. \quad j. \quad (5d) \\
 & : \quad k - \\
 & k - \\
 & i. \\
 & k, \quad k = l.
 \end{aligned}$$

$$\begin{aligned}
 & (5e) \quad p \\
 (5f) \quad (5g) \\
 3 \quad i - k - l - j, Z_{ik} - \quad Y_{kl}^i - \\
 - \quad X_{lj}^i - \quad i. \\
 i, \quad i.
 \end{aligned}$$

$$\begin{aligned}
 & \chi \quad 1 \quad \delta \quad 1, \\
 & a \\
 & \top \\
 C_{ijkm} & = \chi C_{ik} + \alpha C_{km} + \delta C_{mj}.
 \end{aligned}$$

3

e

[10] 1992. , -
 p -
(4) 2.1.
1994. , [11]
:

[2] 1995. [12] 1996.

(Greedy-interchange method). -

p

p

p

[12]

p -
(UMApHSP),

[39] 1996. ,

(4), (4d) (4e) :

$$\sum_m X_{ijkm} H_k, \delta_{i,j,k}, \quad \sum_k X_{ijkm} H_m, \delta_{i,j,m}.$$

o CAB [16], $O(n)$
 (5) 2.1.

(Explicit enumeration method),
 (Branch-and-Bound)

E a a

p. $O(n^p)$, n , p.

1960.

[31].

[17].

500

AP $n = 200$ $p = 3$. M
 $n = 100$ $p = 5$ $n = 200$ $p = 3$.

[5], 2004.

(P, n 50 p 5).

[38] 1999. p -

p -

(1-stop UMApHMP),

CAB

[40], 2008.

1975. [27],

crossover,

AP $n = 200$ $p = 2$. 2010.

[32],

[19], 2021.

$O(n^2)$

(Branch-and-Cut)

1991. [37].

(Cutting planes) [23],

p .

2013. [30] (Electromagnetism-like metaheuristic).

[42] 2017.

p -

[36]

		min-max	-	-
				-
				-
	p -			-
		(Variable Neighbourhood		-
Search, VNS) [35]				-
	[20], 2018.		(Tabu Search)	-
			[22]	-
	[21], 2021.			-
			[9]	-
			p -	-
				-
		[7],		p -
	[6]		p -	-
				-
			NP-	-
				-

4

, 1997.

[35].

- 1.
- 2.
- 3.

(1)

x_{opt}

$N_k, k = 1, \dots, k_{max}$

k_{max}

$N_k(x)$

k

x

N

$x^l \in X$

$x \in N(x^l)$

X

$f(x) < f(x^l)$.

(Local Search)

N .

4.

x_i $f(x_i)$
 $N(x)$ $N(x)$
(Best Improvement).
 $N(x)$
a
(First Improvement).

1: Best improvement

Data: inici alno rešen e x , okolina N

Result: Lokalni minimum u odnosu na okolinu N optimizacionog problema $\min_x f(x)$ počevši iz x

```
1 while  $f(x) > f(x^0)$  do
2   |  $x^0 = x$ ;
3   |  $x = \operatorname{argmin}_{y \in N(x)} f(y)$ ;
4 end
5 return  $x^0$ 
```

2: First improvement

Data: inici alno rešen e x , okolina N

Result: Lokalni minimum u odnosu na okolinu N optimizacionog problema $\min_x f(x)$ počevši iz x

```
1 do
2   |  $x^0 = x$ ;
3   |  $i = 0$ ;
4   do
5     |  $i = i + 1$ ;
6     |  $x = \operatorname{argmin}_{f(x), f(x_i) \in N(x)}$ ;
7     | while  $f(x_i) < f(x)$  and  $i < jN(x)$ ;
8 while  $f(x) < f(x^0)$ ;
9 return  $x^0$ 
```

4.

e [24] e -

(1) – (3).
M (. Variable Neighbourhood Descent, VND)

$k_{max} = 1,$
 $k_{max} = 1$ $k_{max} = 2$ [24]. $k_{max} = 1$
 $k_{max} > 1$
(2).

3: M

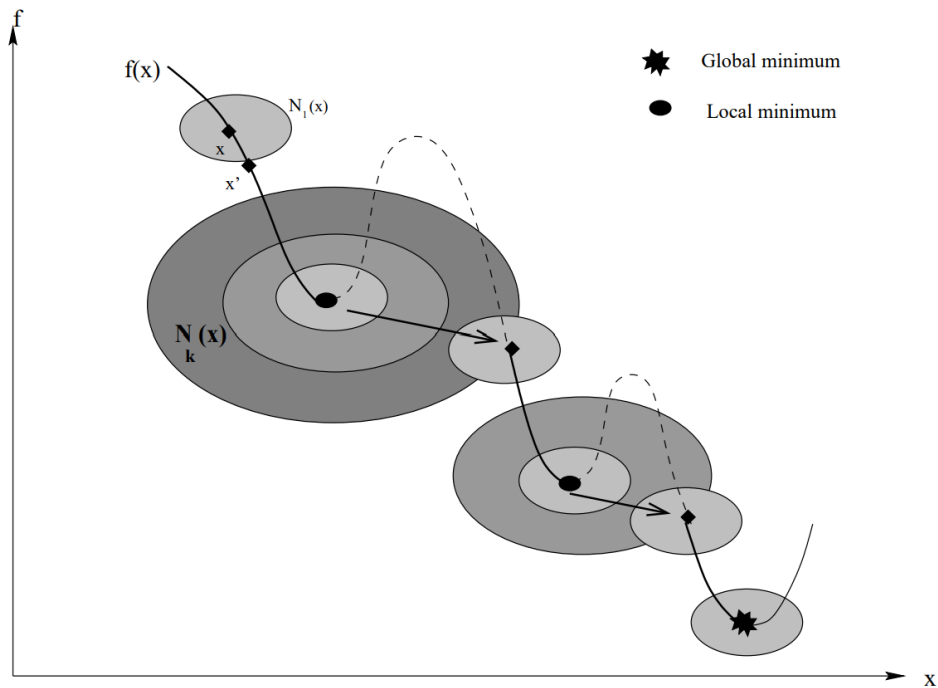
Data: kriterijum zaustavljanja, tipovi okolina $N_k, 1 \leq k \leq k_{max}$

Result: Na bolje pronađeno rešenje optimizacionog problema $\min_x f(x)$

```
1  $x$  inicijalizacija();  
2 while ni e zadovoljen kriterijum zaustavljanja do  
3    $k = 1$ ;  
4   while  $k \leq k_{max}$  do  
5      $x^{\theta} = lokalna\_pretraga(x^{\theta}, N_k(x))$  ;  
6     prelazak_u_narednu_iteraciju( $x, x^{\theta}, k$ ) ;  
7   end  
8 end  
9 return  $x$ 
```

(. Basic Variable Neighbourhood Search, BVNS)

(. shaking),



2:

4:

Data: kriterijum zaustavljanja, tipovi okolina $N_k, 1 \leq k \leq k_{max}$

Result: Na bolje pronađeno rešenje optimizacionog problema $\min_x f(x)$

```

1  $x$  inicijalizacija();
2 while nije zadovoljen kriterijum zaustavljanja do
3    $k = 1$ ;
4   while  $k \leq k_{max}$  do
5      $x^0 = \text{razmrdavanje}(x, N_k(x))$ ;
6      $x^{00} = \text{lokalna\_pretraga}(x^0, N_k(x))$ ;
7     prelazak_u_narednu_iteraciju( $x, x^{00}, k$ );
8   end
9 end
10 return  $x$ 

```

(. Reduced Variable

Neighbourhood Search, RVNS)

k_{max} 2 [24].

[24].

5:

Data: kriterijum zaustavljanja, tipovi okolina $N_k, 1 \leq k \leq k_{max}$
 Result: Na bolje pronađeno rešenje optimizacionog problema $\min_x f(x)$

```

1  $x$  inicijalizacija();
2 while ni e zadovoljen kriterijum zaustavljanja do
3    $k = 1$ ;
4   while  $k \leq k_{max}$  do
5      $x^l = razmrdavanje(x, N_k(x))$ ;
6     prelazak_u_narednu_iteraciju( $x, x^l, k$ );
7   end
8 end
9 return  $x$ 
```

6:

Data: kriterijum zaustavljanja, tipovi okolina $N_k, 1 \leq k \leq k_{max}$
 Result: Na bolje pronađeno rešenje optimizacionog problema $\min_x f(x)$

```

1  $x$  inicijalizacija();
2 while ni e zadovoljen kriterijum zaustavljanja do
3    $k = 1$ ;
4   while  $k \leq k_{max}$  do
5      $x^l = razmrdavanje(x, N_k(x))$ ;
6      $x^{ll} = vnd(x^l, N_k(x))$ ;
7     prelazak_u_narednu_iteraciju( $x, x^{ll}, k$ );
8   end
9 end
10 return  $x$ 
```

Neighbourhood Search, GVNS)

[24].

3, 4, 5 6.

a 7.

7: *prelazak_u_narednu_iteraciju*

```

1 def prelazak_u_narednu_iteraci u(x, x', k):
2   | if  $f(x^0) < f(x^0)$  then
3   |   |  $x = x^0$  ;
4   |   |  $k = 1$  ;
5   | else
6   |   |  $k = k + 1$  ;
7   | end

```

[24], p - [25].

[26], [34], [4], [15], [28],

[41], [33], [7], [14], [13], [42].

4.1

p -

$$I = \{1, 2, \dots, n\}$$

$$H \subseteq I, |H| = p$$

$$O(pn^2), a$$

$$\alpha, \delta, \chi,$$

$$p$$

$$g(h)$$

$$h,$$

$$g(h) = \max_{i \in H, j \in I \setminus H} c_{ij}, \quad h \subseteq I,$$

$$p$$

$$g.$$

$$[42] \quad [8]. \quad 0$$

$$e$$

$$N_k(H) = \sum_{j \in I \setminus H} \binom{p}{k} c_{H^0 j}^k, \quad k \in \{1, 2, 3\}. \quad (6)$$

$$[42] \quad [8].$$

5

3.0 GHz Intel Scalable

16 GB RAM

https://github.com/marijakatic/VNS_for_UMaPHMP.

C,

Python.

ORLIB [3]

AP (Australia Post).

200

$W_{ii} \neq 0,$

$[W_{ij}],$

0

CPLEX

(5). CPLEX

22.1.1.0,

5.1

$p-$

4.1

(. random)

a 4.1.

$$C = \left\{ \frac{2\pi/p}{p} \right\}$$

(. intensification vs. diversification) [24].

$$r \in [0, 1],$$

$$l \in \{1, 2, \dots, pg\}$$

p.

l

$$l = 1, \quad r = 1,$$

$$r \in (0, 1)$$

$$r = 0.5.$$

$$(4.1) \quad k_{max} = 3.$$

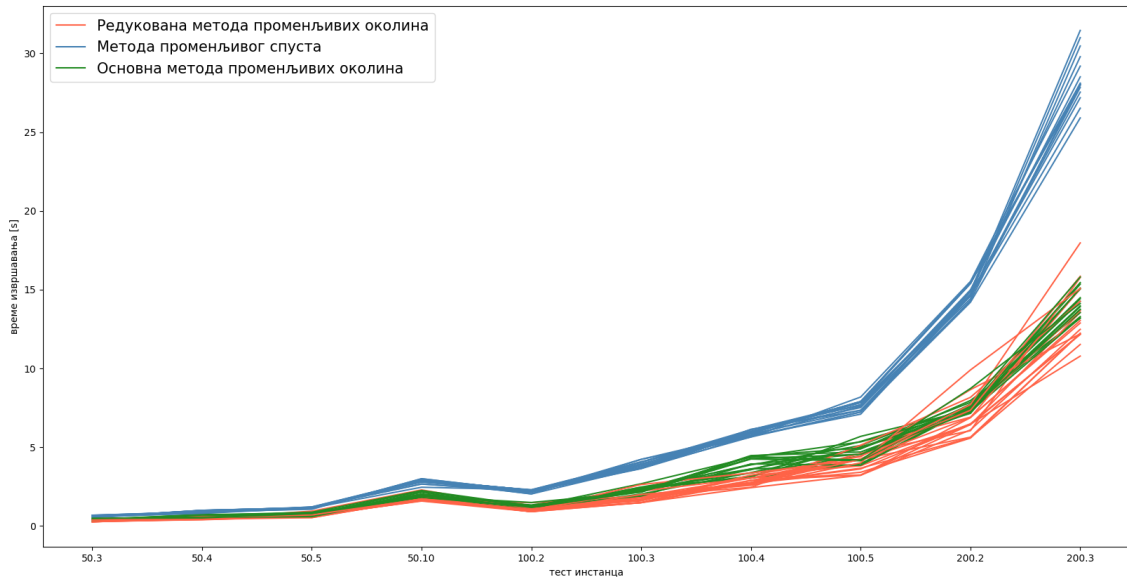
$$r = 1$$

- $LS \in [best, first]$ -
- $r \in [0.5, 1]$ -
- $k_{max} \in [1, 2, 3]$ -
- $init \in [rand, grane, ugovi]$ -

: rand -

, grane - p

, ugovi - p



3:

3

1,

r, k_{max}

16

$n \in \{40, 50, 100, 200\}g,$

RVNS.

RVNS

$r = 1, k_{max} = 1$

4.1.

1: RVNS

	RVNS								
<i>r</i> :	0.5	1	1	0.5	0.5	1	1	1	1
<i>k_{max}</i> :	3	3	1	1	2	2	3	1	1
<i>init</i> :	<i>grane</i>	<i>grane</i>	<i>grane</i>	<i>grane</i>	<i>grane</i>	<i>grane</i>	<i>uglovi</i>	<i>uglovi</i>	<i>rand</i>
n.p	[s]								
40.2	0.12	0.15	0.13	0.14	0.15	0.12	0.14	0.13	0.12
40.3	0.2	0.18	0.24	0.27	0.18	0.19	0.26	0.29	0.26
40.4	0.33	0.31	0.46	0.38	0.33	0.43	0.33	0.29	0.31
40.5	0.39	0.44	0.42	0.46	0.42	0.44	0.48	0.44	0.48
40.10	1.19	1.02	1.09	1.02	1.19	1.24	1.23	1.08	1.07
50.2	0.21	0.24	0.2	0.21	0.24	0.2	0.21	0.22	0.24
50.3	0.36	0.44	0.37	0.46	0.4	0.42	0.46	0.45	0.41
50.4	0.62	0.57	0.65	0.69	0.56	0.55	0.66	0.63	0.57
50.5	0.74	0.77	0.74	0.8	0.73	0.8	0.9	0.84	0.77
50.10	2.02	2.03	2.11	2.1	1.95	2.04	2.02	2.42	2.29
100.2	1.28	1.34	1.27	1.27	1.54	1.4	1.28	1.28	1.47
100.3	2.09	2.31	2.2	2.09	2.42	2.09	2.3	2.73	2.30
100.4	3.17	3.77	3.47	3.17	3.17	3.31	3.51	3.61	4.23
100.5	4.78	4.59	5.23	5.48	5.2	4.79	4.59	4.83	5.60
200.2	8.64	8.19	8.19	8.65	8.22	10.34	8.62	9.47	9.90
200.3	14.39	14.38	14.41	14.35	15.12	14.43	16.55	15.81	20.90
.	2.53	2.54	2.57	2.59	2.61	2.67	2.72	2.78	3.18

5.2

4.1 a a
 a AP $n \in \{10, 20, 25, 40, 50, 100, 200\}$
 $p \in \{2, 3, 4, 5, 8, 10, 15, 20, 25, 30\}$. (2)
 e CPLEX (5).
 CPLEX

5.

300 400 CPLEX , 20 40
 $n = 50,$
 2.5%.
 2.
 2:

AP

n.p	cplex		rvns		
		t[s]		dev[%]	t[s]
10.2	163603.94	0.23	163603.94	0.0	0.01
10.3	131581.79	0.31	131581.79	0.0	0.01
10.4	107354.73	0.26	107354.73	0.0	0.01
10.5	86028.88	0.28	86028.88	0.0	0.01
20.2	168599.79	1.37	168599.79	0.0	0.03
20.3	148048.3	2.0	148048.29	0.0	0.04
20.4	131665.43	1.77	131665.43	0.0	0.06
20.5	118934.97	1.77	118934.97	0.0	0.07
25.2	171298.1	3.43	171298.08	0.0	0.04
25.3	151080.66	4.26	153172.09	1.38	0.07
25.4	135638.58	4.76	135638.58	0.0	0.09
25.5	120581.99	3.43	120581.99	0.0	0.12
40.2	173415.96	24.42	173415.96	0.0	0.12
40.3	155458.61	72.1	159197.6	2.41	0.19
40.4	140682.74	69.48	140682.74	0.0	0.35
40.5	130384.74	61.73	130384.74	0.0	0.41
40.8	109971.92	64.51	110358.25	0.36	0.77
40.10	99452.67	40.81	100345.22	0.9	1.12
50.2	174390.03	82.15	174390.03	0.0	0.2
50.3	156014.73	391.48	156014.73	0.0	0.37
50.4	141153.38	318.4	141153.38	0.0	0.68
50.5	129412.6	234.29	129505.83	0.07	0.79
50.8	109926.6	450.71	110320.04	0.35	1.43
50.10	100508.95	307.66	100644.72	0.14	1.93

(3)

CPLEX

4

[40], [30] [42].

(3)

0.01%.

3:

AP

n.p		rvns		
			dev[%]	t[s]
100.2	176245.38	176245.38	0.000	1.35
100.3	157869.93	157869.93	0.000	2.3
100.4	143004.31	143004.31	0.000	3.75
100.5	133482.57	133482.57	0.000	6.07
100.8	114295.92	115180.52	0.007	9.53
100.10	104794.05	104794.05	0.000	15.83
100.15	88882.05	89037.18	0.002	27.61
100.20	79191.02	79409.05	0.003	51.56
100.25	-	72164.19	0.000	74.13
100.30	-	67200.95	0.000	104.1
200.2	178093.99	178093.99	0.000	8.26
200.3	159725.11	159725.11	0.000	14.46
200.4	144508.20	144508.20	0.000	40.19
200.5	136761.83	136761.83	0.000	33.78
200.8	117709.98	118322.91	0.005	78.35
200.10	107846.82	107846.82	0.000	122.25
200.15	92646.39	92857.68	0.002	270.54
200.20	83385.94	83385.94	0.000	511.86
200.25	-	77052.07	0.000	781.56
200.30	-	72252.28	0.000	941.56

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